Manual for Design and Detailings of Reinforced Concrete to Code of Practice for Structural Use of Concrete 2004

Housing Department

***May* 2008**

(Version 2.3)

Acknowledgement

We would like to express our greatest gratitude to Professor A.K.H. Kwan of The University of Hong Kong who has kindly and generously provided invaluable advice and information during the course of our drafting of the Manual. His advice is most important for the accuracy and completeness of contents in the Manual.

Contents

Appendices

1.0 Introduction

1.1 Promulgation of the Revised Code

A revised concrete code titled "Code of Practice for Structural Use of Concrete 2004" was formally promulgated by the Buildings Department of Hong Kong in late 2004 which serves to supersede the former concrete code titled "The Structural Use of Concrete 1987". The revised Code, referred to as "the Code" hereafter in this Manual will become mandatory by 15 December 2006, after expiry of the grace period in which both the revised and old codes can be used.

1.2 Main features of the Code

As in contrast with the former code which is based on "working stress" design concept, the drafting of the Code is largely based on the British Standard BS8110 1997 adopting the limit state design approach. Nevertheless, the following features of the Code in relation to design as different from BS8110 are outlined :

(a) Provisions of concrete strength up to grade 100 are included;
(b) Stress strain relationship of concrete is different from that of BS8110 for various concrete grades as per previous tests on local concrete;
(c) Maximum design shear stresses of concrete (v_{max}) are raised;
(d) Provisions of r.c. detailings to enhance ductility are added, together with the requirements of design in beam-column joints (Sections 9.9 and 6.8 respectively);
(e) Criteria for dynamic analysis for tall building under wind loads are added (Clause 7.3.2).

As most of our colleagues are familiar with BS8110, a comparison table highlighting differences between BS8110 and the Code is enclosed in Appendix A which may be helpful to designers switching from BS8110 to the Code in the design practice.

1.3 Outline of this Manual

This Practical Design Manual intends to outline practice of detailed design and detailings of reinforced concrete work to the Code. Detailings of individual

types of members are included in the respective sections for the types, though Section 13 in the Manual includes certain aspects in detailings which are common to all types of members. Design examples, charts are included, with derivations of approaches and formulae as necessary. Aspects on analysis are only discussed selectively in this Manual. In addition, as the Department has decided to adopt Section 9.9 of the Code which is in relation to provisions for "ductility" for columns and beams contributing in the lateral load resisting system in accordance with Cl. 9.1 of the Code, conflicts of this section with others in the Code are resolved with the more stringent ones highlighted as requirements in our structural design.

As computer methods have been extensively used nowadays in analysis and design, the contents as related to the current popular analysis and design approaches by computer methods are also discussed. The background theory of the plate bending structure involving twisting moments, shear stresses, and design approach by the Wood Armer Equations which are extensively used by computer methods are also included in the Appendices in this Manual for design of slabs, flexible pile caps and footings.

To make distinctions between the equations quoted from the Code and the equations derived in this Manual, the former will be prefixed by (Ceqn) and the latter by (Eqn).

Unless otherwise stated, the general provisions and dimensioning of steel bars are based on high yield bars with $f_y = 460\,\text{N/mm}^2$.

1.4 Revision as contained in Amendment No. 1 comprising major revisions including (i) exclusion of members not contributing to lateral load resisting system from ductility requirements in Cl. 9.9; (ii) rectification of ε_0 in the concrete stress strain curves; (iii) raising the threshold concrete grade for limiting neutral axis depths to $0.5d$ from grade 40 to grade 45 for flexural members; (iv) reducing the x values of the simplified stress block for concrete above grade 45 are incorporated in this Manual.

2.0 Some highlighted aspects in Basis of Design

2.1 Ultimate and Serviceability Limit states

The ultimate and serviceability limit states used in the Code carry the usual meaning as in BS8110. However, the new Code has incorporated an extra serviceability requirement in checking human comfort by limiting acceleration due to wind load on high-rise buildings (in Clause 7.3.2). No method of analysis has been recommended in the Code though such accelerations can be estimated by the wind tunnel laboratory if wind tunnel tests are conducted. Nevertheless, worked examples are enclosed in Appendix B, based on approximation of the motion of the building as a simple harmonic motion and empirical approach in accordance with the Australian Wind Code AS/NZS 1170.2:2002 on which the Hong Kong Wind Code has based in deriving dynamic effects of wind loads. The relevant part of the Australian Code is Appendix G of the Australian Code.

2.2 Design Loads

The Code has made reference to the "Code of Practice for Dead and Imposed Loads for Buildings" for determination of characteristic gravity loads for design. However, this Load Code has not yet been formally promulgated and the Amendment No. 1 has deleted such reference. At the meantime, the design loads should be therefore taken from HKB(C)R Clause 17. Nevertheless, the designer may need to check for the updated loads by fire engine for design of new buildings, as required by FSD.

The Code has placed emphasize on design loads for robustness which are similar to the requirements in BS8110 Part 2. The requirements include design of the structure against a notional horizontal load equal to 1.5% of the characteristic dead weight at each floor level and vehicular impact loads (Clause 2.3.1.4). The small notional horizontal load can generally be covered by wind loads required for design. Identification of key elements and design for ultimate loads of 34 kPa, together with examination of disproportionate collapse in accordance with Cl. 2.2.2.3 can be exempted if the buildings are provided with ties determined by Cl. 6.4.1. The usual reinforcement provisions as required by the Code for other purposes can generally cover the required ties provisions.

Wind loads for design should be taken from Code of Practice on Wind Effects in Hong Kong 2004.

It should also be noted that there are differences between Table 2.1 of the Code that of BS8110 Part 1 in some of the partial load factors γ_f. The beneficial partial load factor for earth and water load is 1. However, lower values should be used if the earth and water loads are known to be over-estimated.

2.3 Materials – Concrete

Table 3.2 has tabulated a set of Young's Moduli of concrete up to grade 100. The values are generally smaller than that in BS8110 by more than 10% and also slightly different from the former 1987 Code. The stress strain curve of concrete as given in Figure 3.8 of the Code, whose initial tangent is determined by these Young's Moduli values is therefore different from Figure 2.1 of BS8110 Part 1. Furthermore, in order to achieve smooth (tangential) connection between the parabolic portion and straight portion of the stress strain curve, the Code, by its Amendment No. 1, has shifted the ε_0 value to $\dfrac{1.34(f_{cu}/\gamma_m)}{E_c}$ instead of staying at $2.4\times10^{-4}\sqrt{\dfrac{f_{cu}}{\gamma_m}}$ which is the value in BS8110. The stress strain curves for grade 35 by the Code and BS8110 are plotted as an illustration in Figure 2.1.

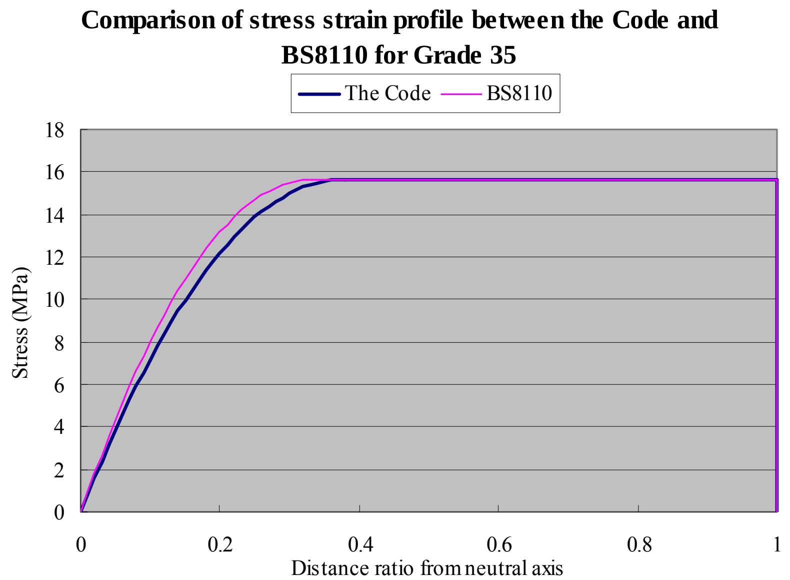


Figure 2.1 - Stress Strain Curves of Grade 35 by the Code and BS8110

From Figure 2.1 it can be seen that stress strain curve by BS8110 envelops that of the Code, indicating that design based on the Code will be slightly less economical. Design formulae for beams and columns based on these stress strain curves by BS8110, strictly speaking, become inapplicable. A full derivation of design formulae and charts for beams, columns and walls are given in Sections 3, 5 and 7, together with Appendices C, F and G of this Manual.

Table 4.2 of the Code tabulated nominal covers to reinforcements under different exposure conditions. However, reference should also be made to the "Code of Practice for Fire Resisting Construction 1996".

To cater for the "rigorous concrete stress strain relation" as indicated in Figure 2.1 for design purpose, a "simplified stress approach" by assuming a rectangular stress block of length 0.9 times the neutral axis depth has been widely adopted, as similar to BS8110. However, the Amendment No. 1 of the Code has restricted the 0.9 factor to concrete grades not exceeding 45. For 45 < f_{cu} ≤ 70 and 70 < f_{cu}, the factors are further reduced to 0.8 and 0.72 respectively as shown in Figure 2.2

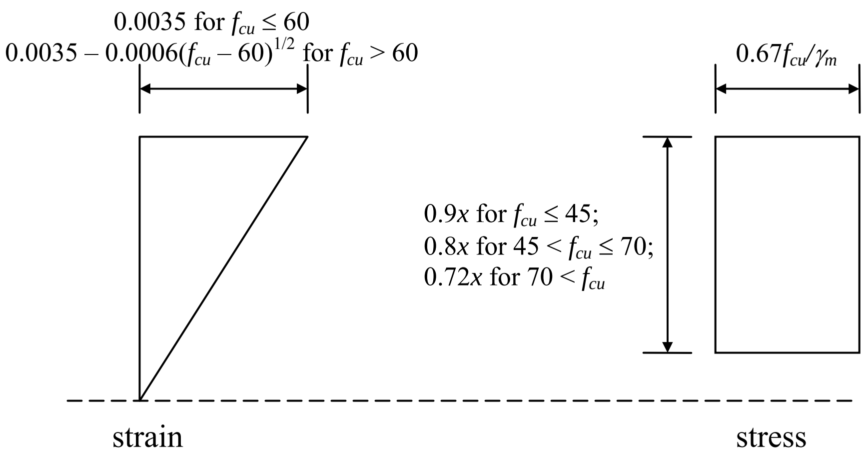


Figure 2.2 – Simplified stress block for ultimate reinforced concrete design

2.4 Ductility Requirements (for beams and columns contributing to lateral load resisting system)

As discussed in para. 1.3, an important feature of the Code is the incorporation of ductility requirements which directly affects r.c. detailings. By ductility we refer to the ability of a structure to undergo "plastic deformation", which is

comparatively larger than the "elastic" one prior to failure. Such ability is desirable in structures as it gives adequate warning to the user for repair or escape before failure. The underlying principles in r.c. detailings for ductility requirements are highlighted as follows :

(i) Use of closer and stronger transverse reinforcements to achieve better concrete confinement which enhances both ductility and strength of concrete against compression, both in columns and beams;

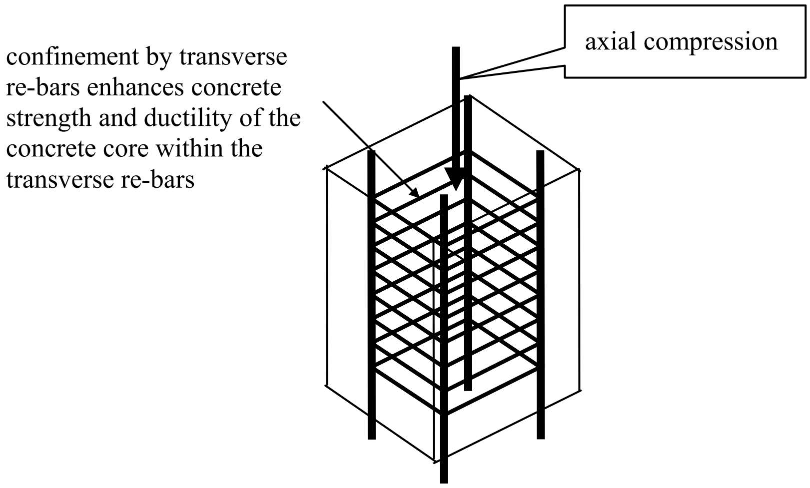
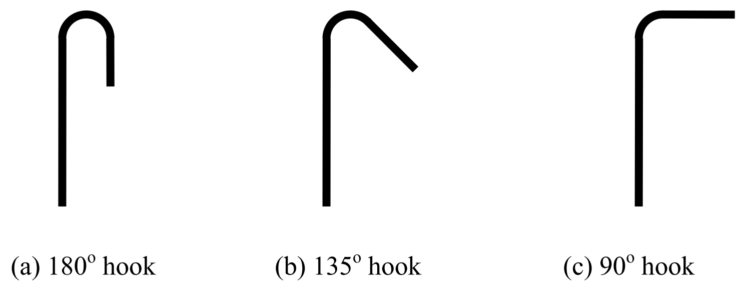


Figure 2.3 – enhancement of ductility by transverse reinforcements

(ii) Stronger anchorage of transverse reinforcements in concrete by means of hooks with bent angles $\geq 135^{o}$ for ensuring better performance of the transverse reinforcements;



Anchorage of link in concrete : (a) better than (b); (b) better than (c)

Figure 2.4 – Anchorage of links in concrete by hooks

(In fact Cl. 9.9.1.2(b) of the Code has stated that links must be adequately anchored by means of 135^{o} or 180^{o} hooks and anchorage by means of 90^{o} hooks is not permitted for beams. Cl. 9.5.2.2, Cl. 9.5.2.3 and 9.9.2.2(c) states that links for columns should have bent angle at

least 135° in anchorage. Nevertheless, for walls, links used to restrain vertical bars in compression should have an included angle of not more than 90° by Cl. 9.6.4 which is identical to BS8110 and not a ductility requirement;

(iii) More stringent requirements in restraining and containing longitudinal reinforcing bars in compression against buckling by closer and stronger transverse reinforcements with hooks of bent angles ≥ 135°;

(iv) Longer bond and anchorage length of reinforcing bars in concrete to ensure failure by yielding prior to bond slippage as the latter failure is brittle;

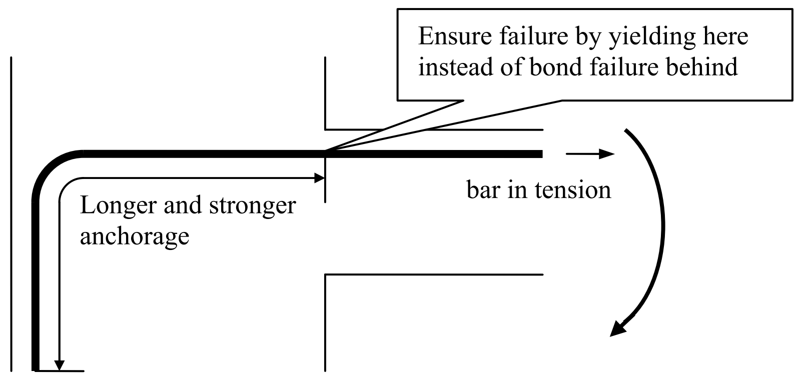


Figure 2.5 – Longer bond and anchorage length of reinforcing bars

(v) Restraining and/or avoiding radial forces by reinforcing bars on concrete at where the bars change direction and concrete cover is thin;

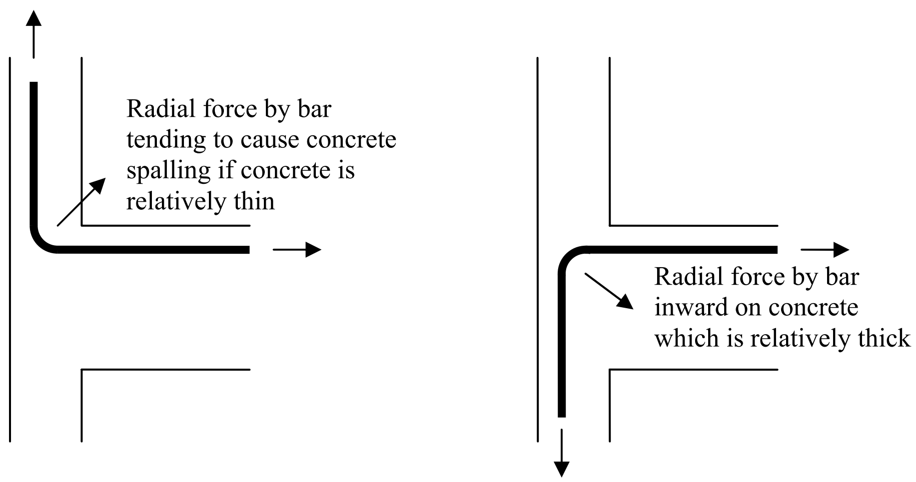
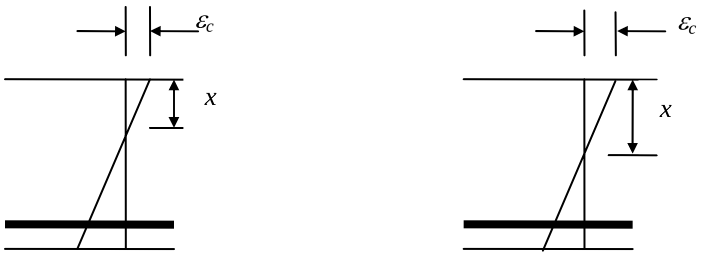


Figure 2.6 – Bars bending inwards to avoid radial forces on thin concrete cover

(vi) Limiting amounts of tension reinforcements in flexural members as over-provisions of tension reinforcements will lead to increase of

neutral axis and thus greater concrete strain and easier concrete failure which is brittle;



Lesser amount of tensile steel, smaller x, smaller ε_c

Greater amount of tensile steel, greater x, greater ε_c

Figure 2.7 – Overprovision of tensile steel may lower ductility

(vii) More stringent requirements on design using high strength concrete such as (a) lowering ultimate concrete strain; (b) restricting percentage of moment re-distribution; and (c) restricting neutral axis depth ratios to below 0.5 as higher grade concrete is more brittle.

Often the ductility requirements specified in the Code are applied to locations where plastic hinges may be formed. The locations can be accurately determined by a "push over analysis" by which a lateral load with step by step increments is added to the structure. Among the structural members met at a joint, the location at which plastic hinge is first formed will be identified as the critical section of plastic hinge formation. Nevertheless, the determination can be approximated by judgment without going through such an analysis. In a column beam frame with relatively strong columns and weak beams, the critical sections of plastic hinge formation should be in the beams at their interfaces with the columns. In case of a column connected into a thick pile cap, footing or transfer plate, the critical section with plastic hinge formation will be in the columns at their interfaces with the cap, footing or transfer plate as illustrated in Figure 2.8.

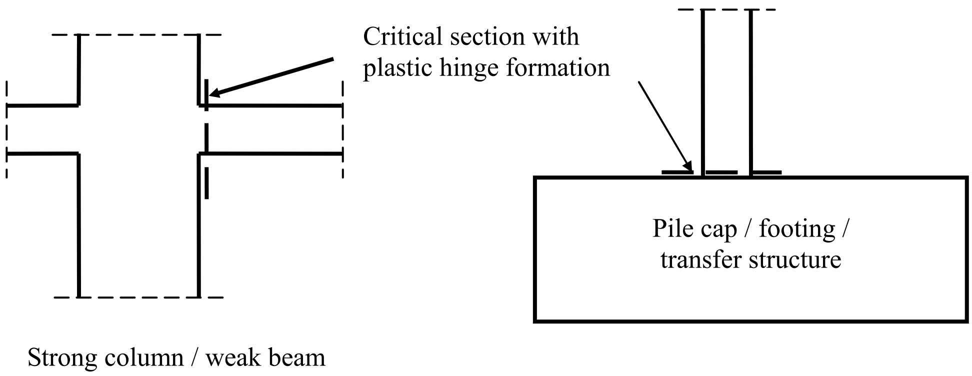


Figure 2.8 – locations of critical section with plastic hinge formation

2.5 Design for robustness

The requirements for design for robustness are identical to BS8110 and more detailed discussions are given in Section 14.

2.6 Definitions of structural elements

The Code has included definitions of slab, beam, column and wall in accordance with their dimensions in Clause 5.2.1.1, 5.4 and 5.5 which are repeated as follows for ease of reference :

(a) Slab : the minimum panel dimension ≥ 5 times its thickness;
(b) Beam : for span ≥ 2 times the overall depth for simply supported span and ≥ 2.5 times the overall depth for continuous span, classified as shallow beam, otherwise : deep beam;
(c) Column : vertical member with section depth not exceeding 4 times its width;
(d) Wall : vertical member with plan dimensions other than that of column.
(e) Shear Wall : wall contributing to the lateral stability of the structure.
(f) Transfer Structure : horizontal element which redistributes vertical loads where there is a discontinuity between the vertical structural elements above and below.

This Manual is based on the above definitions in delineating structural members for discussion.

3.0 Beams

3.1 Analysis (Cl. 5.2.5.1 & 5.2.5.2)

Normally continuous beams are analyzed as sub-frames by assuming no settlements at supports by walls, columns (or beams) and rotational stiffness by supports provided by walls or columns as $4EI/L$ (far end of column / wall fixed) or $3EI/L$ (far end of column / wall pinned).

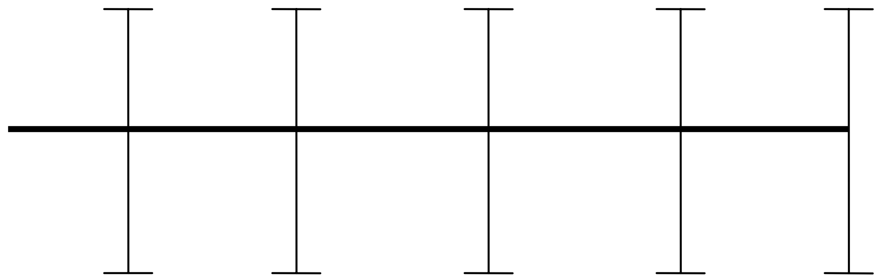


Figure 3.1 – continuous beam analyzed as sub-frame

In analysis as sub-frame, Cl. 5.2.3.2 of the Code states that the following loading arrangements will be adequate for seeking for the design moments :

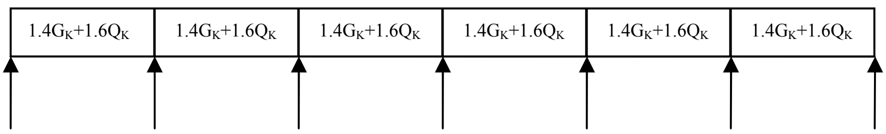


Figure 3.2a – To search for maximum support reactions

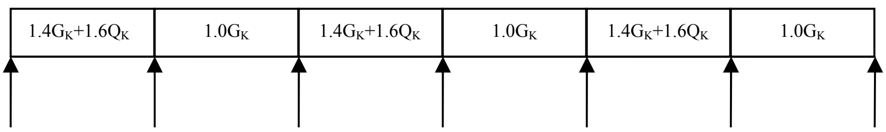


Figure 3.2b – To search for maximum sagging moment in spans with $1.4G_K+1.6Q_K$

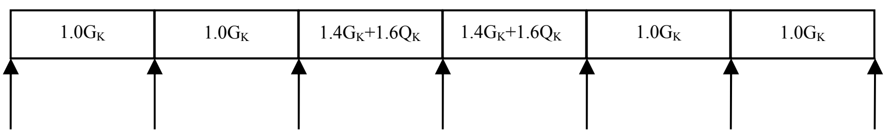


Figure 3.2c – To search for maximum hogging moment at support adjacent to spans with $1.4G_K+1.6Q_K$

However, most of the commercial softwares can actually analyze individual load cases, each of which is having live load on a single span and the effects on itself and others are analyzed. The design value of shears and moments at any location will be the summation of the values of the same sign created by the individual cases. Thus the most critical loads are arrived at easily.

With wind loads, the load cases to be considered will be $1.2(G_K+Q_K+W_K)$ and $1.0G_K+1.4W_K$ on all spans.

3.2 Moment Redistribution (Cl. 5.2.9 of the Code)

Moment redistribution is allowed for concrete grade not exceeding 70 under conditions 1, 2 and 3 as stated in Cl. 5.2.9.1 of the Code. Nevertheless, it should be noted that there would be further limitation of the neutral axis depth ratio x/d if moment redistribution is employed as required by (Ceqn 6.4) and (Ceqn 6.5) of the Code which is identical to the provisions in BS8110. The rationale is discussed in Concrete Code Handbook 6.1.2.

3.3 Highlighted aspects in Determination of Design Parameters of Shallow Beam

(i) Effective span (Cl. 5.2.1.2(b) and Figure 5.3 of the Code)

For simply supported beam, continuous beam and cantilever, the effective span can be taken as the clear span plus the lesser of half of the structural depth and half support width except that on bearing where the centre of bearing should be used to assess effective span;

(ii) Effective flange width of T- and L-beams (Cl. 5.2.1.2(a))

Effective flange width of T- and L-beams are as illustrated in Figure 5.2. of the Code as reproduced as Figure 3.3 of this Manual:

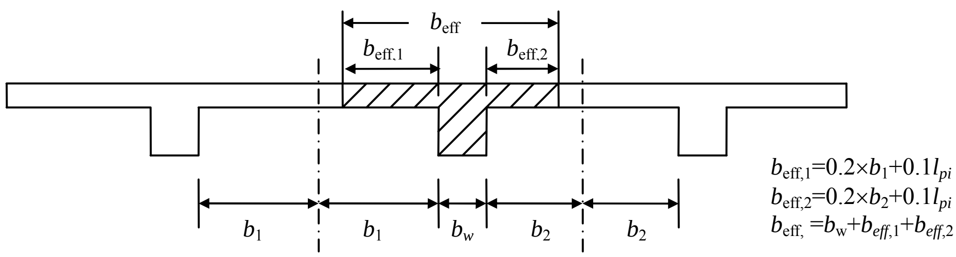


Figure 3.3 – Effective flange Parameters

Effective width (b_{eff}) = width of beam (b_w) + $\sum$(0.2 times of half the centre to centre width to the next beam ($0.2b_i$) + 0.1 times the span of zero moment ($0.1l_{pi}$), with the sum of the latter not exceeding 0.2 times the span of zero moment and l_{pi} taken as 0.7 times the effective span of the beam). An example for illustration as indicated in Figure 3.4 is as indicated :

Worked Example 3.1

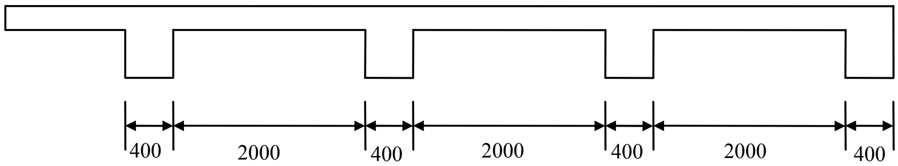


Figure 3.4 – Example illustrating effective flange determination

The effective spans are 5 m and they are continuous beams.
The effective width of the T-beam is, by (Ceqn 5.1) of the Code :
$l_{pi} = 0.7 \times 5000 = 3500$;
$b_{eff,1} = b_{eff,2} = 0.2 \times 1000 + 0.1 \times 3500 = 550$
As $b_{eff,1} = b_{eff,2} = 550 < 0.2 \times 3500 = 700$, $\therefore b_{eff,1} = b_{eff,2} = 550$;
$b_{eff} = 400 + 550 \times 2 = 400 + 1100 = 1500$
So the effective width of the T-beam is 1500 mm.

Similarly, the effective width of the L-beam at the end is

$b_w + b_{eff,1} = 400 + 550 = 950$.

(iii) Support Moment Reduction (Cl. 5.2.1.2 of the Code)

The Code allows design moment of beam (and slab) monolithic with its support providing rotational restraint to be that at support face if the support is rectangular and 0.2Ø if the support is circular with diameter Ø. But the design moment after reduction should not be less than 65% of the support moment. A worked example 3.2 as indicated by Figure 3.5 for illustration is given below :

Worked Example 3.2

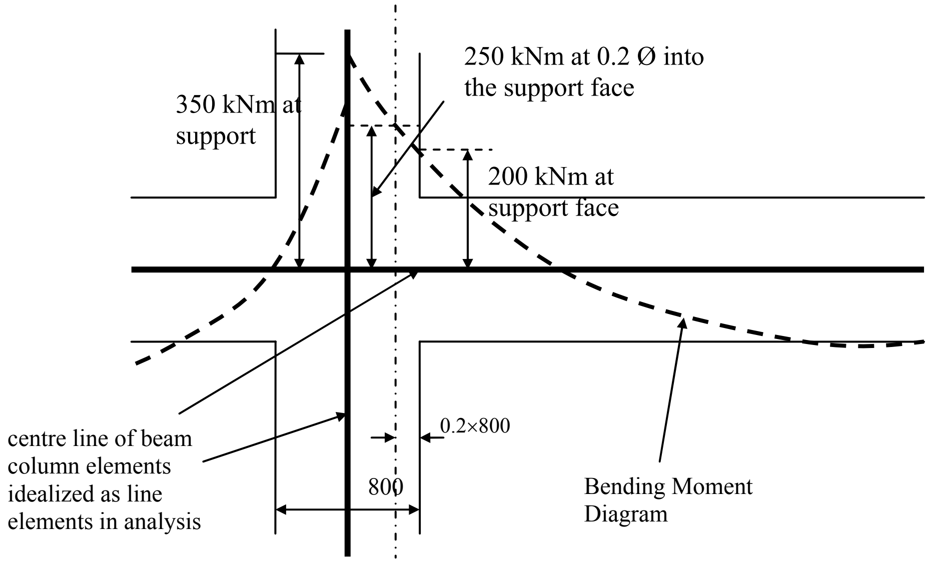


Figure 3.5 – Reduced moment to Support Face for support providing rotational restraint

In Figure 3.5, the bending moment at support face is 200 kNm which can be the design moment of the beam if the support face is rectangular. However, as it is smaller than 0.65×350 = 227.5 kNm. 227.5 kNm should be used for design.

If the support is circular and the moment at 0.2Ø into the support and the bending moment at the section is 250 kNm, then 250 kNm will be the design moment as it is greater than 0.65×350 = 227.5 kNm.

For beam (or slab) spanning continuously over a support considered not providing rotational restraint (e.g. wall support), the Code allows moment reduction by support shear times one eighth of the support width to the moment obtained by analysis. Figure 3.6 indicates a numerical Worked Example 3.3.

<u>Worked Example 3.3</u>

By Figure 3.6, the design support moment at the support under consideration can be reduced to $250 - 200 \times \frac{0.8}{8} = 230$ kNm.

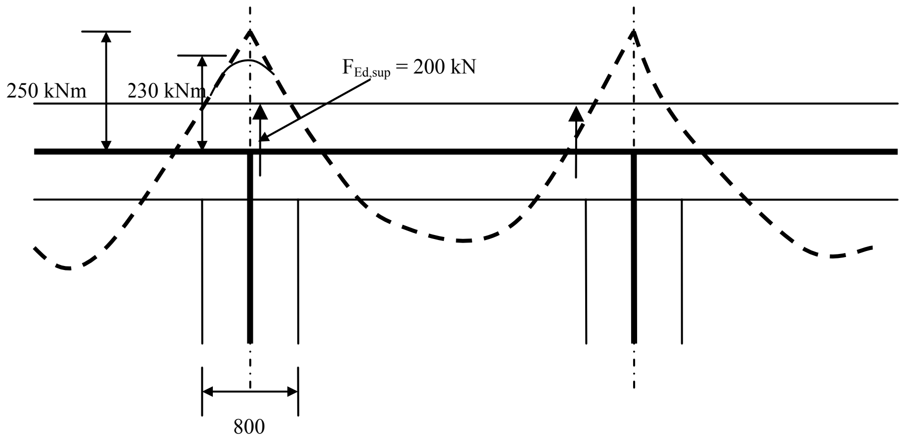


Figure 3.6 – Reduction of support moment by support shear for support considered not providing rotational restraint

(iv) Slenderness Limit (Cl. 6.1.2.1 of the Code)

The provision is identical to BS8110 as

1. Simply supported or continuous beam :
   Clear distance between restraints $\leq 60b_c$ or $250b_c^2/d$ if less; and
2. Cantilever with lateral restraint only at support :
   Clear distance from cantilever to support $\leq 25b_c$ or $100b_c^2/d$ if less where b_c is the breadth of the compression face of the beam and d is the effective depth.

Usually the slenderness limits need be checked for inverted beams or bare beam (without slab).

(v) Span effective depth ratio (Cl. 7.3.4.2 of the Code)

Table 7.3 under Cl. 7.3.4.2 tabulates basic span depth ratios for various types of beam / slab which are deemed-to-satisfy requirements against deflection. The table has provisions for "slabs" and "end spans" which are not specified in BS8110 Table 3.9. Nevertheless, calculation can be carried out to justify deflection limits not to exceed span / 250. In addition, the basic span depth ratios can be modified due to provision of tensile and compressive steels as given in Tables 7.4 and 7.5 of the Code which are identical to BS8110. Modification of the factor by 10/span for

span > 10 m except for cantilever as similar to BS8110 is also included.

Support condition	Rectangular Beam	Flanged Beam $b_w/b < 0.3$	One or two-way spanning solid slab
Cantilever	7	5.5	7
Simply supported	20	16	20
Continuous	26	21	26
End span	23	18.5	$23^{(2)}$

Note :
1. The values given have been chosen to be generally conservative and calculation may frequently show shallower sections are possible;
2. The value of 23 is appropriate for two-way spanning slab if it is continuous over one long side;
3. For two-way spanning slabs the check should be carried out on the basis of the shorter span.

Table 3.1 – effective span / depth ratio

(vi) Maximum spacing between bars in tension near surface, by Cl. 9.2.1.4 of the Code, should be such that the clear spacing between bar is limited by clear spacing $\leq \dfrac{70000\beta_b}{f_y} \leq 300$ mm where β_b is the ratio of moment redistribution. Or alternatively, clear spacing $\leq \dfrac{47000}{f_s} \leq 300$ mm. So the simplest rule is $\dfrac{70000\beta_b}{f_y} = \dfrac{70000 \times 1}{460} = 152$ mm when using high yield bars and under no moment redistribution.

(vii) Concrete covers to reinforcements (Cl. 4.2.4 and Cl. 4.3 of the Code)

Cl. 4.2.4 of the Code indicates the nominal cover required in accordance with Exposure conditions. However, we can, as far as our building structures are concerned, roughly adopt condition 1 (Mild) for the structures in the interior of our buildings (except for bathrooms and kitchens which should be condition 2), and to adopt condition 2 for the external structures. Nevertheless, the “Code of Practice for Fire Resisting Construction 1996” should also be checked for different fire resistance periods (FRP). So, taking into account our current practice of using concrete not inferior than grade 30 and maximum aggregate sizes not exceeding 20 mm, we may generally adopt the provision in our DSEG Manual (DSEDG-104 Table 1) with updating by the Code except for compartment of 4 hours FRP. The recommended covers are summarized in the following table :

Description	Nominal Cover (mm)
Internal	30 (to all rebars)
External	40 (to all rebars)
Simply supported (4 hours FRP)	80 (to main rebars)
Continuous (4 hours FRP)	60 (to main rebars)

Table 3.2 – Nominal Cover of Beams

3.4 Sectional Design for Rectangular Beam against Bending

3.4.1 Design in accordance with the Rigorous Stress Strain curve of Concrete

The stress strain block of concrete as indicated in Figure 3.8 of the Code is different from Figure 2.1 of BS8110. Furthermore, in order to achieve smooth connection between the parabolic and the straight line portions, the Concrete Code Handbook has recommended to shift the ε_0 to the right to a value of $\frac{1.34 f_{cu}}{\gamma_m E_c}$, which has been adopted in Amendment No. 1. With the values of Young's Moduli of concrete, E_c, as indicated in Table 3.2 of the Code, the stress strain block of concrete for various grades can be determined. The stress strain curve of grade 35 is drawn as shown in Figure 3.7.

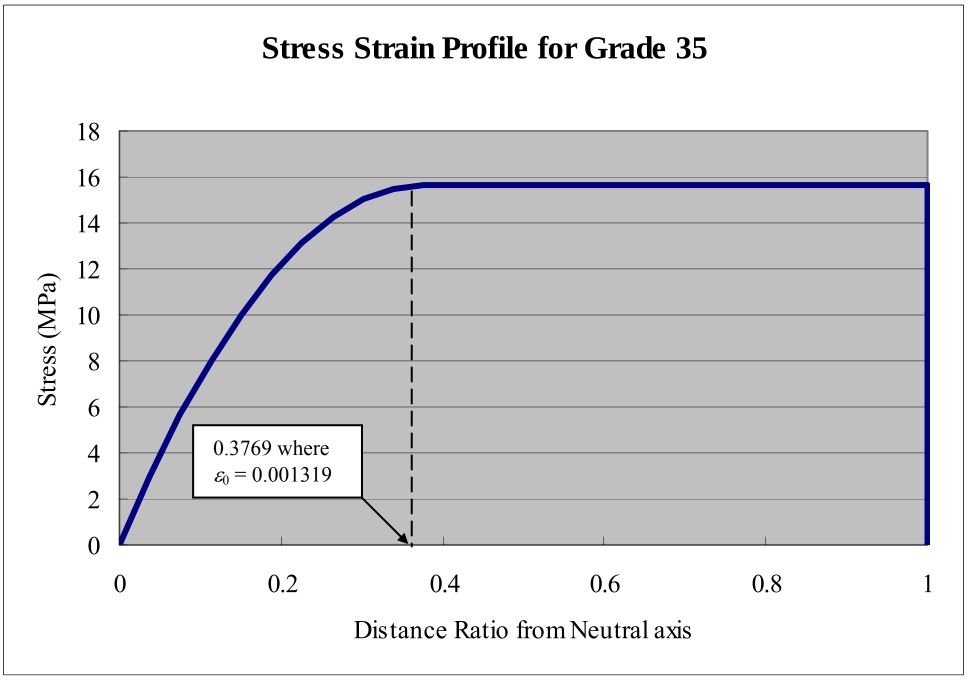


Figure 3.7 – Stress strain block of grades 35

Based on this rigorous concrete stress strain block, design formulae for beam

can be worked out as per the strain distribution profile of concrete and steel as indicated in Figure 3.8.

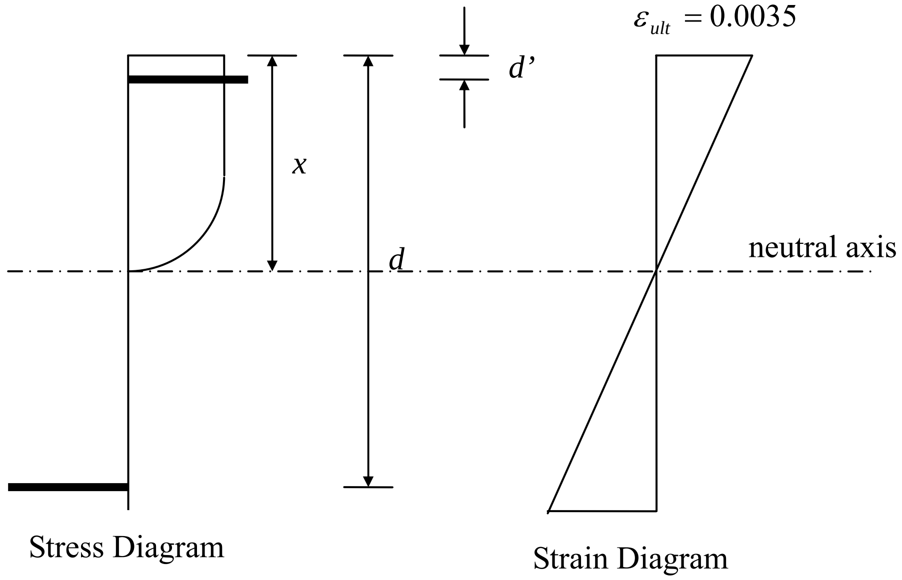


Figure 3.8 – Stress Strain diagram for Beam

The solution for the neutral axis depth ratio $\frac{x}{d}$ for singly reinforced beam is the positive root of the following quadratic equation where $\varepsilon_{ult} = 0.0035$ for concrete grades not exceeding 60 (Re Appendix C for detailed derivation) :

$$\frac{0.67 f_{cu}}{\gamma_m}\left[-\frac{1}{2}+\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{d}\right)^2+\frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{d}-\frac{M}{bd^2}=0$$

(Eqn 3-1)

With neutral axis depth ratio determined, the steel ratio can be determined by

$$\frac{A_{st}}{bd}=\frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{d} \qquad \text{(Eqn 3-2)}$$

As $\frac{x}{d}$ is limited to 0.5 for singly reinforcing sections for grades up to 45 under moment redistribution not greater than 10% (Clause 6.1.2.4 of the Code), by (Eqn 3-1), $\frac{M}{bd^2 f_{cu}}$ will be limited to K' values as in

$K' = 0.154$ for grade 30; $K' = 0.152$ for grade 35;

$K' = 0.151$ for grade 40; $K' = 0.150$ for grade 45

which are all smaller than 0.156 under the simplified stress block.

However, for grades exceeding 45 and below 70 where neutral axis depth ratio is limited to 0.4 for singly reinforced sections under moment redistribution not

greater than 10% (Clause 6.1.2.4 of the Code), again by (Eqn 3-1) $\frac{M}{bd^2 f_{cu}}$ will be limited to

$K' = 0.125$ for grade 50; $K' = 0.123$ for grade 60;

$K' = 0.121$ for grade 70.

which are instead above 0.120 under the simplified stress block as Amendment No. 1 has reduce the x/d factor to 0.8. Re discussion is in Appendix C.

It should be noted that the x/d ratio will be further limited if moment redistribution exceeds 10% by (Ceqn 6.4) and (Ceqn 6.5) of the Code (with revision by Amendment No. 1) as

$\frac{x}{d} \le (\beta_b - 0.4)$ for $f_{cu} \le 45$; and

$\frac{x}{d} \le (\beta_b - 0.5)$ for $45 < f_{cu} \le 70$

where β_b us the ratio of the moment after and before moment redistribution.

When $\frac{M}{bd^2 f_{cu}}$ exceeds the limited value for single reinforcement, compression reinforcements at d' from the surface of the compression side should be added. The compression reinforcements will take up the difference between the applied moment and $K'bd^2 f_{cu}$ and the compression reinforcement ratio is

$$\frac{A_{sc}}{bd} = \frac{\left(\frac{M}{bd^2 f_{cu}} - K'\right) f_{cu}}{0.87 f_y \left(1 - \frac{d'}{d}\right)} \qquad \text{(Eqn 3-3)}$$

And the same amount of reinforcement will be added to the tensile reinforcement :

$$\frac{A_{st}}{bd} = \frac{1}{0.87 f_y} \frac{0.67 f_{cu}}{\gamma_m} \left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\eta + \frac{\left(\frac{M}{bd^2 f_{cu}} - K'\right) f_{cu}}{0.87 f_y \left(1 - \frac{d'}{d}\right)} \qquad \text{(Eqn 3-4)}$$

where η is the limit of neutral axis depth ratio which is 0.5 for $f_{cu} \le 45$, 0.4 for $45 < f_{cu} \le 70$ and 0.33 for $70 < f_{cu} \le 100$ where moment redistribution does not exceed 10%.

It follows that more compressive reinforcements will be required for grade 50 than 45 due to the limitation of neutral axis depth ratio, as illustrated by the following Chart 3-1 in which compression reinforcement decreases from grade

30 to 40 for the same $\frac{M}{bd^2}$, but increases at grade 45 due to the change of the limit of neutral axis depth ratio from 0.5 to 0.4 with moment redistribution not exceeding 10%. The same phenomenon applies to tensile steel also. With moment redistribution exceeding 10%, the same trend will also take place.

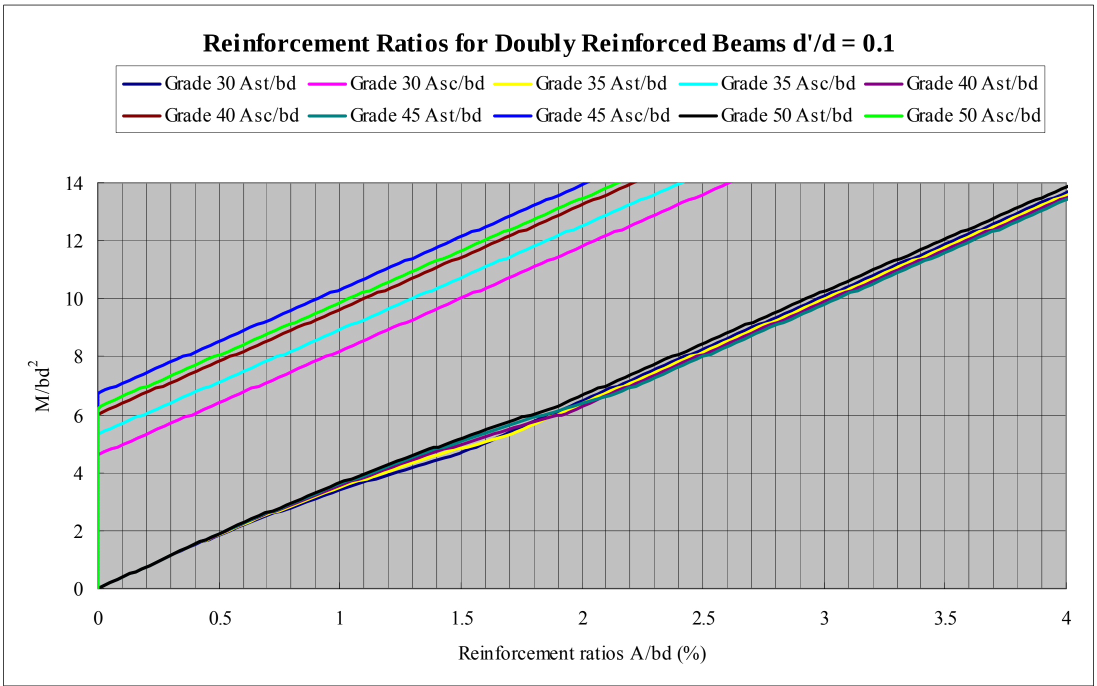


Chart 3-1 – Reinforcement Ratios of Doubly Reinforced Beams for Grade 30 to 50 with Moment Redistribution limited to 10% or below

As similar to BS8110, there is an upper limit of "lever arm ratio" $\frac{z}{d}$ which is the depth of the centroid of the compressive force of concrete to the effective depth of the beam section of not exceeding 0.95. Thus for calculated values of $\frac{z}{d} \geq 0.95$ or $\frac{x}{d} \leq 0.111$ in accordance with the simplified stress block approach, $\frac{A_{st}}{bd} = \frac{M}{0.87 f_y (0.95d)bd}$

Design Charts for grades 30 to 50 comprising tensile steel and compression steel ratios $\frac{A_{st}}{bd}$ and $\frac{A_{sc}}{bd}$ are enclosed at the end of Appendix C.

3.4.2 Design in accordance with the Simplified Stress Block

The design will be simpler and sometimes more economical if the simplified

rectangular stress block as given by Figure 6.1 of the Code is adopted. The design formula becomes :

For singly reinforced sections where $K = \frac{M}{f_{cu}bd^2} \le K'$ where $K' = 0.156$ for grades 45 and below and $K' = 0.120$ for $45 < f_{cu} \le 70$; $K' = 0.094$ for $70 < f_{cu} \le 100$.

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \le 0.95;$$

$$\frac{x}{d} = \left(1 - \frac{z}{d}\right)\frac{1}{0.45} = \left(0.5 - \sqrt{0.25 - \frac{K}{0.9}}\right)\frac{1}{0.45}; \quad A_{st} = \frac{M}{0.87 f_y z} \qquad \text{(Eqn 3-5)}$$

For doubly reinforced sections $K = \frac{M}{f_{cu}bd^2} > K'$,

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K'}{0.9}} \qquad \frac{x}{d} = \left(1 - \frac{z}{d}\right)\frac{1}{0.45}$$

$$A_{sc} = \frac{(K - K')f_{cu}bd^2}{0.87 f_y (d - d')} \qquad A_{st} = \frac{K' f_{cu}bd^2}{0.87 f_y z} + A_{sc} \qquad \text{(Eqn 3-6)}$$

3.4.3 Ductility Requirement on amounts of compression reinforcement

In accordance with Cl. 9.9.1.1(a) of the Code, at any section of a beam (participating in lateral load resisting system) within a “critical zone” the compression reinforcement should not be less than one-half of the tension reinforcement at the same section. A “critical zone” is understood to be a zone where a plastic hinge is likely to be formed and thus generally include sections near supports or at mid-span. The adoption of the clause will likely result in providing more compression reinforcements in beams (critical zones).

3.4.4 Worked Examples for Determination of steel reinforcements in Rectangular Beam with Moment Redistribution < 10%

Unless otherwise demonstrated in the following worked examples, the requirement in Cl. 9.9.1.1(a) of the Code as discussed in para. 3.4.3 by requiring compression reinforcements be at least one half of the tension reinforcement is not included in the calculation of required reinforcements.

Worked Example 3.4

Section : 500 (h) × 400 (w), $f_{cu} = 35\,\text{MPa}$
cover = 40 mm (to main reinforcement)

(i) $M_1 = 286\,\text{kNm}$;

$d = 500 - 40 - 16 = 444$

$$\varepsilon_0 = \frac{1.34 f_{cu}}{\gamma_m E_c} = \frac{1.34 \times 35}{1.5 \times 23700} = 0.0013192 \qquad \frac{\varepsilon_0}{\varepsilon_{ult}} = 0.3769$$

$$\frac{M_1}{f_{cu} bd^2} = \frac{286 \times 10^6}{35 \times 400 \times 444^2} = 0.104 < 0.152 \text{, so singly reinforced}$$

Solving the neutral axis depth ratio by (Eqn 3-1) $\frac{x}{d}$

$$\frac{0.67 f_{cu}}{\gamma_m}\left[-\frac{1}{2} + \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right] = -60.38\,;$$

$$\frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right) = 13.669\,; \qquad -\frac{M}{bd^2} = \frac{286 \times 10^6}{400 \times 444^2} = -3.627$$

$$\frac{x}{d} = \frac{-13.699 + \sqrt{13.699^2 - 4 \times (-60.38) \times (-3.627)}}{2 \times (-60.38)} = 0.307 \le 0.5$$

$$\frac{A_{st}}{bd} = \frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{d} = \frac{1}{0.87 \times 460} \times 13.699 \times 0.307 = 0.0105$$

$\Rightarrow A_{st} = 1865\,\text{mm}^2$ Use 2T32 + 1T25

(ii) $M_2 = 486\,\text{kNm}$;

$d = 500 - 40 - 20 = 440$

$$\varepsilon_0 = \frac{1.34 f_{cu}}{\gamma_m E_c} = \frac{1.34 \times 35}{1.5 \times 23700} = 0.0013192 \qquad \frac{\varepsilon_0}{\varepsilon_{ult}} = 0.3769$$

$$\frac{M_2}{f_{cu} bd^2} = \frac{486 \times 10^6}{35 \times 400 \times 440^2} = 0.179 > 0.152 \text{, so doubly reinforced}$$

$d' = 40 + 10 = 50 \quad \frac{d'}{d} = \frac{50}{440} = 0.114$ (assume T20 bars)

By (Eqn 3-3) $$\frac{A_{sc}}{bd} = \frac{\left(\frac{M}{bd^2 f_{cu}} - K\right) f_{cu}}{0.87 f_y \left(1 - \frac{d'}{d}\right)} = \frac{(0.179 - 0.152) \times 35}{0.87 \times 460 \times (1 - 0.114)} = 0.267\,\%$$

$A_{sc} = 0.00267 \times 400 \times 440 = 469\,\text{mm}^2$ Use 2T20

By (Eqn 3-4) $\dfrac{A_{st}}{bd}=\dfrac{1}{0.87f_y}\dfrac{0.67f_{cu}}{\gamma_m}\left(1-\dfrac{1}{3}\dfrac{\varepsilon_0}{\varepsilon_{ult}}\right)\eta+\dfrac{\left(\dfrac{M}{bd^2 f_{cu}}-K\right)f_{cu}}{0.87f_y\left(1-\dfrac{d'}{d}\right)}$

$$\frac{A_{st}}{bd}=\frac{1}{0.87\times 460}13.699\times 0.5+0.00267=1.978\,\%$$

$A_{st}=0.01978\times 400\times 440=3481\,\text{mm}^2$ Use 3T40

Worked Example 3.5

(i) and (ii) of Worked Example 3.4 are re-done in accordance with Figure 6.1 of the Code (the simplified stress) block by (Eqn 3-5) and (Eqn 3-6)

(i) $\dfrac{z}{d}=0.5+\sqrt{0.25-\dfrac{K}{0.9}}=0.5+\sqrt{0.25-\dfrac{286\times 10^6}{35\times 400\times 444^2\times 0.9}}=0.867$

$$\frac{A_{st}}{bd}=\frac{M}{bd^2\times 0.87f_y(z/d)}=\frac{286\times 10^6}{400\times 444^2\times 0.87\times 460\times 0.867}=0.01045$$

$\Rightarrow A_{st}=1856\,\text{mm}^2$ Use 2T32 + 1T25

(ii) $K=\dfrac{M}{f_{cu}bd^2}=\dfrac{486\times 10^6}{35\times 400\times 440^2}=0.179>0.156$, so doubly reinforcing section required, $\dfrac{z}{d}=1-0.5\times 0.9\times 0.5=0.775$

$A_{sc}=\dfrac{(K-K')f_{cu}bd^2}{0.87f_y(d-d')}=\dfrac{(0.179-0.156)\times 35\times 400\times 440^2}{0.87\times 460\times(440-50)}=399$ mm^2 > 0.2% in accordance with Table 9.1 of the Code, Use 2T16

$$A_{st}=\frac{K' f_{cu}bd^2}{0.87f_y z}+A_{sc}=\frac{0.156\times 35\times 400\times 440^2}{0.87\times 460\times 0.775\times 440}+399=3498\,\text{mm}^2$$

Use 3T40

(Note : If the beam is contributing in lateral load resisting system and the section is within "critical zone", compressive reinforcements has to be at least half of that of tension reinforcements $A_{sc}=3498/2=1749\,\text{mm}^2$ by Cl. 9.9.1.1(a) in the Code (D). So use 2T25 + 1T32.)

Results of comparison of results from Worked Examples 3.4 and 3.5 (with the omission of the requirement in Cl. 9.9.1.1(a) that compressive reinforcements be at least half of that of tension reinforcements) are summarized in Table 3.3, indicating differences between the "Rigorous Stress" and "Simplified Stress" Approach :

	Singly Reinforced	Doubly Reinforced		
	A_{st} (mm^2)	A_{sc} (mm^2)	A_{st} (mm^2)	Total (mm^2)
Based on Rigorous Stress Approach	1865	469	3481	3950
Based on Simplified stress Approach	1856	399	3498	3897

Table 3.3 – Summary of Results for comparison of Rigorous stress and simplified stress Approaches.

Results by the two approaches are very close. The approach based on the simplified stress block are slightly more economical.

3.4.5 Worked Example 3.6 for Rectangular Beam with Moment Redistribution > 10%

If the Worked Example 3.4 (ii) has undergone a moment redistribution of 20% > 10%, i.e. $\beta_b = 0.8$, by (Ceqn 6.4) of the Code, the neutral axis depth is limited to $\frac{x}{d} \le (\beta_b - 0.4) \Rightarrow \frac{x}{d} \le 0.8 - 0.4 = 0.4$,

and the lever arm ratio becomes $\frac{z}{d} = 1 - 0.4 \times 0.9 \times 0.5 = 0.82$.

So the $K = \frac{M}{bd^2 f_{cu}}$ value become $0.5 + \sqrt{0.25 - \frac{K}{0.9}} = 0.82 \Rightarrow K = 0.132$

$$A_{sc} = \frac{(K - K')f_{cu}bd^2}{0.87 f_y (d - d')} = \frac{(0.176 - 0.132) \times 35 \times 400 \times 440^2}{0.87 \times 460 \times (440 - 50)} = 764 \text{ mm}^2 > 0.2\ \%$$

as required by Table 9.1 of the Code;

$$A_{st} = \frac{K' f_{cu} bd^2}{0.87 f_y z} + A_{sc} = \frac{0.132 \times 35 \times 400 \times 440^2}{0.87 \times 460 \times 0.82 \times 440} + 764 = 3242 \text{ mm}^2$$

So total amount of reinforcement is greater.

3.5 Sectional Design of Flanged Beam against Bending

3.5.1 Slab structure adjacent to the beam, if in flexural compression, can be used to act as part of compression zone of the beam, thus effectively widen the structural width of the beam. The use of flanged beam will be particularly useful in eliminating the use of compressive reinforcements, as in addition to

reducing tensile steel due to increase of lever arm. The principle of sectional design of flanged beam follows that rectangular beam with an additional flange width of $b_{eff} - b_w$ as illustrated in Figure 3.9.

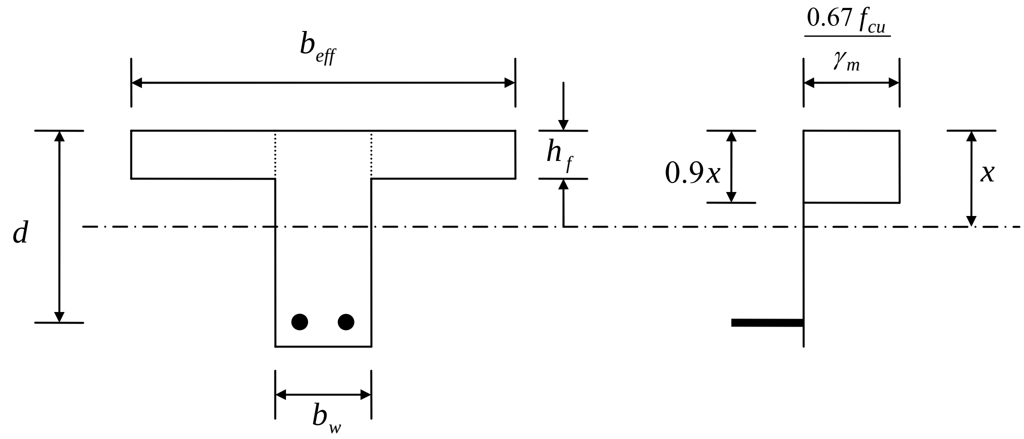


Figure 3.9 – Analysis of a T or L beam section

Design formulae based on the simplified stress block are derived in Appendix C which are summarized as follows :

(i) Singly reinforcing section where η × neutral axis depth is inside flange depth by checking where $\eta = 0.9$ for $f_{cu} \le 45$; $\eta = 0.8$ for $45 < f_{cu} \le 70$; $\eta = 0.72$ for $70 < f_{cu} \le 100$.

$$\eta \frac{x}{d} = 1 - \sqrt{1 - \frac{K}{0.225}} \le \frac{h_f}{d} \quad \text{where } K = \frac{M}{f_{cu} b_{eff} d^2} \qquad \text{(Eqn 3-7)}$$

If so, carry out design as if it is a rectangular beam of width b_{eff}.

(ii) Singly reinforcing section where η × neutral axis depth is outside flange depth, i.e. $\eta \frac{x}{d} > \frac{h_f}{d}$ and

$$\frac{M}{b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m} \left(\frac{b_{eff}}{b_w} - 1 \right) \frac{h_f}{d} \left(1 - \frac{1}{2} \frac{h_f}{d} \right) + \frac{0.67 f_{cu}}{\gamma_m} \left(\eta \frac{x}{d} \right) \left(1 - \frac{\eta}{2} \frac{x}{d} \right)$$

$\frac{x}{d}$ be solved by the quadratic equation :

$$\frac{0.67 f_{cu}}{\gamma_m} \frac{\eta^2}{2} \left(\frac{x}{d} \right)^2 - \frac{0.67 f_{cu}}{\gamma_m} \eta \frac{x}{d} + \frac{M - M_f}{b_w d^2} = 0 \qquad \text{(Eqn 3-8)}$$

where $$\frac{M_f}{b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m} \frac{h_f}{d} \left(\frac{b_{eff}}{b_w} - 1 \right) \left(1 - \frac{1}{2} \frac{h_f}{d} \right) \qquad \text{(Eqn 3-9)}$$

And $\dfrac{A_{st}}{b_w d} = \dfrac{0.67 f_{cu}}{\gamma_m 0.87 f_y}\left[\left(\dfrac{b_{eff}}{b_w} - 1\right)\dfrac{h_f}{d} + \eta \dfrac{x}{d}\right]$ (Eqn 3-10)

(iii) Doubly reinforcing section :

By following the procedure in (ii), if $\dfrac{x}{d}$ obtained by (Eqn 3-8) exceeds φ where $\varphi = 0.5$ for $f_{cu} > 45$; 0.4 for $f_{cu} > 70$ and 0.33 for $f_{cu} > 100$, then double reinforcements will be required with required A_{sc} and A_{st} as

$$\frac{A_{sc}}{b_w d} = \frac{1}{0.87 f_y (1 - d'/d)}\left[\frac{M}{b_w d^2} - \frac{0.67 f_{cu}}{\gamma_m}\left[\left(\frac{b_{eff}}{b_w} - 1\right)\frac{h_f}{d}\left(1 - \frac{1}{2}\frac{h_f}{d}\right) + \eta\varphi\left(1 - \frac{1}{2}\varphi\right)\right]\right]$$

(Eqn 3-11)

$$\frac{A_{st}}{b_w d} = \frac{0.67 f_{cu}}{\gamma_m 0.87 f_y}\left[\left(\frac{b_{eff}}{b_w} - 1\right)\frac{h_f}{d} + \eta\varphi\right] + \frac{A_{sc}}{b_w d}$$ (Eqn 3-12)

3.5.2 Worked Examples for Flanged Beam, grade 35 ($\eta = 0.9$)

(i) Worked Example 3.7 : Singly reinforced section where $0.9\dfrac{x}{d} \le \dfrac{h_f}{d}$

Consider the previous example done for a rectangular beam 500 (h) × 400 (w), $f_{cu} = 35$ MPa, under a moment 486 kNm, with a flanged section of width = 1200 mm and depth = 150 mm :

$b_w = 400$, $d = 500 - 40 - 20 = 440$, $b_{eff} = 1200$ $h_f = 150$

First check if $0.9\dfrac{x}{d} \le \dfrac{h_f}{d}$ based on beam width of 1200,

$$K = \frac{M}{f_{cu} b_{eff} d^2} = \frac{486 \times 10^6}{35 \times 1200 \times 440^2} = 0.0598$$

By (Eqn 3-5), $\dfrac{x}{d} = \left(0.5 - \sqrt{0.25 - \dfrac{K}{0.9}}\right)\dfrac{1}{0.45} = 0.159$;

$\therefore 0.9\dfrac{x}{d} = 0.143 < \dfrac{h_f}{d} = \dfrac{150}{440} = 0.341$. $\dfrac{z}{d} = 1 - 0.45\dfrac{x}{d} = 0.928$; Thus

$$\frac{A_{st}}{b_{eff} d} = \frac{M}{b_{eff} d^2 \times 0.87 f_y (z/d)} = \frac{486 \times 10^6}{1200 \times 440^2 \times 0.87 \times 460 \times 0.928} = 0.00563$$

> 0.18% (minimum for $\dfrac{b_w}{b_{eff}} = \dfrac{400}{1200} = 0.33 < 0.4$ in accordance with Table 9.1 of the Code)

$\therefore A_{st} = 2974\,\text{mm}^2$. Use 2T40 + 1T25

As in comparison with the previous example based on rectangular

section, it can be seen that there is saving in tensile steel (2974 mm^2 vs 3498 mm^2) and the compression reinforcements are eliminated.

(ii) Worked Example 3.8 – Singly reinforced section where $\eta \frac{x}{d} > \frac{h_f}{d}$, and $\eta = 0.9$ for grade 35.

Beam Section : 1000 (h) × 600 (w), flange width = 2000 mm, flange depth = 150 mm $f_{cu} = 35$ MPa under a moment 4000 kNm

$b_w = 600$, $d = 1000 - 50 - 60 = 890$, $b_{eff} = 2000$ $h_f = 150$

$$\frac{h_f}{d} = \frac{150}{890} = 0.169; \qquad \frac{b_{eff}}{b_w} = \frac{2000}{600} = 3.333$$

First check if $0.9\frac{x}{d} \le \frac{h_f}{d}$ based on beam width of $b_w = b_{eff} = 2000$

$$K = \frac{M}{f_{cu} b_{eff} d^2} = \frac{4000 \times 10^6}{35 \times 2000 \times 890^2} = 0.0721$$

By (Eqn 3-7)

$$0.9\frac{x}{d} = 2\left(0.5 - \sqrt{0.25 - \frac{K}{0.9}}\right) = 0.176 > \frac{h_f}{d} = \frac{150}{890} = 0.169$$

So 0.9 × neutral axis depth extends below flange.

$$\frac{M_f}{b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m} \frac{h_f}{d}\left(\frac{b_{eff}}{b_w} - 1\right)\left(1 - \frac{1}{2}\frac{h_f}{d}\right) \Rightarrow M_f = 2675.65 \text{ kNm}$$

Solve $\frac{x}{d}$ by (Eqn 3-8) with $\eta = 0.9$.

$$0.1809 f_{cu}\left(\frac{x}{d}\right)^2 - 0.402 f_{cu}\frac{x}{d} + \frac{M - M_f}{b_w d^2} = 0$$

$$\Rightarrow 0.1809 \times 35\left(\frac{x}{d}\right)^2 - 0.402 \times 35\frac{x}{d} + \frac{(4000 - 2675.65) \times 10^6}{600 \times 890^2} = 0;$$

$$\Rightarrow \frac{x}{d} = 0.2198;$$

By (Eqn 3-10)

$$\frac{A_{st}}{b_w d} = \frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left[\left(\frac{b_{eff}}{b_w} - 1\right)\frac{h_f}{d} + 0.9 \times 0.2198\right] = 0.02309$$

$A_{st} = 12330$ mm^2, Use 10-T40 in 2 layers

(iii) Worked Example 3.9 – Doubly reinforced section

Beam Section : 1000 (h) × 600 (w), flange width = 1250 mm, flange depth = 150 mm $f_{cu} = 35$ MPa under a moment 4000 kNm

$b_w = 600$, $d = 1000 - 50 - 60 = 890$, $b_{eff} = 1250$ $h_f = 150$

$$\frac{h_f}{d}=\frac{150}{890}=0.169\,; \qquad \frac{b_{eff}}{b_w}=\frac{1250}{600}=2.083\,; \qquad \eta=0.9$$

First check if $\eta\frac{x}{d}\le\frac{h_f}{d}$ based on beam width of $b_{eff}=1250$

$$K=\frac{M}{f_{cu}b_{eff}d^2}=\frac{4000\times10^6}{35\times1250\times890^2}=0.115$$

By (Eqn 3-7)

$$0.9\frac{x}{d}=1-\sqrt{1-\frac{0.115}{0.225}}=0.302>\frac{h_f}{d}=\frac{150}{890}=0.169$$

So 0.9 × neutral axis depth extends below flange.

$$\frac{M_f}{b_w d^2}=\frac{0.67f_{cu}}{\gamma_m}\frac{h_f}{d}\left(\frac{b_{eff}}{b_w}-1\right)\left(1-\frac{1}{2}\frac{h_f}{d}\right)\Rightarrow M_f=1242.26\,\text{kNm}$$

Solve $\frac{x}{d}$ by (Eqn 3-8) with $\eta=0.9$

$$0.1809f_{cu}\left(\frac{x}{d}\right)^2-0.402f_{cu}\frac{x}{d}+\frac{M-M_f}{b_w d^2}=0$$

$$\Rightarrow 0.1809f_{cu}\left(\frac{x}{d}\right)^2-0.402\times35\frac{x}{d}+\frac{(4000-1242.26)\times10^6}{600\times890^2}=0$$

$\frac{x}{d}=0.547>0.5$. Double reinforcement required. $d'=50+20=70$

By (Eqn 3-11)

$$\frac{A_{sc}}{b_w d}=\frac{1}{0.87f_y(1-d'/d)}\left[\frac{M}{b_w d^2}-\frac{0.67f_{cu}}{\gamma_m}\left[\left(\frac{b_{eff}}{b_w}-1\right)\frac{h_f}{d}\left(1-\frac{1}{2}\frac{h_f}{d}\right)+\eta\varphi\left(1-\frac{1}{2}\varphi\right)\right]\right]$$

$=0.001427=0.143\,\%$

$A_{sc}=763\,\text{mm}^2$ > 0.4% on flange as per Table 9.1 of the Code which is $0.004\times1250\times150=750\,\text{mm}^2$. Use 6T20

By (Eqn 3-12)

$$\frac{A_{st}}{b_w d}=\frac{0.67f_{cu}}{\gamma_m 0.87f_y}\left[\left(\frac{b_{eff}}{b_w}-1\right)\frac{h_f}{d}+\eta\varphi\right]+\frac{A_{sc}}{b_w d}=0.02614$$

$\Rightarrow A_{st}=13958\,\text{mm}^2$, Use 10-T40 + 2-T32 in 2 layers (2.65%)

3.6 Detailings of longitudinal steel for bending

The followings should be observed in placing of longitudinal steel bars for bending. Re Cl. 9.2.1 and 9.9.1 of the Code. The requirements arising from "ductility" requirements are marked with "D" for beams contributing in lateral load resisting system:

(i) Minimum tensile steel percentage : For rectangular beam, 0.13% in accordance with Table 9.1 of the Code and 0.3% in accordance with Cl.

9.9.1 of the Code (D); except for beams subject to pure tension which requires 0.45% as in Table 9.1 of the Code;

(ii) Maximum tension steel percentage : 2.5% (Cl. 9.9.1.1(a)) for beams contributing in lateral load resisting system(D); and 4% for others (Cl. 9.2.1.3 of the Code);

(iii) Minimum compressive steel percentage : When compressive steel is required for ultimate design, Table 9.1 of the Code should be followed by providing 0.2% for rectangular beam and different percentages for others. In addition, at any section of a beam within a critical zone (e.g. a potential plastic hinge zone as discussed in Section 2.4) the compression reinforcement ≥ one-half of the tension reinforcement in the same region (Cl. 9.9.1.1(a) of the Code) (D);

(iv) For flanged beam, Figure 3.10 is used to illustrate the minimum percentages of tension and compression steel required in various parts of flanged beams (Table 9.1 of the Code), but not less than 0.3% in accordance with Cl. 9.9.1.1(a) of the Code (D);

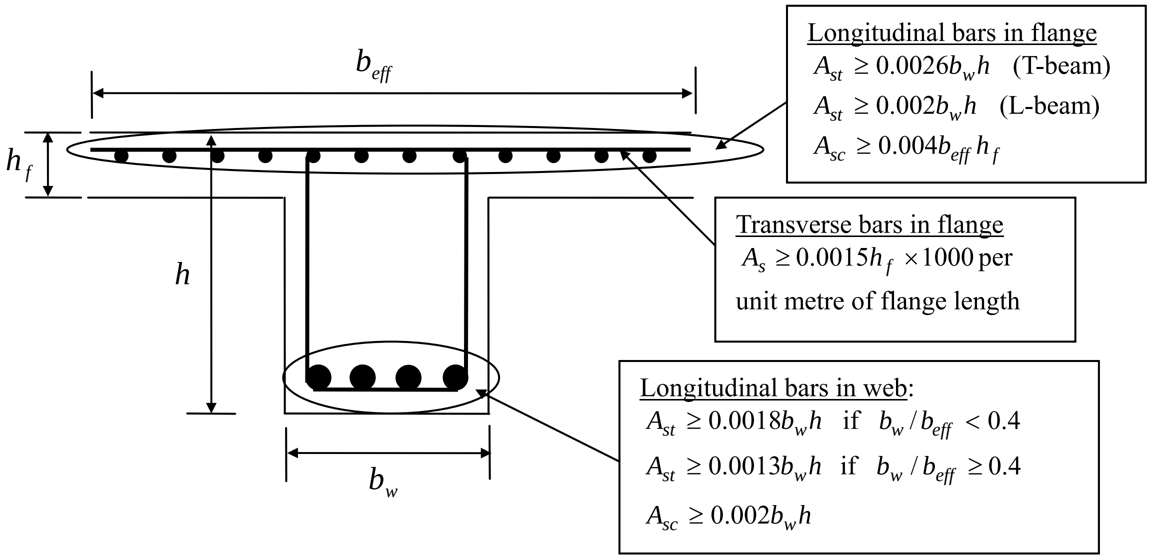


Figure 3.10 – Minimum steel percentages in various parts of flanged beams

(v) For beams contributing in lateral load resisting system, calculation of anchorage lengths of longitudinal bars anchored into exterior columns, bars must be assumed to be fully stressed as a ductility requirement according to Cl 9.9.1.1(c) of the Code. That is, stresses in the steel should be f_y instead of $0.87 f_y$ in the assessment of anchorage length. As such, the anchorage and lap lengths as indicated in Tables 8.4 and 8.5 of the Code should be increased by 15% as per (Ceqn 8.4)

of the Code in which $l_b \geq \dfrac{f_y \phi}{4 f_{bu}}$ which is a modification (by changing $0.87 f_y$ to f_y) where $f_{bu} = \beta \sqrt{f_{cu}}$ and β is 0.5 for tension anchorage and 0.63 for compression anchorage for high yield bars in accordance with Table 8.3 of the Code. Lap lengths can be taken as identical to anchorage lengths (D);

(vi) Full strength welded splices may be used in any location according to Cl. 9.9.1.1(d) of the Code;

(vii) For beams contributing in lateral load resisting system, no portion of the splice (laps and mechanical couplers) shall be located within the beam/column joint region or within one effective depth of the member from the critical section of a potential plastic hinge (discussed in Section 2.4) in a beam where stress reversals in lapped bars could occur (Cl. 9.9.1.1(d) of the Code). However, effects due to wind load need not be considered as creating stress reversal (D);

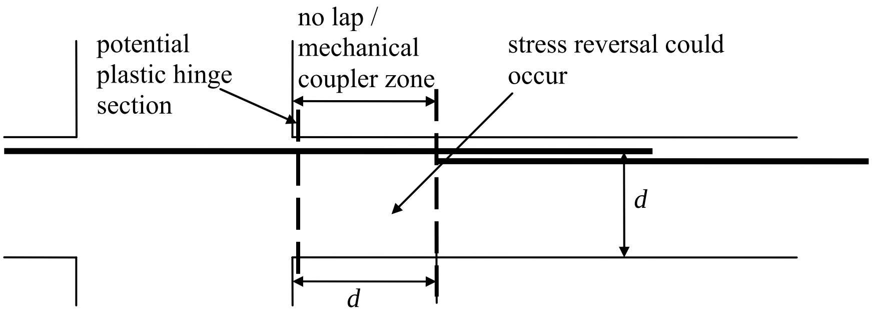


Figure 3.11 – Location of no lap / mechanical coupler zone in beam contributing to load resisting system

(viii) For beams contributing in lateral load resisting system, longitudinal bars shall not be lapped in a region where reversing stresses at the ultimate state may exceed $0.6 f_y$ in tension or compression unless each lapped bar is confined by links or ties in accordance with (Ceqn 9.6) reproduced as follows (D) :

$$\frac{A_{tr}}{s} \geq \frac{\phi \cdot f_y}{48 f_{yt}} \qquad \text{(Eqn 3-13)}$$

According to the definitions in the Code, ϕ is the diameter of the longitudinal bar; A_{tr} is the smaller of area of transverse reinforcement within the spacing s crossing a plane of splitting

normal to the concrete surface containing extreme tension fibres, or total area of transverse reinforcements normal to the layer of bars within a spacing s, divided by n (no. of longitudinal bars) in mm^2; s is the maximum spacing of transverse reinforcement within the lap length, f_{yt} is the characteristic yield strength of the transverse reinforcement.

As the "just adequate" design is by providing steel so that the reinforcing bars are at stress of $0.87f_y$, overprovision of the section by 0.87/0.6 – 1 = 45% of reinforcing bars at the laps should fulfill the requirement for lapping in regions with reversing stress. Or else, transverse reinforcement by (Ceqn 9.6) will be required. Figure 3.12 shows the occurrence of the plane of splitting at lapping.

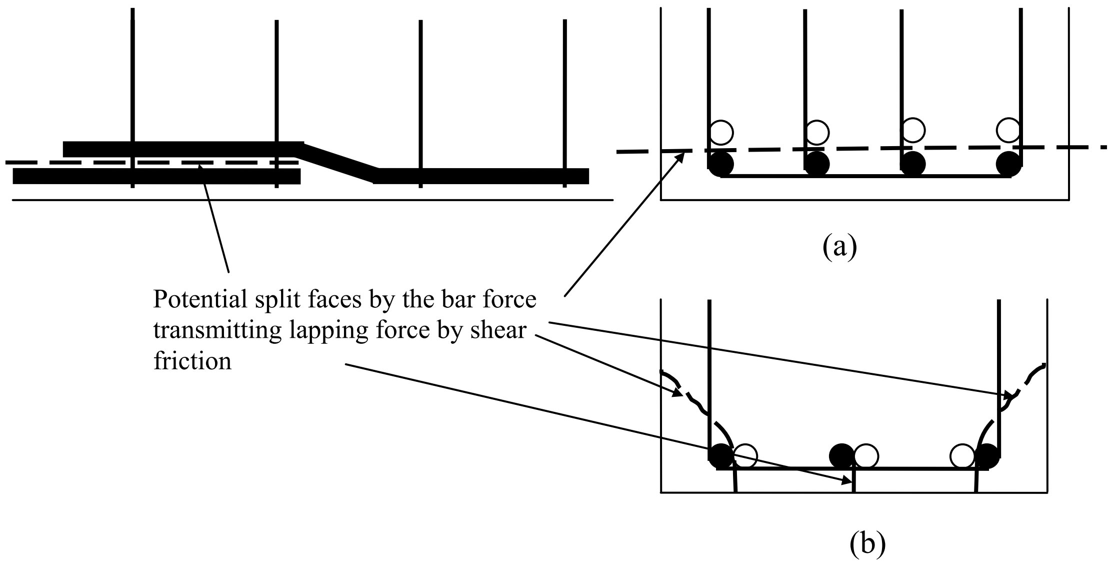


Figure 3.12 – splitting of concrete by shear friction in lapping of bars

Consider the example (a) illustrated in Figure 3.12, transverse reinforcement required will simply be $\frac{A_{tr}}{s} \geq \frac{\phi \cdot f_y}{48 f_{yt}} = \frac{\phi}{48}$ if high yield bars are used for both longitudinal and transverse reinforcements. If $\phi = 40$ (i.e. the longitudinal bars are T40), $\frac{A_{tr}}{s} \geq \frac{40}{48} = 0.833$. The total area of transverse reinforcement is $\sum A_{tr} = 4A_{tr}$ as there are 4

no. of bars. So $\frac{\sum A_{tr}}{s} \geq 0.833 \times 4 = 3.333$. Using T12 – 4 legs – 125 is adequate as $\frac{\sum A_{tr}}{s}$ provided is 3.619. It should be noted that case (b) is generally not the controlling case.

(ix) At laps in general, the sum of reinforcement sizes in a particular layer should not exceed 40% of the beam width as illustrated by a numerical example in Figure 3.13 (Cl. 9.2.1.3 of the Code);

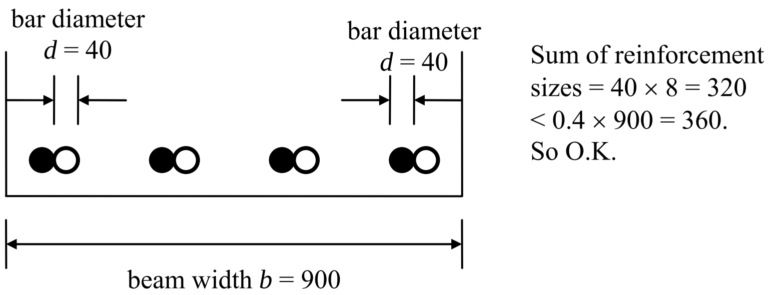


Figure 3.13 – Illustration of sum of reinforcement sizes at laps ≤ 0.4 of beam width

(x) Minimum clear spacing of bars should be the greatest of bar diameter, 20 mm and aggregate size + 5 mm (Cl. 8.2 of the Code);

(xi) Maximum clear spacing between adjacent bars near tension face of a beam ≤ $70000\beta_b/f_y$ ≤ 300 mm where β_b is the ratio of moment redistribution (ratio of moment after redistribution to moment before redistribution) or alternatively ≤ $47000/f_s$ ≤ 300 mm where $f_s = \frac{2 f_y A_{s,req}}{3A_{s,prov}} \times \frac{1}{\beta_b}$. Based on the former with β_b = 1 (no redistribution), the maximum clear spacing is 152 mm (Cl. 9.2.1.4 of the Code);

(xii) Requirements for containment of compression steel bars is identical to that of columns (Cl. 9.5.2.2 of the Code) : Every corner bar and each alternate bar (and bundle) in an outer layer should be supported by a link passing around the bar and having an included angle ≤ 135°. Links should be adequately anchored by means of hook through a bent angle ≥ 135°. No bar within a compression zone be more than 150 mm from a restrained bar (anchored by links of included angle ≥ 135°) as illustrated in Figure 3.14;

(xiii) No tension bars should be more than 150 mm from a vertical leg which is also illustrated in Figure 3.14 (Cl. 9.2.2 of the Code);

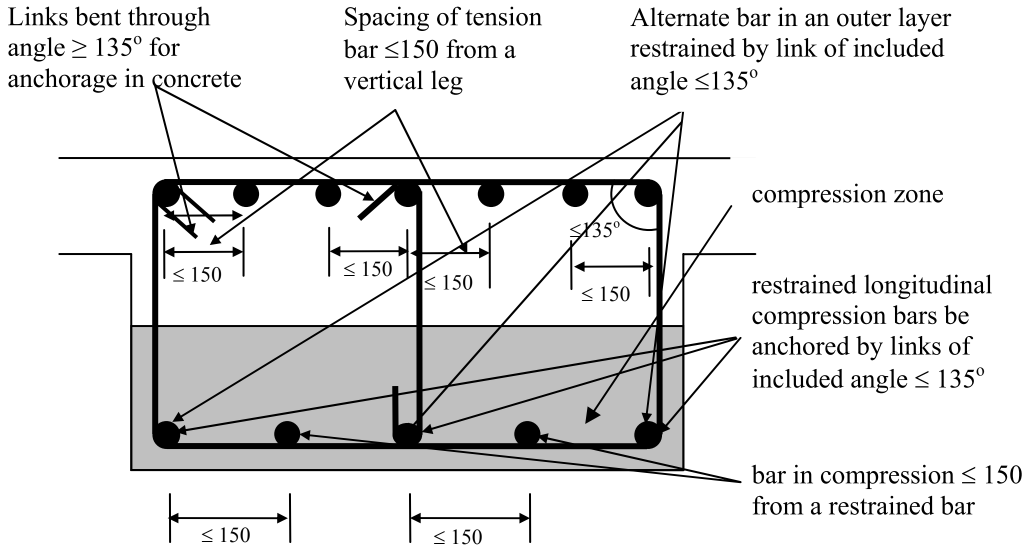


Figure 3.14 – Anchorage of longitudinal bar in beam section

(xiv) At an intermediate support of a continuous member, at least 30% of the calculated mid-span bottom reinforcement should be continuous over the support as illustrated in Figure 3.15 (Cl. 9.2.1.8 of the Code);

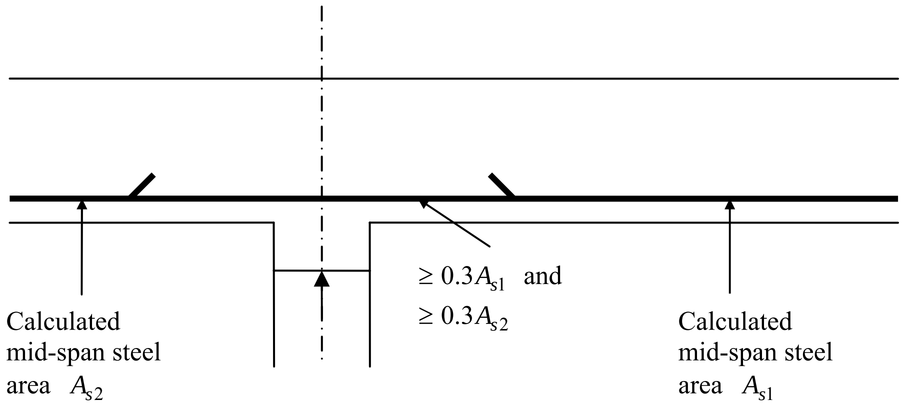


Figure 3.15 – At least 30% of the calculated mid-span bottom bars be continuous over intermediate support

(xv) In monolithic construction, simple supports should be designed for 15% of the maximum moment in span as illustrated in Figure 3.16 (Cl. 9.2.1.5 of the Code);

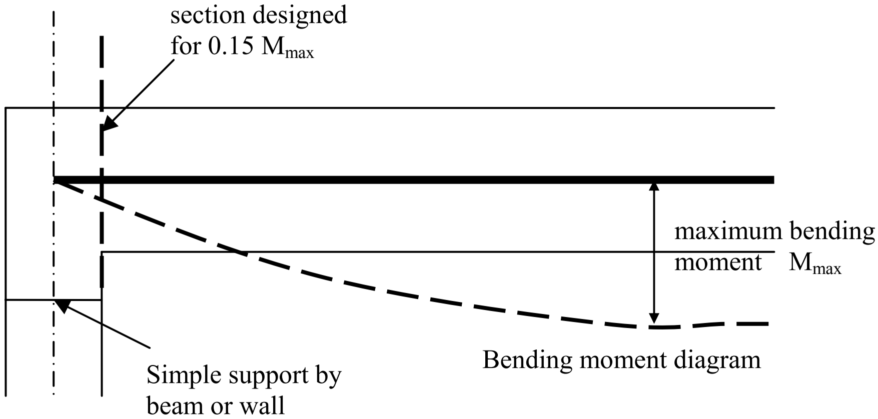


Figure 3.16 – Simple support be designed for 15% of the maximum span moment

(xvi) For flanged beam over intermediate supports, the total tension reinforcements may be spread over the effective width of the flange with at least 50% inside the web as shown in Figure 3.17 reproduced from Figure 9.1 of the Code;

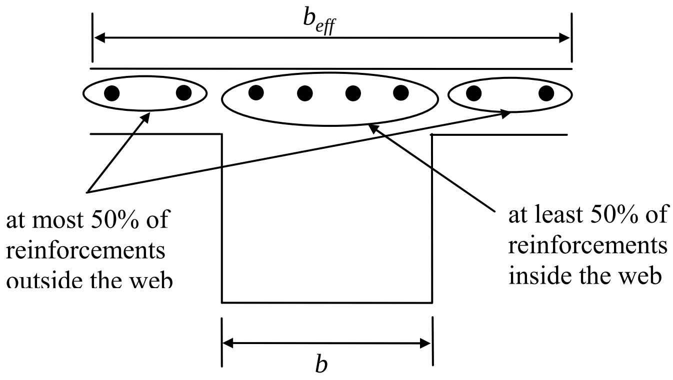
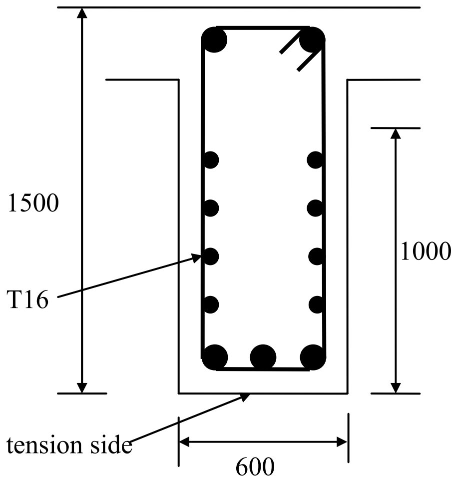


Figure 3.17 – distribution of tension rebars of flanged beam over support

(xvii) For beam with depths > 750 mm, provision of sides bars of size (in mm) ≥ $\sqrt{s_b b / f_y}$ where s_b is the side bar spacing (in mm) and b is the lesser of the beam breadth (in mm) under consideration and 500 mm. f_y is in N/mm^2. In addition, it is required that $s_b \le 250$ mm and side bars be distributed over two-thirds of the beam's overall depth measured from its tension face. Figure 3.18 illustrate a numerical example (Cl. 9.2.1.2 of the Code);



b is the lesser of 600 and 500, so
$b = 500$
s_b chosen to be 200 mm ≤ 250mm,
So size of side bar is
$\sqrt{s_b b / f_y} = \sqrt{200 \times 500 / 460}$
$= 14.74$
Use T16.
The side bars be distributed over
$\frac{2}{3} \times 1500 = 1000$ from bottom
which is the tension side.

Figure 3.18 – Example of determination of side bars

(xviii) When longitudinal bars of beams contributing to lateral load resisting system are anchored in cores of exterior columns or beam studs, the anchorage for tension shall be deemed to commence at the lesser of 1/2 of the relevant depth of the column or 8 times the bar diameter as indicated in Figure 3.19. In addition, notwithstanding the adequacy of the anchorage of a beam bar in a column core or a beam stud, no bar shall be terminated without a vertical 90° standard book or equivalent anchorage device as near as practically possible to the far side of the column core, or the end of the beam stud where appropriate, and not closer than 3/4 of the relevant depth of the column to the face of entry. Top beam bars shall be bent down and bottom bars must be bent up as indicated in Figure 3.19. (Cl. 9.9.1.1(c) of the Code) (D);

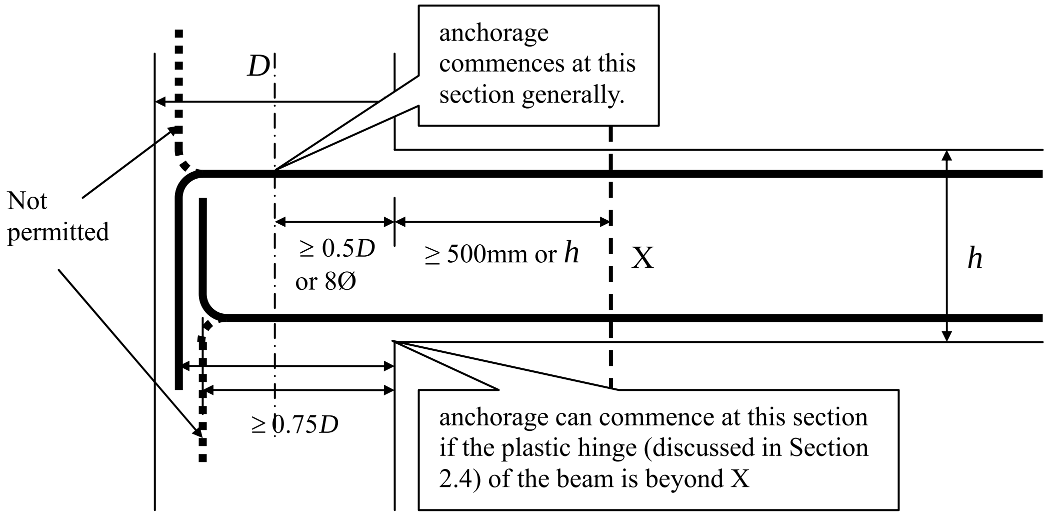


Figure 3.19 – Anchorage of reinforcing bars at support for beams contributing to lateral load resisting system

(xix) Beam should have a minimum support width by beam, wall, column as shown in Figure 3.20 as per Cl. 8.4.8 of the Code;

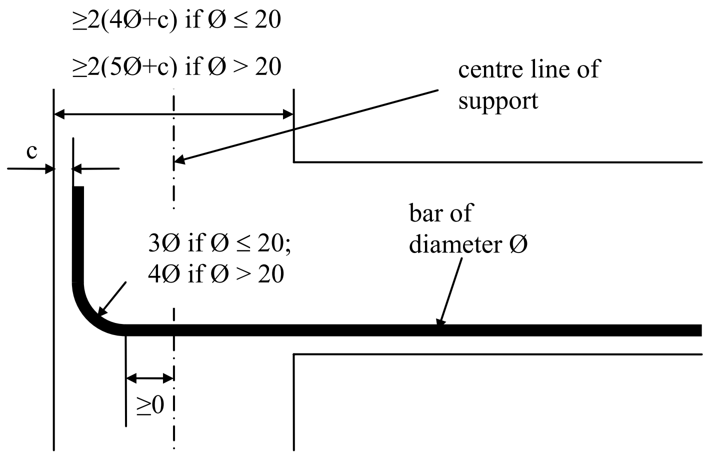


Figure 3.20 – Support width requirement

(xx) Curtailment of tension reinforcements except at end supports should be in accordance with Figure 3.21 (Cl. 9.2.1.6 of the Code).

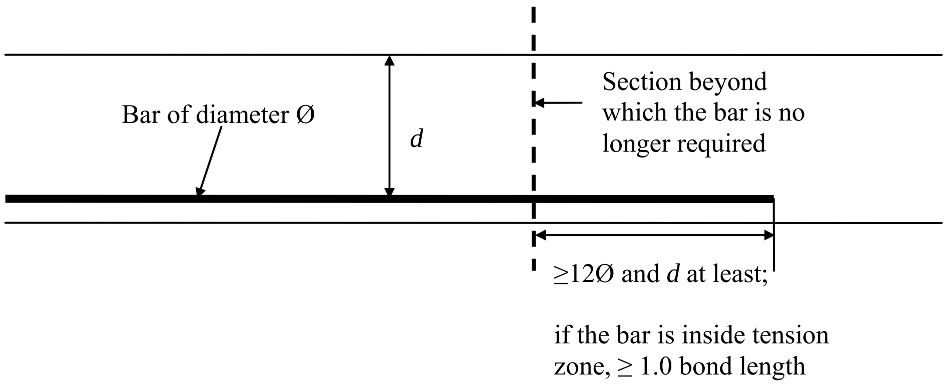


Figure 3.21 – curtailment of reinforcement bars

<u>Worked Example 3.10</u>

Worked example 3.10 is used to illustrate the arrangement of longitudinal bars and the anchorages on thin support for the corridor slab beam of a typical housing block which functions as coupling beam between the shear walls on both sides. Plan, section and dimensions are shown in Figure 3.22(a). Concrete grade is 35.

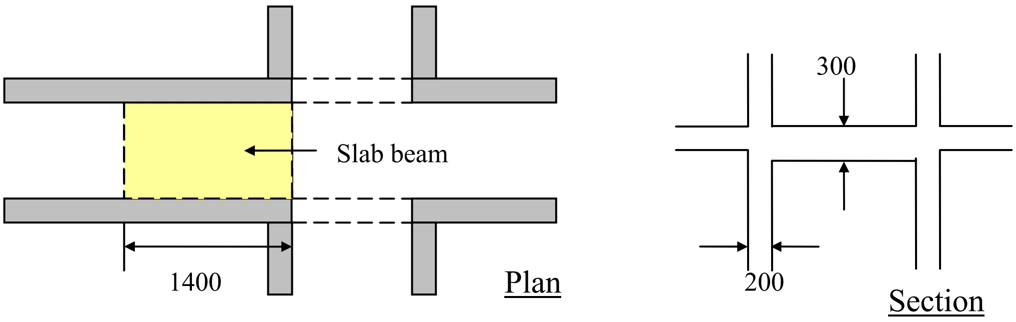


Figure 3.22(a) – Layout of the slab beam in Worked Example 3.10

The designed moment is mainly due to wind loads which is 352 kNm, resulting in required longitudinal steel area of 3910 mm^2 (each at top and bottom). The 200 mm thick wall can accommodate at most T16 bars as $2(4\times16+25)=178<200$ as per 3.6(xix). So use 20T16 (A_{st} provided is 4020 mm^2. Centre to centre bar spacing is $(1400-25\times2-16)/19=70$ mm.

For anchorage on support, lap length should be $34\times16=544$ mm. The factor 34 is taken from Table 8.4 of the Code which is used in assessing anchorage length. Anchorage details of the longitudinal bars at support are shown in Figure 3.22(b);

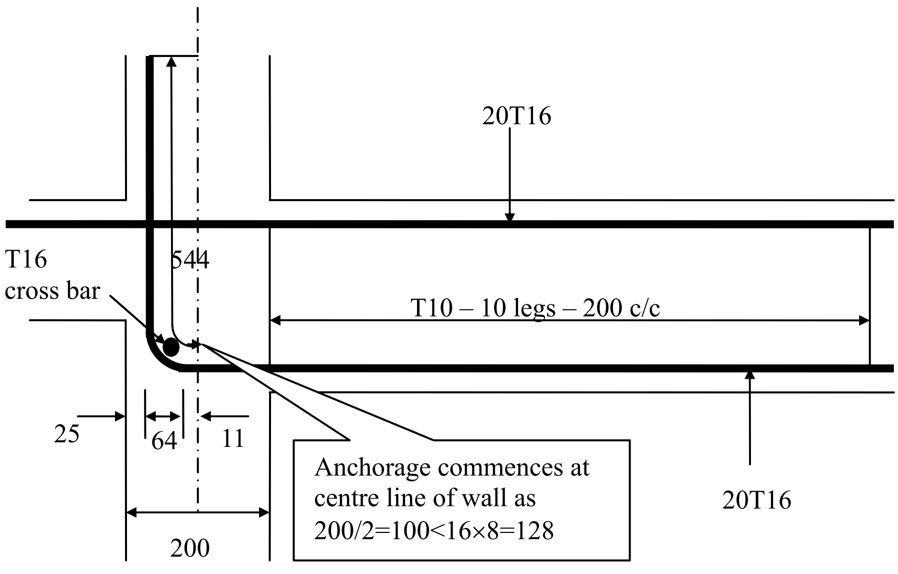


Figure 3.22(b) – Anchorage Details at Support for Worked Example 3.10

3.7 Design against Shear

3.7.1 Checking of Shear Stress and provision of shear reinforcements

Checking of shear in beam is based on the averaged shear stress calculated from (Ceqn 6.19)

$$v = \frac{V}{b_v d}$$

where v is the average shear stress, V is the ultimate shear, d is the effective depth of the beam and b_v is beam width. b_v should be taken as the averaged width of the beam below flange in case of flanged beam)

If v is greater than the values of v_c, termed "design concrete shear stress" in Table 6.3 of the Code which is determined by the formula

$v_c = 0.79\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}\left(\frac{100A_s}{b_v d}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}}\frac{1}{\gamma_m}$ listed in Table 6.3 of the Code with the following limitations :

(i) $\gamma_m = 1.25$;

(ii) $\frac{100A_s}{b_v d}$ should not be taken as greater than 3;

(iii) $\left(\frac{400}{d}\right)^{\frac{1}{4}}$ should not be taken as less than 0.67 for member without shear reinforcements and should not be taken as less than 1 for members with minimum links. For others, calculate according to the expression;

Then shear links of $\frac{A_{sv}}{s_v} = \frac{b_v(v - v_c)}{0.87 f_{yv}} \geq \frac{b_v v_r}{0.87 f_{yv}}$ should be provided (Table 6.2 of the Code) where $v_r = 0.4$ for $f_{cu} \leq 40$ MPa and $0.4(f_{cu}/40)^{2/3}$ for $f_{cu} > 40$, or alternatively, less than half of the shear resistance can be taken up by bent up bars by $0.5V \geq V_b = A_{sb}(0.87 f_{yv})(\cos\alpha + \sin\alpha \cot\beta)\frac{d - d'}{s_b}$ as per (Ceqn 6.20) and Cl. 6.1.2.5(e) of the Code and the rest by vertical links.

Maximum shear stress not to exceed $v_{tu} = 0.8\sqrt{f_{cu}}$ or 7 MPa, whichever is the lesser by Cl. 6.1.2.5(a).

3.7.2 Minimum shear reinforcements (Table 6.2 of the Code)

If $v < 0.5v_c$, no shear reinforcement is required;

If $0.5v_c < v < (v_c + v_r)$, minimum shear links of $\frac{A_{sv}}{s_v} = \frac{b_v v_r}{0.87 f_{yv}}$ along the

whole length of the beam be provided where $v_r = 0.4$ for $f_{cu} \le 40$ and $0.4(f_{cu}/40)^{2/3}$ for $f_{cu} > 40$, but not greater than 80;

3.7.3 Enhanced shear strength close to support (Cl. 6.1.2.5(g))

At sections of a beam at distance $a_v \le 2d$ from a support, the shear strength can be increased by a factor $\frac{2d}{a_v}$, bounded by the absolute maximum of $v_{tu} = 0.8\sqrt{f_{cu}}$ or 7 MPa as illustrated by Figure 3.23.

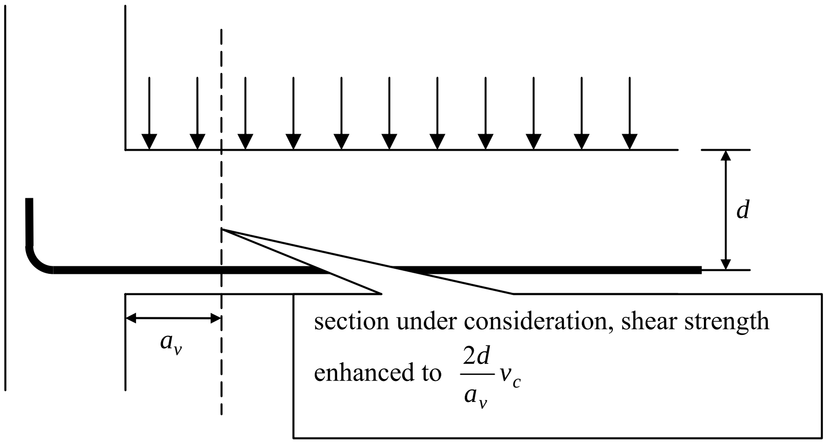


Figure 3.23 – Shear enhancement near support

3.7.4 Where load is applied to the bottom of a section, sufficient vertical reinforcement to carry the load should be provided in addition to any reinforcements required to carry shear as per Cl. 6.1.2.5(j);

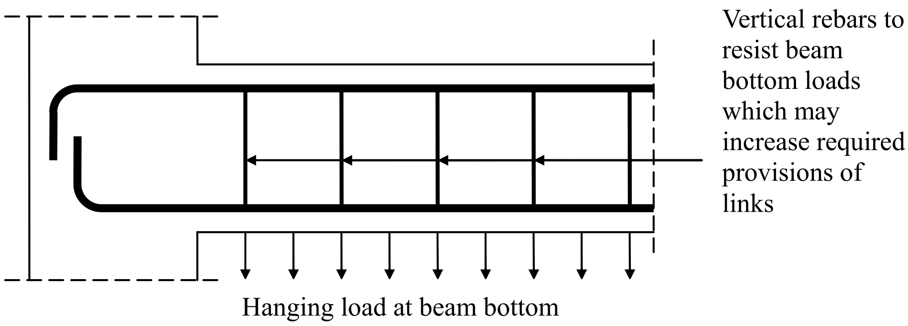


Figure 3.24 – Vertical rebars to resist hanging load at beam bottom (e.g. inverted beam)

3.7.5 Worked Examples for Shears

(i) Worked Example 3.11 – shear design without shear enhancement in concrete

Section : $b = 400$ mm;

$d = 700 - 40 - 16 = 644$ mm; $\dfrac{100A_{st}}{bd} = 1.5$; $f_{cu} = 35$ MPa;

$V = 700$ kN;

$$v_c = 0.79\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}\left(\frac{100A_s}{b_v d}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}}\frac{1}{\gamma_m} = 0.81\,\text{MPa};$$

where $\left(\dfrac{400}{d}\right)^{\frac{1}{4}}$ be kept as unity for $d > 400$.

$$v = \frac{700\times10^3}{400\times644} = 2.72\text{ MPa},$$

$$\frac{A_{sv}}{s_v} = \frac{b(v - v_c)}{0.87f_{yv}} = \frac{400(2.72 - 0.81)}{0.87\times460} = 1.91;$$ Use T12 – 200 c/c d.s.

(ii) Worked Example 3.12 – shear design with shear enhancement in concrete.

Re Figure 3.25 in which a section of 0.75 m from a support, as a heavy point load is acting so that from the support face to the point load along the beam, the shear is more or less constant at 700 kN.

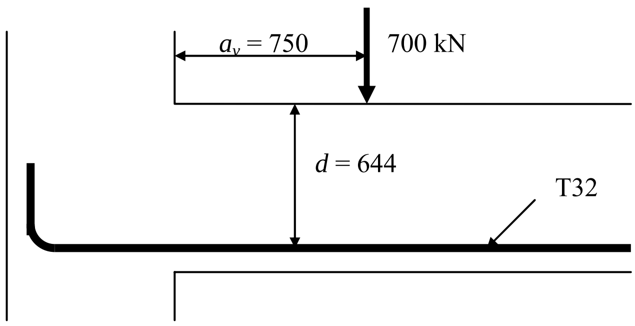


Figure 3.25 – Worked Example 3.11

Section : $b = 400$ mm; cover to main reinforcement = 40 mm;

$d = 700 - 40 - 16 = 644$ mm; $\dfrac{100A_{st}}{bd} = 1.5$; $f_{cu} = 35$ MPa; $V = 700$ kN;

$$v_c = 0.79\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}\left(\frac{100A_s}{b_v d}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}}\frac{1}{\gamma_m} = 0.81\,\text{MPa}$$ as in Worked Example 3.11.

Concrete shear strength enhanced to $\frac{2d}{a_v}v_c = \frac{2\times 644}{750}\times 0.81 = 1.39\,\text{MPa} <$ 7 MPa and $0.8\sqrt{f_{cu}} = 0.8\sqrt{35} = 4.7\,\text{MPa}$

$$v = \frac{700\times 10^3}{400\times 644} = 2.72\,\text{MPa},$$

$$\frac{A_{sv}}{s_v} = \frac{b(v - v_c)}{0.87 f_{yv}} = \frac{400(2.72 - 1.39)}{0.87\times 460} = 1.33;$$ Use T12 – 150 c/c s.s

(iii) Worked Example 3.13 – inclusion of bent-up bars (Cl. 6.1.25(e) of the Code)

If half of the shear resisted by steel in Worked Example 3.11 is taken up by bent-up bars, i.e. $0.5\times(2.72 - 0.81)\times 400\times 644\times 10^{-3} = 246\,\text{kN}$ to be taken up by bent-up bars as shown in Figure 3.26.

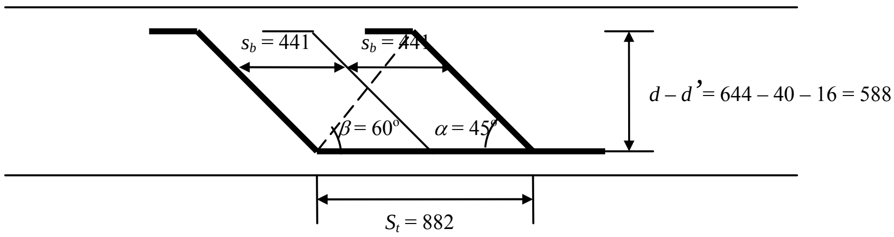


Figure 3.26 – Worked Example 3.12

By (Ceqn 6.20) of the Code,

$$V_b = A_{sb}(0.87 f_{yv})(\cos\alpha + \sin\alpha\cot\beta)\frac{d - d'}{s_b}$$

$$\Rightarrow A_{sb} = \frac{246000\times 441}{0.87\times 460(\cos 45^0 + \sin 45^0\cot 60^0)\times 588} = 413\,\text{mm}^2$$

Use 6 nos. of T10 at spacing of s_b = 441 mm as shown.

3.8 Placing of Shear reinforcements

The followings should be observed for the placing of shear reinforcements :

(i) Bar size ≥ the greater of 1/4 of the bar size of the maximum longitudinal bar and 6 mm (BS8110 Cl. 3.12.7.1);

(ii) The minimum provision of shear reinforcements (links or bent up bars) in beams should be given by $A_{sv} \geq \frac{v_r b_v s_v}{0.87 f_{yv}}$ where $v_r = 0.4$ for $f_{cu} \leq 40$ and $v_r = 0.4(f_{cu}/40)^{2/3}$ for $80 \geq f_{cu} > 40$ (Cl. 6.1.2.5(b) of the Code);

(iii) At least 50% of the necessary shear reinforcements be in form of links (Cl. 6.1.2.5(e) of the Code);

(iv) The maximum spacing of links in the direction of span of the beam should be the least of the followings as illustrated by a numerical example in Figure 3.27 (Cl. 6.1.2.5(d), 9.2.1.10, 9.5.2.1 (BS8110 Cl. 3.12.7.1), 9.5.2.2, 9.9.1.2(a) of the Code) :

(a) $0.75d$;

(b) the least lateral dimension of the beam (D);

(c) 16 times the longitudinal bar diameter (D);

(d) 12 times the smallest longitudinal bar diameter for containment of compression reinforcements.

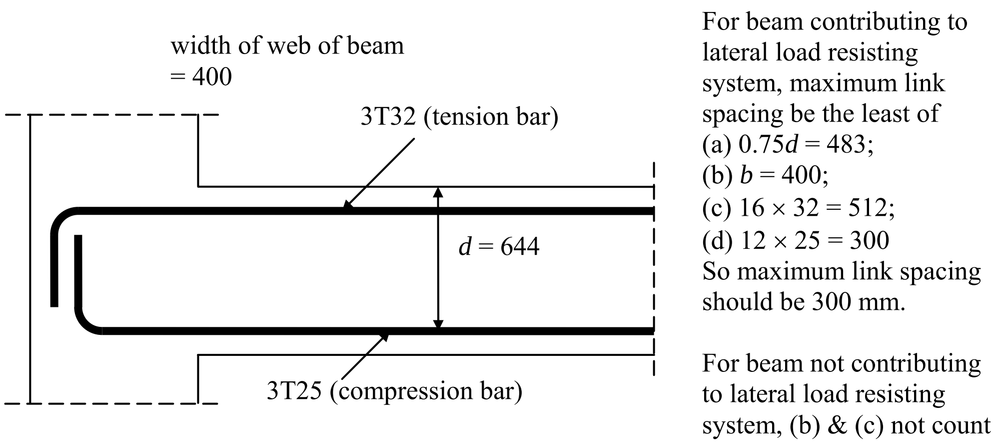


Figure 3.27 – Maximum spacing of shear links in the span direction of beam

(v) At right angle of the span, the horizontal spacing of links should be such that no longitudinal tension bar should be more than 150 mm from a vertical leg and $\leq d$ as per Cl. 6.1.2.5(d) of the Code and shown in Figure 3.28;

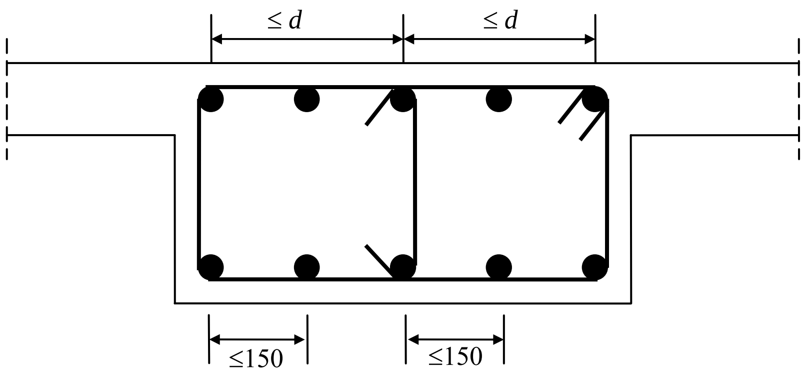


Figure 3.28 – Maximum spacing of shear links at right angle to the span direction of beam

(vi) Links or ties shall be arranged so that every corner and alternate longitudinal bar that is required to function as compression reinforcement shall be restrained by a leg as illustrated Figure 3.14;

(vii) By Cl. 9.9.1.2(b) of the Code, links in beams contributing to lateral load resisting system should be adequately anchored by means of 135^{o} or 180^{o} hooks in accordance with Cl. 8.5 of the Code as shown in Figure 3.29 (D);

(viii) Anchorage by means of 90^{o} hook is only permitted for tensile steel in beams not contributing to lateral load resisting system;

(ix) Links for containment of compression longitudinal bars in general must be anchored by hooks with bent angle ≥ 135^{o} in accordance with Cl. 9.2.1.10 and 9.5.2.1 of the Code. Links with different angles of hooks are shown in Figure 3.29. (Reference to Cl. 9.5.2.1 should be included in Cl. 9.2.1.10 as per Cl. 3.12.7.1 of BS8110)

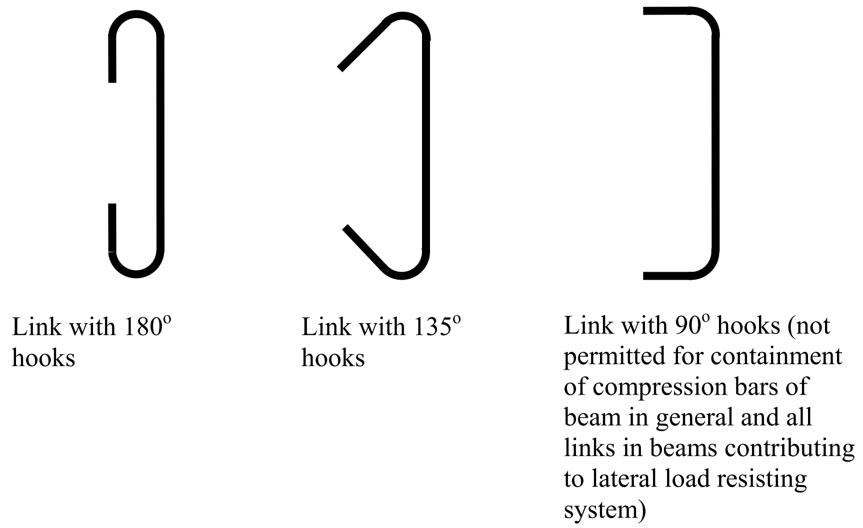


Figure 3.29 – Links with hooks with different bent angles

3.9 Design against Torsion

3.9.1 By Cl. 6.3.1 of the Code, in normal slab-and-beam and framed construction, checking against torsion is usually not necessary. However, checking needs be carried out if the design relies entirely on the torsional resistance of a member such as that indicated in Figure 3.30.

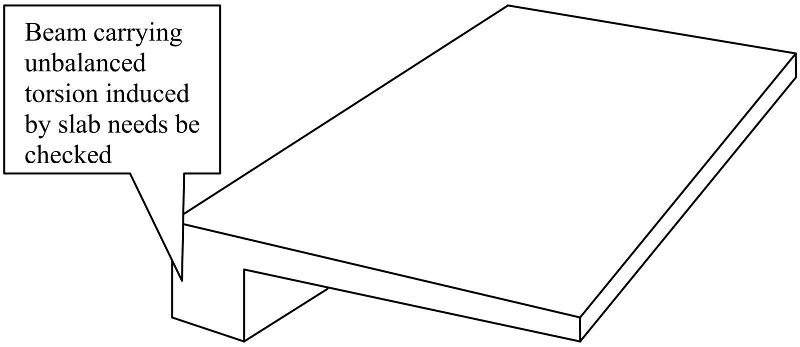


Figure 3.30 – Illustration for necessity of checking against torsion

3.9.2 Calculation of torsional rigidity of a rectangular section for analysis (in grillage system) is by (Ceqn 6.64) of the Code

$C = \frac{1}{2}\beta h_{min}{}^3 h_{max}$ where β is to be read from Table 6.16 of the Code reproduced as Table 3.4 of this Manual.

h_{max}/h_{min}	1	1.5	2	3	5	>5
β	0.14	0.20	0.23	0.26	0.29	0.33

Table 3.4 – Values of coefficient β

3.9.3 Calculation of torsional shear stress

Upon arrival of the torsion on the rectangular section, the torsional shear stress is calculated by (Ceqn 6.65) of the Code

$$v_t = \frac{2T}{h_{min}{}^2\left(h_{max} - \frac{h_{min}}{3}\right)}$$

and in case of a section such as T or L sections made of rectangles, the section should be broken up into two or more rectangles such that the $\sum h_{min}{}^3 h_{max}$ is maximized and the total Torsional moment T be apportioned to each rectangle in accordance with (Ceqn 6.66) of the Code as $T \times \left(\frac{h_{min}{}^3 h_{max}}{\sum h_{min}{}^3 h_{max}}\right)$.

If the torsional shear stress exceeds $0.067\sqrt{f_{cu}}$ (but not more than 0.6MPa), torsional reinforcements will be required (Table 6.17 of the Code).

Furthermore, the torsional shear stress should be added to the shear stress induced by shear force to ensure that the absolute maximum $v_{tu} = 0.8\sqrt{f_{cu}}$ or 7MPa is not exceeded, though for small section where y_1 (the larger centre-to-centre dimension of a rectangular link) < 550mm, v_{tu} will be decreased by a factor $y_1/550$. Revision of section is required if the absolute maximum is exceeded (Table 6.17 of the Code).

3.9.4 Calculation of torsional reinforcements

Torsional reinforcement in forms of close rectangular links and longitudinal bars are to be calculated by (Ceqn 6.67) and (Ceqn 6.68) of the Code as

$$\frac{A_{sv}}{s_v} = \frac{T}{0.8x_1y_1(0.87f_{yv})} \qquad \text{(Ceqn 6.67)}$$

(A_{sv} is the area of the 2 legs of the link)

$$A_s = \frac{A_{sv}f_{yv}(x_1+y_1)}{s_vf_y} \qquad \text{(Ceqn 6.68)}$$

It should be noted that there is no reduction by shear strength (v_c) of concrete. The derivation of the design formula (Ceqn 6.67) of the Code for close rectangular links is under the assumption of a shear rupture length of stirrup width + stirrup depth x_1+y_1 as shown in Figure 3.31. A spiral torsional failure face is along the heavy dotted line in the figure. It is also shown in the figure that the torsional moment of resistance by the stirrups within the Regions X and Y are identical and is the total resistance is therefore $\dfrac{A_{sv}0.87f_yx_1y_1}{s_v}$. So $T = \dfrac{A_{sv}0.87f_yx_1y_1}{s_v} \Rightarrow \dfrac{A_{sv}}{s_v} = \dfrac{T}{0.87f_yx_1y_1}$. An additional factor of 0.8 is added and the equation becomes (Ceqn 6.67) by which $\dfrac{A_{sv}}{s_v} = \dfrac{T}{0.8x_1y_1(0.87f_y)}$. The derivation of the longitudinal bars is based on the use of same quantity of longitudinal bars as that of stirrups with even distribution along the inside of the stirrups. Nevertheless, the Code allows merging of the flexural steel with these longitudinal bars by using larger

diameter of bars as will be illustrated in the Worked Example 3.14.

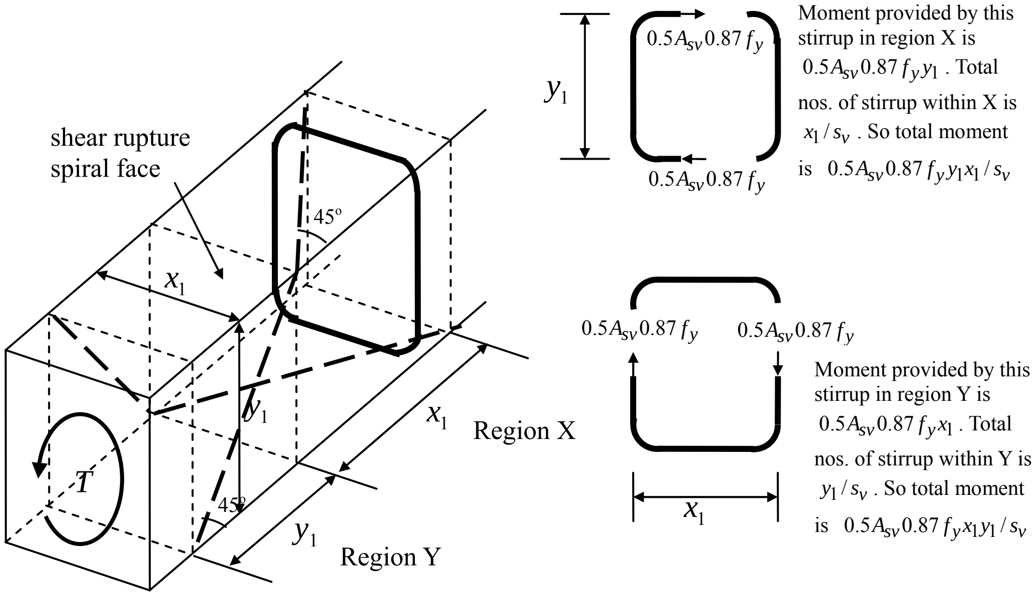


Figure 3.31 – Derivation of Formulae for torsional reinforcements

3.9.5 Worked Example 3.14 – Design for T-beam against torsion

A total torsion of $T = 200$ kNm on a T-section as shown in Figure 3.32 with an average vertical shear stress on the web of 0.82 N/mm^2. The section is also under bending requiring flexural steel area of 2865 mm^2 at bottom. Concrete grade is 35.

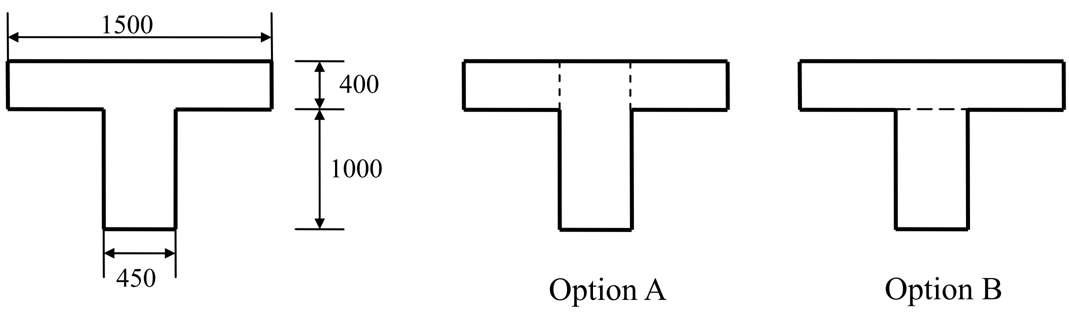


Figure 3.32 – Section of a T section resisting torsion for Worked Example 3.14

For vertical shear, taking $\dfrac{100A_s}{b_v d} = \dfrac{2865 \times 100}{450 \times 1334} = 0.477$

$$v_c = 0.79\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}\left(\frac{100A_s}{b_v d}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}}\frac{1}{\gamma_m} = 0.55$$, again taking $\left(\frac{400}{d}\right)^{\frac{1}{4}}$ as unity.

$$\frac{A_{sv}}{s_v} = \frac{b(v - v_c)}{0.87 f_{yv}} = \frac{450(0.82 - 0.55)}{0.87 \times 460} = 0.3$$

For torsion, Option A is made up of two rectangles of 525×400 and one rectangle of 450×1400.

$$\therefore \left(\sum h_{\min}{}^3 h_{\max}\right)_{optionA} = 2 \times 525 \times 400^3 + 1400 \times 450^3 = 1.94775 \times 10^{11}\,\text{mm}^4$$

Option B is made up of one rectangle of 1500×400 and one rectangle of 450×1000.

$$\therefore \left(\sum h_{\min}{}^3 h_{\max}\right)_{optionB} = 1500 \times 400^3 + 1000 \times 450^3 = 1.87125 \times 10^{11}\,\text{mm}^4$$

As Option A has a larger torsional stiffness, it is adopted for design.

The torsional moment is apportioned to the three rectangles of Option A as :

For the two 525×400 rectangles $T_1 = 200 \times \frac{525 \times 400^3}{1.94775 \times 10^{11}} = 34.50\,\text{kNm}$;

Torsional shear stress is

$$v_{t1} = \frac{2T_1}{h_{\min}{}^2\left(h_{\max} - \frac{h_{\min}}{3}\right)} = \frac{2 \times 34.5 \times 10^6}{400^2\left(525 - \frac{400}{3}\right)} = 1.101\,\text{N/mm}^2$$

$> 0.067\sqrt{f_{cu}} = 0.396\,\text{N/mm}^2$ (< 0.6 N/mm^2)

So torsional shear reinforcement is required

$x_1 = 400 - 40 \times 2 - 6 \times 2 = 308$; $y_1 = 525 - 40 \times 2 - 6 \times 2 = 433$

$$\frac{A_{sv}}{s_v} = \frac{T_1}{0.8x_1 y_1(0.87 f_{yv})} = \frac{34.5 \times 10^6}{0.8 \times 308 \times 433 \times 0.87 \times 460} = 0.808$$

Use <u>T12 – 200 C.L.</u> $x_1 = 308$, $y_1/2 = 525/2 = 262.5$; use $s_v = 200 \le 200$; $\le x_1$ and $\le y_1/2$ as per Cl. 6.3.7 of the Code.

$$A_s = \frac{A_{sv} f_{yv}(x_1 + y_1)}{s_v f_y} = \frac{0.808 \times 460 \times (308 + 525)}{460} = 673\,\text{mm}^2$$

Use <u>4T16</u>

For the 1400×450 rectangle $T_2 = 200 \times \dfrac{1400 \times 450^3}{1.94775 \times 10^{11}} = 131\,\text{kNm}$

$$v_{t1} = \frac{2T_2}{h_{\min}{}^2\left(h_{\max} - \dfrac{h_{\min}}{3}\right)} = \frac{2 \times 131 \times 10^6}{450^2\left(1400 - \dfrac{450}{3}\right)} = 1.035\,\text{N/mm}^2$$

The total shear stress is $1.035 + 0.82 = 1.855\,\text{N/mm}^2 < v_{tu} = 4.73\,\text{MPa}$

As $1.035 > 0.067\sqrt{f_{cu}} = 0.396$ N/mm^2, torsional shear reinforcement is required.

$x_1 = 450 - 40 \times 2 - 6 \times 2 = 358\,\text{mm}$; $y_1 = 1400 - 40 \times 2 - 6 \times 2 = 1308\,\text{mm}$

$$\frac{A_{sv}}{s_v} = \frac{T_2}{0.8x_1y_1(0.87f_{yv})} = \frac{131 \times 10^6}{0.8 \times 358 \times 1308 \times 0.87 \times 460} = 0.87\,\text{mm}$$

Adding that for vertical shear, total $\dfrac{A_{sv}}{s_v} = 0.87 + 0.3 = 1.17$

Use <u>T12 – 175 C.L.</u> $x_1 = 358$, $y_1/2 = 1308/2 = 654$; use $s_v = 175 \le 200$; $\le x_1$ and $\le y_1/2$ as per Cl. 6.3.7 of the Code.

It should be noted that the torsional shear link should be closed links of shape as indicated in Figure 9.3 of the Code.

$$A_s = \frac{A_{sv}f_{yv}(x_1 + y_1)}{s_v f_y} = \frac{0.87 \times 460 \times (358 + 1308)}{460} = 1449\,\text{mm}^2.$$ Use <u>13T12</u>

Incorporating the bottom 3T12 into the required flexural steel, the bottom steel area required is $2865 + 113.1 \times 3 = 3205\,\text{mm}^2$. So use <u>4T32</u> at bottom and 10T12 at sides.

The sectional details is shown in Figure 3.33.

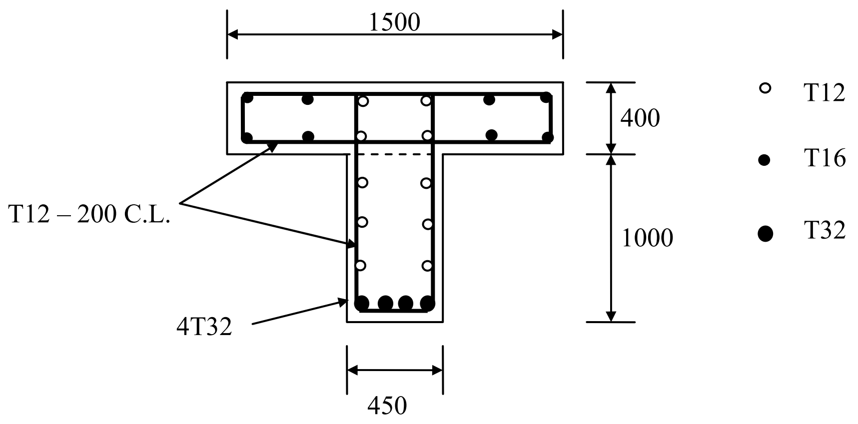


Figure 3.33 – Arrangement of torsional reinforcements

It should be borne in mind that these torsional reinforcements are in addition to others required for flexure and shear etc.

3.10 Placing of Torsional reinforcements

The followings (in Cl. 6.3.7, Cl. 6.3.8 and Cl. 9.2.3 of the Code) should be observed for the placing of shear reinforcements :

(i) The torsional shear link should form the closed shape as in Figure 9.1 of the Concrete Code Handbook reproduced as Figure 3.34. It should be noted that the second configuration in the Figure is not included in Figure 9.3 of the Code though it should also be acceptable;

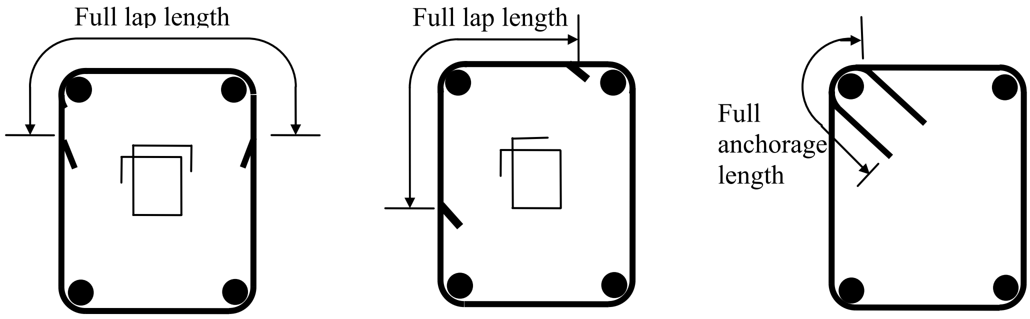


Figure 3.34 – Shape of Torsional shear links

(ii) The value s_v for the closed link should not exceed the least of x_1, $y_1/2$ or 200 mm as per Cl. 6.3.7 of the Code;

(iii) In accordance with Cl. 9.2.3 of the Code, provision of the longitudinal torsion reinforcement should comply the followings :

(a) The bars distributed should be evenly round the inside perimeter of the links as illustrated in Figure 3.33;

(b) Clear distance of the bars not to exceed 300 mm;

(c) Additional longitudinal bars required at the level of the tension or compression reinforcements may be provided by using larger bars than those required for bending alone, as illustrated in Worked Example 3.14 and Figure 3.33;

(d) The longitudinal bars should extend a distance at least equal to the largest dimension of the section beyond where it theoretically ceases to be required.

4.0 Slabs

4.1 Types of Slabs

Slabs can be classified as “one way slab”, “two way slab”, “flat slab”, “ribbed slab” with definition in Cl. 5.2.1.1 of the Code.

4.1.1 One way slab is defined by the Code as one subjected predominantly to u.d.l. either

(i) it possesses two free and parallel edge; or

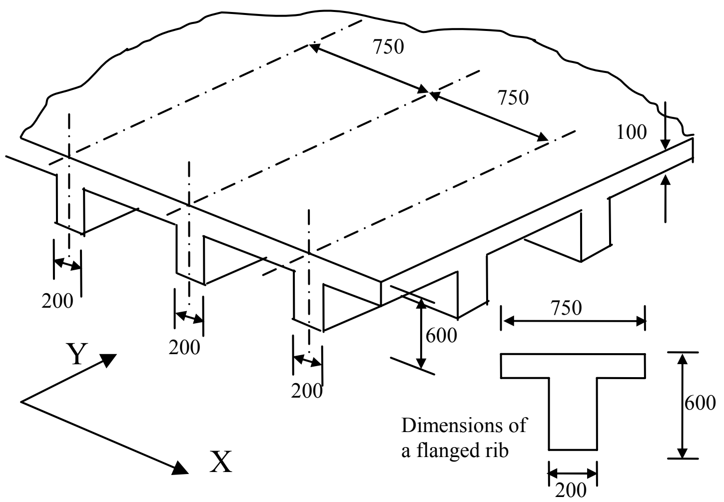
(ii) it is the central part of a rectangular slab supported on four edges with a ratio of the longer to the shorter span greater than 2.

4.1.2 Two way slab is a rectangular one supported on four sides with length to breadth ratio smaller than 2.

4.1.3 Flat slab is a slab supported on columns without beams.

4.1.4 Ribbed or Waffled Slab is a slab with topping or flange supported by closely spaced ribs. The Code allows idealization of the ribbed slab or waffled slab as a single slab without treatment as discretized ribs and flanges in analysis in Cl. 5.2.1.1(d) of the Code. If the stiffness of the ribbed or waffled slab is required for input, the bending stiffness in the X and Y directions can be easily found by summing the total bending stiffness of the composite ribs and flange structure per unit width as illustrated in Figure 4.1. The twisting stiffness is more difficult to assess. However, it should be acceptable to set the twisting stiffness to zero which will end up with pure bending in the X and Y directions as the slab, with its ribs running in the X and Y directions are clearly predominantly strong in bending in the two directions.

Figure 4.1 illustrates the computation of “I” value of a waffle slab about the X-direction which is the total stiffnesses of the nos. of “flanged ribs” within one metre. “I” value in the Y-directions can be worked out similarly.



Centroid of a flanged rib is at

$$\frac{0.55\times0.1^2/2+0.2\times0.6^2/2}{0.55\times0.1+0.2\times0.6}$$

=0.2214m from top

I of a rib is

$$\frac{1}{12}\left(0.55\times0.1^3+0.2\times0.6^3\right)$$
$$+0.55\times0.1\times(0.2214-0.05)^2$$
$$+0.2\times0.6\times(0.3-0.2214)^2$$
$$=0.006\text{m}^4$$

Within one metre, there will be 1000/750 = 1.333 nos. of ribs. So the I per metre width is $0.006\times1.333 = 0.008$ m^4/m

Figure 4.1 – Illustration of calculation of I value about X-direction of a waffle slab

4.2 Analysis of Slabs without the use of computer method

4.2.1 One way slab can be analyzed as if it is a beam, either continuous or single span. As we aim at simple analysis for the slab, we tend to treat it as a single element without the necessity to consider the many loading cases for continuous spans, Cl. 6.1.3.2(c) of the Code allows the design against moment and shear arising from the single-load case of maximum design load on all spans provided that :

(i) the area of each bay (defined in Figure 6.5 of the Code and reproduced in Figure 4.2) > 30 m^2;

(ii) the ratio of the characteristic imposed load to characteristic dead load ≤ 1.25; and

(iii) the characteristic imposed load ≤ 5 kN/m^2 excluding partitions.

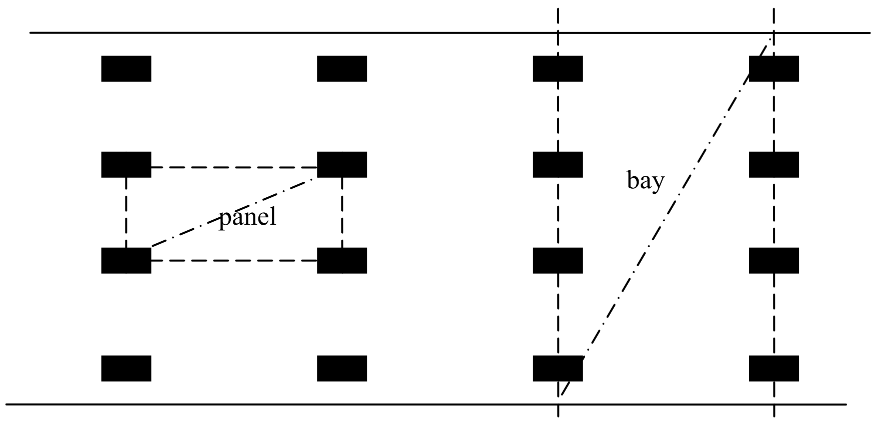


Figure 4.2 – Definition of panels and bays

4.2.2 Two way rectangular slab is usually analyzed by treating it as if it is a single slab in the absence of computer method. Bending moment coefficients for calculation of bending moments are presented in Table 6.6 of the Code for different support restraint conditions. Nevertheless, simplified formulae for the bending coefficients in case of rectangular simply supported two way slab are available in the Code (Ceqn 6.26 and 6.27) and reproduced as follows :
$m_x = \alpha_{sx} n l_x^{\,2}$ and $m_y = \alpha_{sy} n l_x^{\,2}$ where n is the u.d.l. l_x and l_y are respectively the shorter and longer spans and

$$\alpha_{sx} = \frac{(l_y / l_x)^4}{8\left[1 + (l_y / l_x)^4\right]}; \quad \alpha_{sy} = \frac{(l_y / l_x)^2}{8\left[1 + (l_y / l_x)^4\right]}.$$

4.2.3 Flat slabs, if of regular arrangement, can be analyzed as frames in the transverse and longitudinal directions by such methods as moment distribution method as if they are separate frames. Analyzed moments and shears should be apportioned to the "column strip" and "Middle strip" as per Table 6.10 of the Code. In addition, the bending moment and shear force coefficients for the one way slab can also be used as a simplified approach.

4.2.4 More bending moment and shear force coefficients of rectangular slabs with various different support and loading conditions can be found from other published handbooks, the most famous one being "Tables for the Analysis of Plates, Slabs and Diaphragms based on the Elastic Theory".

4.3 Analysis of Slabs with the use of the computer method

Analysis of slabs with the use of the computer method is mainly by the finite element method in which the slab is idealized as an assembly of discrete "plate bending elements" joined at nodes. The support stiffnesses by the supporting walls and columns are derived as similar to that for beams as "sub-frames". A complete set of results including bending moments, twisting moment, shear force per unit width (known as "stress" in finite element terminology) can be obtained after analysis for design purpose. The design against flexure is most commonly done by the Wood Armer Equations which calculate design moments in two directions (conveniently in two perpendicular directions) and they are adequate to cater for the complete set of bending and twisting moments. The design based on node forces / moments should be avoided due to its inadequacy to cater for twisting effects which will result in under-design. A discussion of the plate bending theory and the design approach by the Wood

Armer Equations is enclosed in Appendix D, together with the "stress approach" for checking and designing against shear in the same appendix. An example of the mathematical modeling of a floor slab by the software SAFE and results of subsequent analysis is illustrated in Figure 4.3.

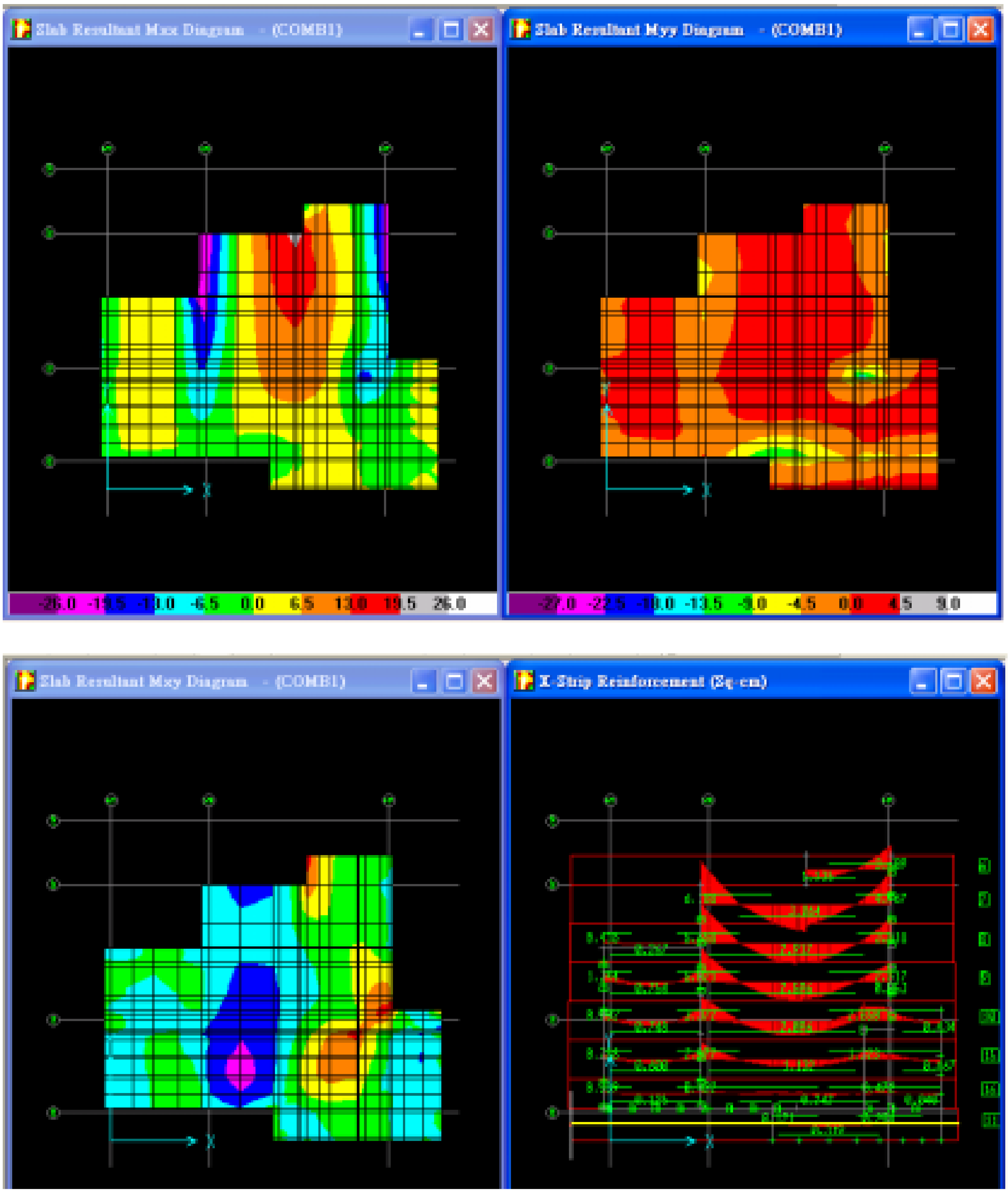


Figure 4.3 – Modeling of an irregular floor slab as 2-D mathematical model, subsequent analytical results of bending moments and twisting moment, design of reinforcements by the Wood Armer Equations.

The finite element mesh of the mathematical model is often very fine. So it is a practice of "lumping" the design reinforcements of a number of nodes over certain widths and evenly distributing the total reinforcements over the widths,

as is done by the popular software "SAFE". However, care must be taken in not taking widths too wide for "lumping" as local effects may not be well captured.

4.4 Detailing for Solid Slabs

Generally considerations in relation to determination of "effective span", "effective span depth ratio", "moment redistribution", "reduced design moment to support", "maximum and minimum steel percentages", "concrete covers" as discussed in Section 3.3 for design of beam are applicable to design of slab. Nevertheless, the detailing considerations for slabs are listed as follows with h as the structural depth of the slab (Re 9.3.1.1 of the Code) :

(i) Minimum steel percentage (Cl. 9.3.1.1(a) of the Code):
Main Reinforcing bars:

0.24% for $f_y = 250$ MPa and 0.13% for $f_y = 460$ MPa;

Distribution bars in one way slab ≥ 20% of the main reinforcements

(ii) Maximum reinforcements spacing (Cl. 9.3.1.1(b) of the Code):
  (a) In general areas without concentrated loads :
  the principal reinforcement, max. spacing $\leq 3h \leq 400$ mm; and the secondary reinforcement, max. spacing $\leq 3.5h \leq 450$ mm.
  (b) In areas with concentrated loads or areas of maximum moment:
  the principal reinforcement, max. spacing $\leq 2h \leq 250$ mm; and for the secondary reinforcement, max. spacing $\leq 3h \leq 400$ mm.

(iii) In addition to (ii), if either :
  (a) $h \leq 250$ mm (grade 250 steel);
  (b) $h \leq 200$ mm (grade 460 steel); or
  (c) the percentage of required tension reinforcement is less than 0.3%.

no direct crack widths check by calculation is required. If none of conditions in (a), (b) & (c) is satisfied, bar spacing to comply with Cl. 9.2.1.4 of the Code as discussed in 3.3(vi) of this Manual if steel percentage > 1%. Otherwise, increase the spacing by 1/percentage;

(iv) Requirements pertaining to curtailment and anchoring of tension reinforcements should be similar to that of beams;

(v) Reinforcements at end supports (Cl. 9.3.1.3 of the Code)
  (a) At least 50% of the span reinforcements should be provided and

well anchored on supports of simply supported slabs and end supports of continuous slabs as illustrated in Figure 4.4;

(b) If support shear stress $v < 0.5v_c$, the arrangement in Figure 4.4 can be considered as effective anchorage.

(vi) Minimum bottom reinforcements at internal supports : 40% of the calculated mid-span bottom reinforcements as illustrated in Figure 4.4. (Cl. 9.3.1.4 of the Code)

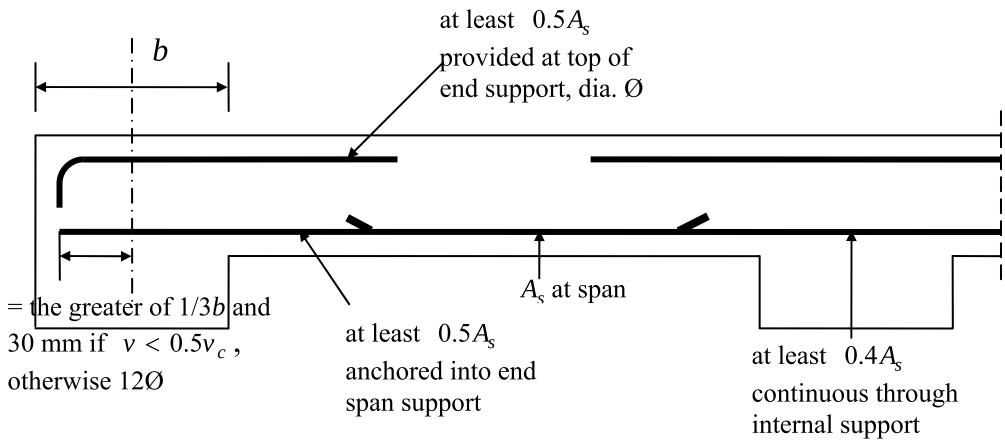


Figure 4.4 – Anchorage of bottom reinforcements into supports

(vii) Reinforcements at free edge should be as shown in Figure 4.5 (Cl. 9.3.1.6 of the Code)

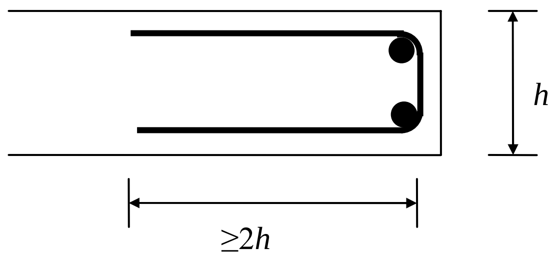


Figure 4.5 – Free edge reinforcements for Slabs

(viii) Shear reinforcements not to be used in slabs < 200 mm. (Cl. 9.3.2 of the Code)

4.5 Structural Design of Slabs

The structural design of slab against flexure is similar to that of beam. The determination of reinforcements should be in accordance with Section 3.4 of

this Manual listing the options of either following the rigorous or simplified "stress strain" relationship of concrete. Design against shear for slabs under line supports (e.g. one-way or two-way) is also similar to that of beam. However for a flat slab, the checking should be based on punching shear in accordance with the empirical method of the Code or based on shear stresses revealed by the finite element method. They are demonstrated in the Worked Examples in the following sub-Section 4.6 :

4.6 Worked Examples

Worked Example 4.1 – One Way Slab

A one-way continuous slab with the following design data :

(i) Live Load = 4.0 kN/m^2;
(ii) Finishes Load = 1 kN/m^2;
(iii) Concrete grade : 35 with cover 25 mm;
(iv) Slab thickness : 200 mm;
(v) Fire rating : 1 hour, mild exposure;
(vi) Span : 4 m

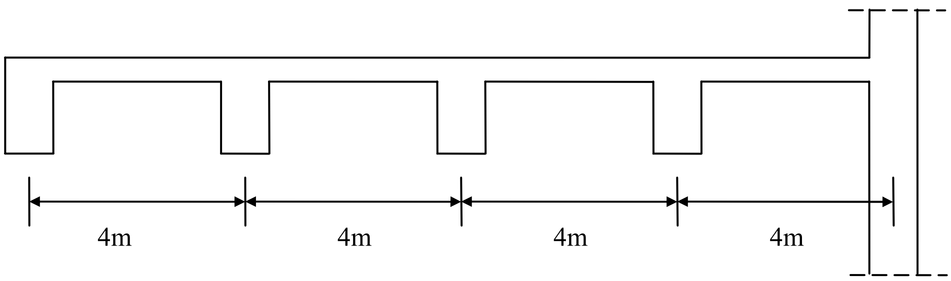


Figure 4.6 – Slab in Worked Example 4.1

Sizing : Limiting Span depth ratio = $23 \times 1.1 = 25.3$ (by Table 7.3 and Table 7.4 of the Code, assuming modification by tensile reinforcement to be 1.1 as the slab should be lightly reinforced). Assuming 10mm dia. bars under 25mm concrete cover, effective depth is $d = 200 - 25 - 5 = 170$. Span effective depth ratio is $4000/170 = 23.5 < 25.3$. So OK.

Loading :	D.L.	O.W.	$0.2 \times 24 =$	4.8kN/m^2
		Fin.		1.0 kN/m^2
		Total		5.8 kN/m^2
	L.L.			4.0 kN/m^2

The factored load on a span is $F = (1.4 \times 5.8 + 1.6 \times 4.0) \times 4 = 58.08$ kN/m.

Based on coefficients of shear and bending moment in accordance with Table 6.4 of the Code listed as follows :

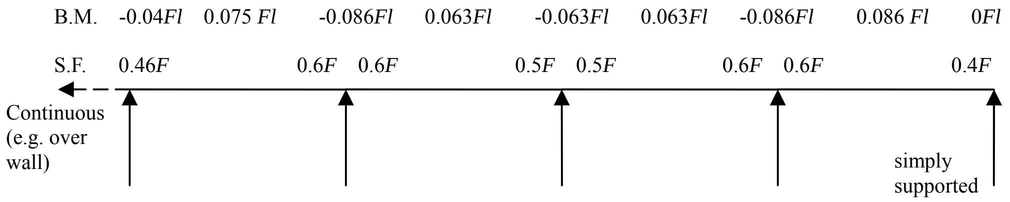


Figure 4.7 – Bending Moment and Shear Force coefficients for Continuous Slab

(a) End span support moment (continuous) $= 0.04 \times 58.08 \times 4 = 9.29$ kNm/m

$$K = \frac{M}{f_{cu} bd^2} = \frac{9.29 \times 10^6}{35 \times 1000 \times 170^2} = 0.0092$$

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} = 0.989 > 0.95$$

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{9.29 \times 10^6}{1000 \times 170^2 \times 0.87 \times 460 \times 0.95} = 0.08\% < 0.13\%$$

$A_{st} = 0.13 \div 100 \times 1000 \times 170 = 221\,\text{mm}^2$ Use T10 – 300

(A_{st} provided = 262mm^2)

(b) End span span moment = $0.086 \times 58.08 \times 4 = 19.98$ kNm/m

$$K = \frac{M}{f_{cu} bd^2} = \frac{19.98 \times 10^6}{35 \times 1000 \times 170^2} = 0.0198$$

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} = 0.978 > 0.95$$

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{19.98 \times 10^6}{1000 \times 170^2 \times 0.87 \times 460 \times 0.95} = 0.18\% > 0.13\%$$

$A_{st} = 0.18 \div 100 \times 1000 \times 170 = 309\,\text{mm}^2$ Use T10 – 250

(A_{st} provided = 314mm^2)

(c) First interior support moment = $0.086 \times 58.08 \times 4 = 19.98$ kNm/m, same reinforcement as that of end span reinforcement.

(d) Interior span or support moment = $0.063 \times 58.08 \times 4 = 14.64$ kNm/m;

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{14.64 \times 10^6}{1000 \times 170^2 \times 0.87 \times 460 \times 0.95} = 0.133\% > 0.13\%$$

$A_{st} = 0.133 \div 100 \times 1000 \times 170 = 227\,\text{mm}^2$ Use T10 – 300

(A_{st} provided = 261mm^2)

(e) End span span moment to continuous support

$= 0.075 \times 58.08 \times 4 = 17.42$ kNm/m

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{17.42 \times 10^6}{1000 \times 170^2 \times 0.87 \times 460 \times 0.95} = 0.159\% > 0.13\%$$

$A_{st} = 0.159 \div 100 \times 1000 \times 170 = 270$ mm^2. Use T10 – 250

(A_{st} provided = 314mm^2)

(f) Check Shear

Maximum shear = $0.6 \times 58.08 = 34.85$ kN/m.

By Table 6.3 of the Code $v_c = 0.79\left(\frac{100A_s}{bd}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}}\frac{1}{\gamma_m}\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}$

$v_c = 0.79(0.13)^{\frac{1}{3}}\left(\frac{400}{170}\right)^{\frac{1}{4}}\frac{1}{1.25}\left(\frac{35}{25}\right)^{\frac{1}{3}} = 0.44$ N/mm^2, based on minimum steel 0.13%; $v = \frac{34850}{1000 \times 170} = 0.205$ N/mm^2 < $v_c = 0.44$ N/mm^2.

No shear reinforcement required.

Worked Example 4.2 – Two Ways Slab (4 sides simply supported)

A two-way continuous slab with the following design data :

(i) Live Load = 4.0 kN/m^2;
(ii) Finishes Load = 1 kN/m^2;
(iii) Concrete grade : 35;
(iv) Slab thickness : 200 mm
(v) Fire rating : 1 hour, mild exposure, cover = 25mm;
(vi) Span : Long way : 4 m, Short way, 3 m

Sizing : Limiting Span depth ratio = 20 (by Table 7.3). So effective depth taken as $d = 200 - 25 - 5 = 170$ as 3000/170 = 17.65 < 20.

Loading :	D.L.	O.W.	$0.2 \times 24 =$	4.8kN/m^2
		Fin.		1.0 kN/m^2
		Total		5.8 kN/m^2
	L.L.			4.0 kN/m^2

The factored load is $F = (1.4 \times 5.8 + 1.6 \times 4.0) = 14.52$ kN/m^2

(Ceqn 6.26) and (Ceqn 6.27) of the Code are used to calculate the bending

moment coefficients along the short and long spans :

$$\alpha_{sx} = \frac{(\ell_y/\ell_x)^4}{8\left[1+(\ell_y/\ell_x)^4\right]} = 0.095; \qquad \alpha_{sy} = \frac{(\ell_y/\ell_x)^2}{8\left[1+(\ell_y/\ell_x)^4\right]} = 0.053$$

So the bending moment along the short span is

$$M_x = 0.095 \times 14.52 \times 3^2 = 12.41\,\text{kNm/m}$$

$$K = \frac{M}{f_{cu}bd^2} = \frac{12.41\times10^6}{35\times1000\times170^2} = 0.0123$$

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} = 0.986 > 0.95$$

$$\frac{A_{st}}{bd} = \frac{M}{bd^2 \times 0.87 f_y z/d} = \frac{12.41\times10^6}{1000\times170^2\times0.87\times460\times0.95} = 0.113\% < 0.13\%$$

$A_{st} = 0.13 \div 100 \times 1000 \times 170 = 221\,\text{mm}^2$ Use T10 – 300

(A_{st} provided = 262mm^2)

$M_y < M_x$, so same provision, despite the slight reduction of effective depth.

(A_{st} provided = 262mm^2)

Worked Example 4.3 – Two Ways Slab (3 sides supported)

A two-way slab with the following design data :

(i) Live Load = 4.0 kN/m^2;
(ii) Finishes Load = 1 kN/m^2;
(iii) Concrete grade : 35;
(iv) Slab thickness : 200 mm
(v) Span : Long way : 5 m, Short way, 4 m
(iv) Fire rating : 1 hour, mild exposure, cover = 25mm;

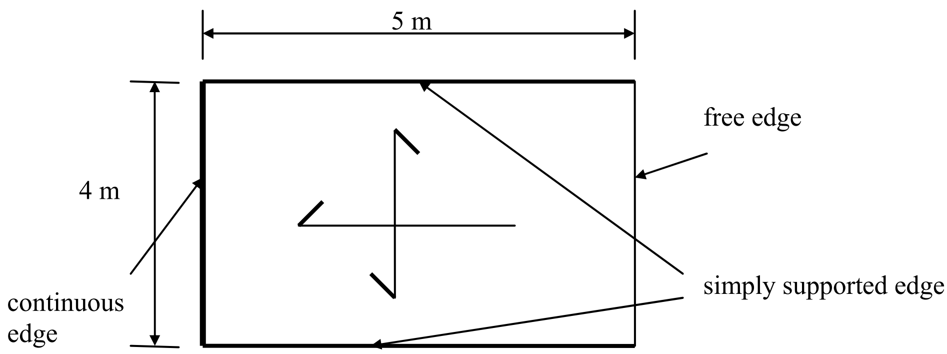


Figure 4.8 – Plan of 3-sides supported slab for Worked Example 4.3

Loading :	D.L.	O.W.	$0.2\times 24=$	4.8kN/m^2
		Fin.		1.0 kN/m^2
		Sum		5.8 kN/m^2
	L.L.			4.0 kN/m^2

The factored load is $F=(1.4\times 5.8+1.6\times 4.0)=14.52\,\text{kN/m}^2$

From Table 1.38 of "Tables for the Analysis of Plates, Slabs and Diaphragms based on Elastic Theory" where $\gamma=4/5=0.8$, the sagging bending moment coefficient for short way span is maximum at mid-span of the free edge which 0.1104 (linear interpolation between $\gamma=0.75$ and $\gamma=1.0$). The coefficients relevant to this example are interpolated and listed in Appendix E.

$$M_x=0.1104\times 14.52\times 4^2=25.65\,\text{kNm/m}$$

$$K=\frac{M}{f_{cu}bd^2}=\frac{25.65\times 10^6}{35\times 1000\times 170^2}=0.0254$$

$$\frac{z}{d}=0.5+\sqrt{0.25-\frac{K}{0.9}}=0.971>0.95$$

$$\frac{A_{st}}{bd}=\frac{M}{bd^2\times 0.87f_y z/d}=\frac{25.65\times 10^6}{1000\times 170^2\times 0.87\times 460\times 0.95}=0.233\%>0.13\%$$

$A_{st}=0.233\div 100\times 1000\times 170=397\text{ mm}^2$ Use T10 – 175

(A_{st} provided = 449mm^2)

At 2 m and 4 m from the free edge, the sagging moment reduces to $0.0844\times 14.52\times 4^2=19.608$ kNm/m and $0.0415\times 14.52\times 4^2=9.64$ kNm/m and A_{st} required are reduces to 303 mm^2 and 149 mm^2.
Use T10 – 250 and T10 – 300 respectively.

The maximum hogging moment (bending along long-way of the slab) is at mid-way along the supported edge of the short-way span

$$M_y=0.0729\times 14.52\times 5^2=26.46\,\text{kNm/m}$$

$$K=\frac{M}{f_{cu}bd^2}=\frac{26.46\times 10^6}{35\times 1000\times 170^2}=0.0262$$

$$\frac{z}{d}=0.5+\sqrt{0.25-\frac{K}{0.9}}=0.97>0.95$$

$$\frac{A_{st}}{bd}=\frac{M}{bd^2\times 0.87f_y z/d}=\frac{26.46\times 10^6}{1000\times 170^2\times 0.87\times 460\times 0.95}=0.241\%>0.13\%$$

$A_{st}=0.241\div 100\times 1000\times 170=409\text{ mm}^2$ Use T10 – 175

(A_{st} provided = 449mm^2)

The maximum sagging moment along the long-way direction is at 2 m from the free edge which is
$M_y = 0.0188 \times 14.52 \times 5^2 = 6.82$ kNm/m. The moment is small.

Use T10 – 300

Back-check compliance of effective span ratio (Re Tables 7.3 and 7.4 of the Code) by considering only the short span which is simply supported,

$$f_s = \frac{2 f_y A_{st,req}}{3A_{st,prov}} \times \frac{1}{\beta_b} = \frac{2 \times 460 \times 397}{3 \times 449} \times \frac{1}{1} = 271 \text{ N/mm}^2;$$

The modification factor (Table 7.4) is

$$0.55 + \frac{(477 - f_s)}{120\left(0.9 + \frac{M}{bd^2}\right)} = 0.55 + \frac{(477 - 271)}{120(0.9 + 0.0254 \times 35)} = 1.51$$

Allowable effective span depth ratio is $1.51 \times 20 = 30.2$ > $\frac{4000}{170} = 23.5$. O.K.

Finally the reinforcement arrangement on the slab is (Detailed curtailment, top support reinforcements at simple supports ($0.5A_s$) omitted for clarity.)

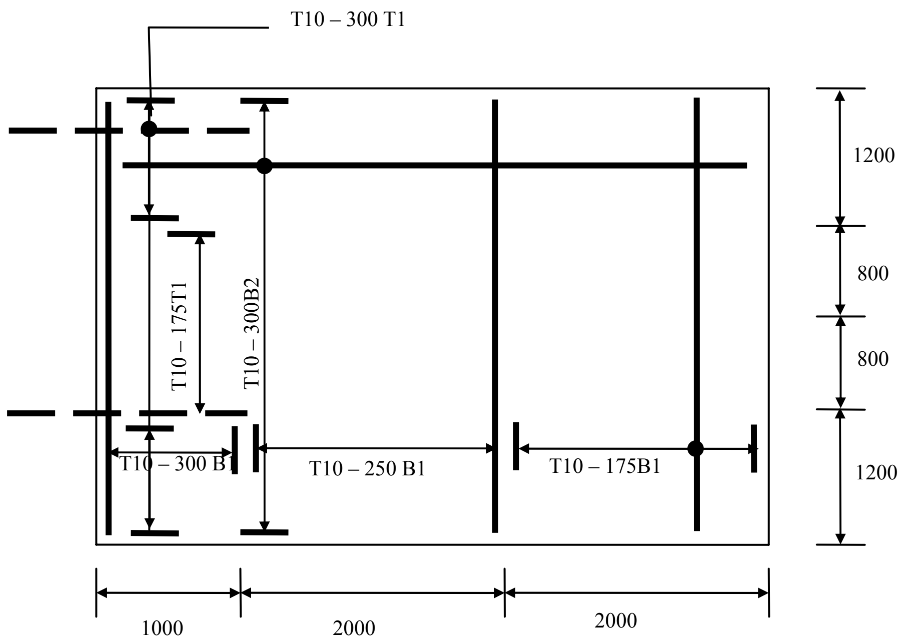


Figure 4.9 – Reinforcement Details for Worked Example 4.3

<u>Worked Example 4.4 – Flat Slab by Simplified Method (Cl. 6.1.5.2(g))</u>

Flat slab arrangement on rectangular column grid of 7.5 m and 6 m as shown in Figure 4.10 with the following design data :

(i) Finish Load = 1.5 kPa

(ii) Live Load = 5.0 kPa.

(iii) Column size = 550 × 550

(iv) Column Drop size = 3000 × 3000 with d_h = 200 mm

(v) Fire rating : 1 hour, mild exposure, cover = 25 mm

(vi) Concrete grade 35;

As the number of panel is more than 3 and of equal span, the simplified method for determining moments in accordance with Cl. 6.1.5.2(g) of the Code is applicable and is adopted in the following analysis.

Effective dimension of column head $l_{h\max} = l_c + 2(d_h - 40)$ (Ceqn 6.37)

$= 550 + 2(200 - 40) = 870$ mm

Effective diameter of column head (Cl. 6.1.5.1(c))

$$h_c = \sqrt{\frac{870^2 \times 4}{\pi}} = 982 \text{ mm} < \frac{1}{4} \times 6000 = 1500 \text{ mm}$$

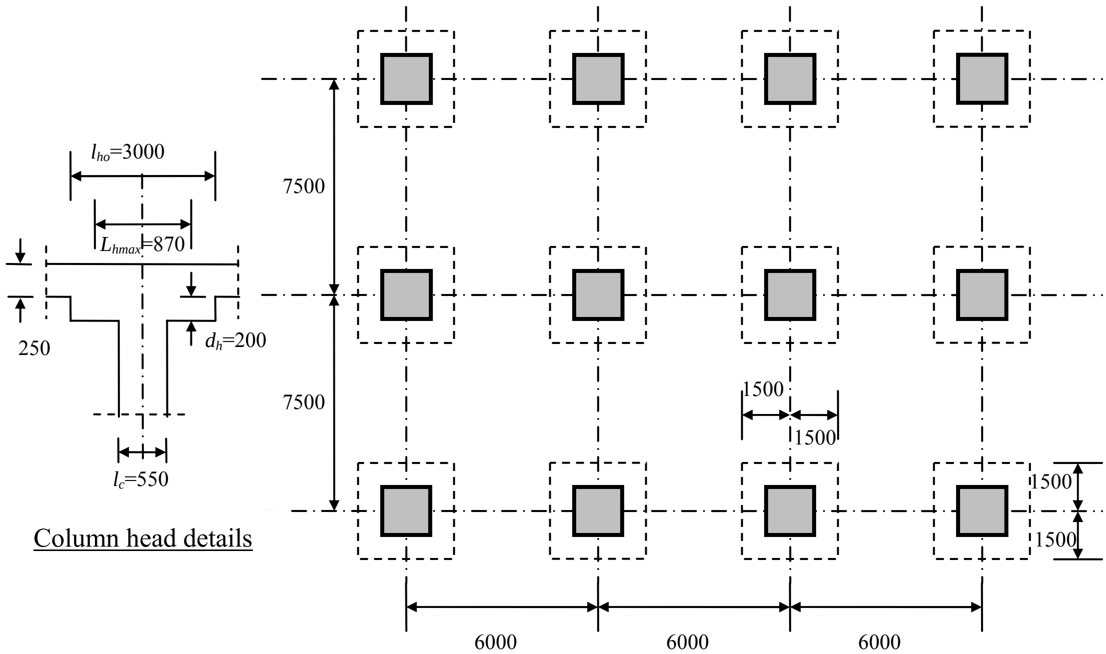


Figure 4.10 – Flat Slab Plan Layout for Worked Example 4.4

In the simplified method, the flat slab is effectively divided into (i) "column strips" containing the columns and the strips of the linking slabs and of "strip widths" equal to the widths of the column drops; and (ii) the "middle strips"

between the "column strips". (Re Figure 6.9 of the Code.) They are designed as beams in flexural design with assumed apportionment of moments among the strips. However, for shear checking, punching shears along successive "critical" perimeters of column are carried out instead.

Sizing : Based on the same limiting span depth ratio for one way and two way slab which is $26\times1.15=30$ (by Table 7.3 and Table 7.4 of the Code, assuming modification by tensile reinforcement to be 1.15), $d=\dfrac{6000}{30}=200$.

As cover = 25 mm, assuming T12 bars, structural depth should at least be $200+25+12\div2=231$ mm, so use structural depth of slab of 250 mm.

Loading :	D.L.	O.W.	$0.25\times24=$	6.0kN/m^2
		Fin.		1.5 kN/m^2
		Total		7.5 kN/m^2
	L.L.			5.0 kN/m^2

The factored load is $F=(1.4\times7.5+1.6\times5.0)=18.5$ kN/m^2

<u>Design against Flexure (Long Way)</u>

The bending moment and shear force coefficients in Table 6.4 will be used as per Cl. 6.1.5.2(g) of the Code. Total design ultimate load on the full width of panel between adjacent bay centre lines is $F=18.5\times7.5\times6=832.5$ kN. Thus the reduction to support moment for design, as allowed by Cl. 6.1.5.2(g) of the Code, is $0.15Fh_c=0.15\times832.5\times0.982=122.63$ kNm for internal support and $0.15Fh_c=0.15\times832.5\times0.982/2=61.32$ kNm for outer support.

The design moment at supports are :

Total moment at outer support is $0.04\times832.5\times7.5=249.75$ kNm, which can be reduced to $249.75-61.32=188.43$ kNm;

Total moment at first interior support is $0.086\times832.5\times7.5=536.96$ kNm, which can be reduced to $536.96-122.63=414.33$ kNm

Total moment at interior support is $0.063\times832.5\times7.5=393.36$ kNm, which can be reduced to $393.36-122.63=270.73$ kNm

The flat slab is divided into column and mid strips in accordance with Figure 6.9 of the Code which is reproduced as Figure 4.11 in this Manual.

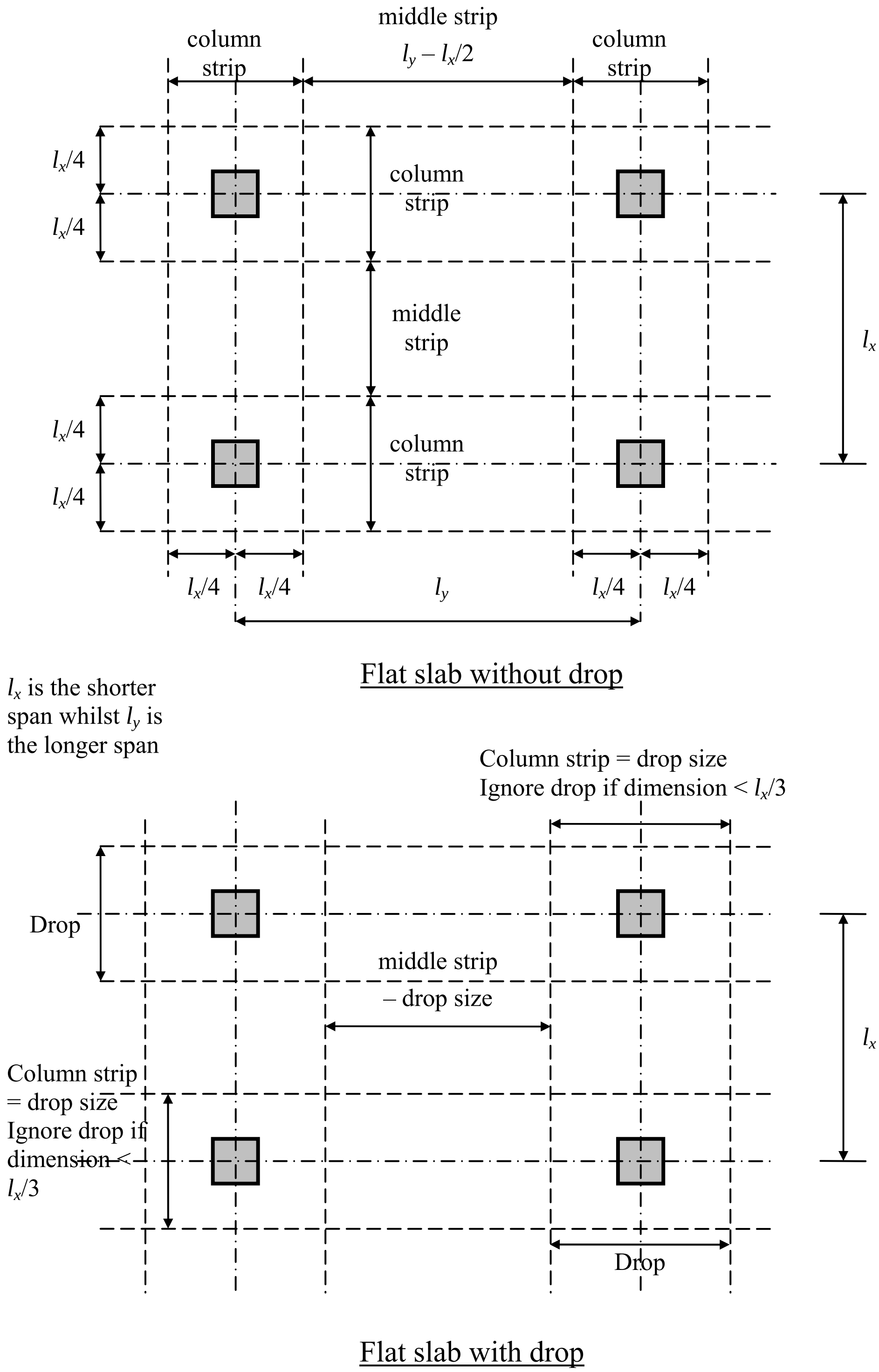


Figure 4.11 – Division of panels

The total support moments as arrived for the whole panel are to be apportioned to the middle and column strips with the percentages of 75% and

25% respectively as per Table 6.10 of the Code,

	Column Strip (75%)		Mid Strip (25%)	
	Total Mt	Mt/width	Total Mt	Mt/width
Outer Support	141.32	47.11	47.11	15.70
1st interior support	310.75	103.58	103.58	34.53
Middle interior support	203.05	67.68	67.68	22.56

The reinforcements – top steel are worked out as follows, (minimum of 0.13% in brackets) ($d = 450 - 25 - 6 = 419$ over column support and $d = 250 - 25 - 6 = 219$ in other locations)

	Column Strip (75%)		Mid Strip (25%)	
	Area (mm^2)/m	Steel	Area (mm^2)/m	Steel
Outer Support	281 (548)	T12 – 200	178 (548)	T12 – 200
1st interior support	618	T12 – 150	392 (548)	T12 – 200
Middle interior support	404 (548)	T12 – 200	256 (548)	T12 – 200

Sagging Moment :

Total moment near middle of end span is $0.075 \times 832.5 \times 7.5 = 468.28$ kNm

Total moment near middle of interior span $0.063 \times 832.5 \times 7.5 = 393.36$ kNm

These moments are to be apportioned in the column and mid strips in accordance with the percentages of 55% and 45% respectively as per Table 6.10, i.e.

	Column Strip (55%)		Mid Strip (45%)	
	Total Mt	Mt/width	Total Mt	Mt/width
Middle of end span	257.55	85.85	210.73	70.24
Middle of interior span	216.35	72.12	177.01	59.00

The reinforcements – bottom steel are worked out as follows :

	Column Strip (55%)		Mid Strip (45%)	
	Area (mm^2)/m	Steel	Area (mm^2)/m	Steel
Middle of end span	980	T12 – 100	801	T12 – 125
Middle of interior span	823	T12 – 125	673	T12 – 150

Design in the short way direction can be carried out similarly.

<u>Design against Shear</u>

Design of shear should be in accordance with Cl. 6.1.5.6 of the Code which is against punching shear by column. For the internal column support, in the

absence of frame analysis, the shear for design will be $V_{eff} = 1.15V_t$ where V_t is the design shear transferred to column calculated on the assumption of all adjacent panels being fully loaded by Cl. 6.1.5.6(b) of the Code.

$V_t = 7.5 \times 6 \times 18.5 = 832.5$ kN; $\quad V_{eff} = 1.15V_t = 957.38$ kN

Check on column perimeter as per Cl. 6.1.5.6(d) of the Code :

$\dfrac{V_{eff}}{ud} \le 0.8\sqrt{f_{cu}}$ or 7 $\quad \dfrac{957.38 \times 10^3}{(4 \times 550) \times 419} = 1.04 \le 0.8\sqrt{f_{cu}} = 4.73$ MPa; O.K.

Check on 1st critical perimeter – $1.5d$ from column face, i.e.
$1.5 \times 0.419 = 0.6285$. So side length of the perimeter is
$(550 + 628.5 \times 2) = 1807$ mm
Length of perimeter is $4 \times 1807 = 7228$ mm
Shear force to be checked can be the maximum shear 957.38 kN after deduction of the loads within the critical perimeter which is
$957.38 - 18.5 \times 1.807^2 = 896.97$ kN

Shear stress = $\dfrac{896.97 \times 10^3}{7228 \times 419} = 0.296$ N/mm^2. < $v_c = 0.48$ N/mm^2 in accordance with Table 6.3 ($0.43 \times (35/25)^{1/3} = 0.48$). No shear reinforcement is required.

No checking on further perimeter is required.

Worked Example 4.5 – Design for shear reinforcement

(Ceqn 6.44) and (Ceqn 6.45) of the Code gives formulae for reinforcement design for different ranges of values of v.

For $v \le 1.6v_c$, $\quad \sum A_{sv} \sin\alpha \ge \dfrac{(v - v_c)ud}{0.87 f_y}$

For $1.6v_c < v \le 2.0v_c$, $\quad \sum A_{sv} \sin\alpha \ge \dfrac{5(0.7v - v_c)ud}{0.87 f_y}$

As a demonstration, if $v = 0.7$ N/mm^2 in the first critical perimeter which is < $1.6v_c = 0.77$ N/mm^2 but > $v_c = 0.48$ N/mm^2 in the Example 4.4. By Table 6.8 of the Code, as $v - v_c < 0.4$, $v_r = 0.4$

$\sum A_{sv} \sin\alpha \ge \dfrac{v_r ud}{0.87 f_y} = \dfrac{0.4 \times 7228 \times 419}{0.87 \times 460} = 3027$ mm^2. If vertical links is chosen as shear reinforcement, $\alpha = 90^0 \Rightarrow \sin\alpha = 1$. So the 3027 mm^2 should be distributed within the critical perimeter as shown in Figure 4.12.

In distributing the shear links within the critical perimeter, there are recommendations in Cl. 6.1.5.7(f) of the Code that

(i) at least two rows of links should be used;

(ii) the first perimeter should be located at approximately $0.5d$ from the face of the loaded area (i.e. the column in this case) and should contain not less than 40% of the calculated area of reinforcements.

So the first row be determined at 200 mm from the column face with total row length $950 \times 4 = 3800$. Using T10 – 225 spacing along the row, the total steel area will be $\left(10^2/4\right)\pi \times 3800/225 = 1326\,\text{mm}^2$ > 40% of 3027 mm^2.

The second row be at further 300 mm ($\leq 0.75d = 314$) away where row length is $1550 \times 4 = 6200$. Again using T10 – 225 spacing along the row, the total steel area will be $\left(10^2/4\right)\pi \times 6200/225 = 2164\,\text{mm}^2$ > 60% of 3027 mm^2.

Total steel area is 1326 + 2164 = 3490 mm^2 for shear. The arrangement is illustrated in Figure 4.12.

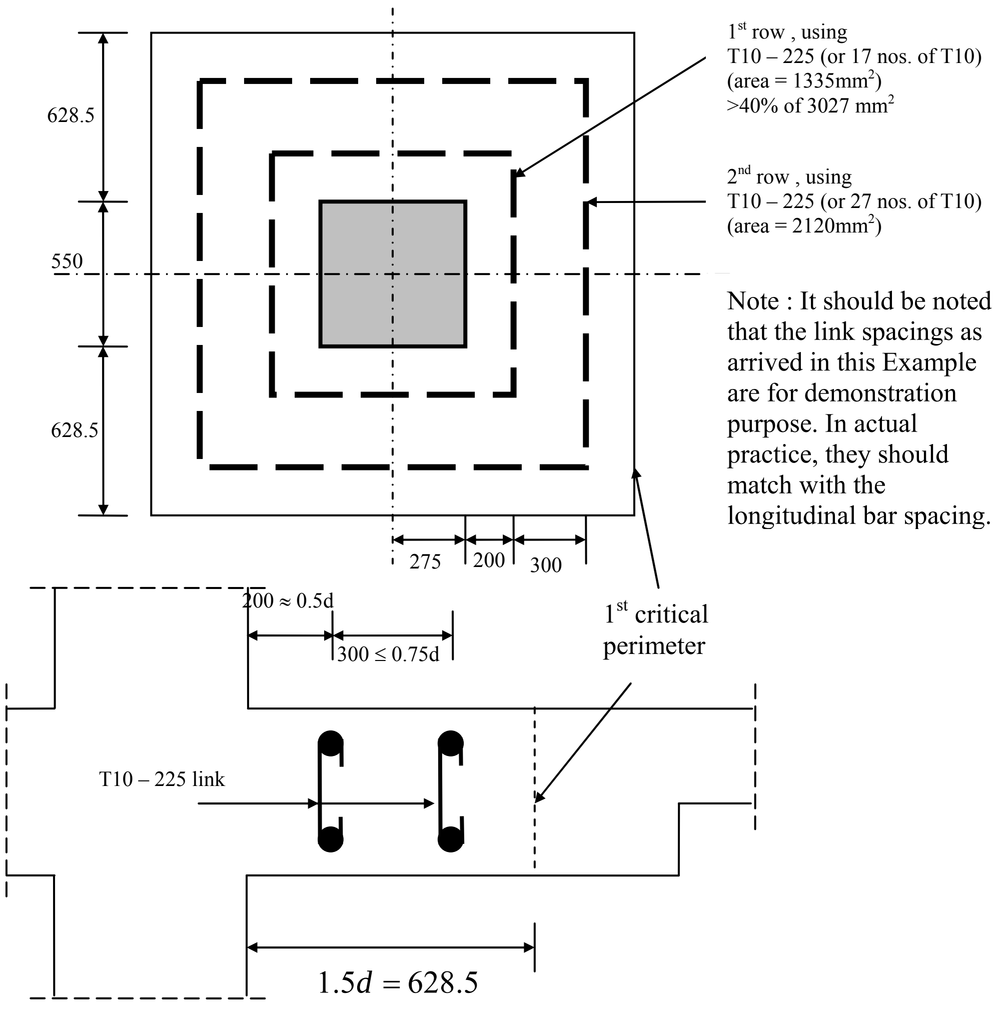


Figure 4.12 – Shear links arrangement in Flat Slab for Worked Example 4.5

Design for Shear when ultimate shear stress exceeds $1.6v_c$

It is stated in (Ceqn 6.43) in Cl. 6.1.5.7(e) that if $1.6_c < v \le 2.0v_c$, $\sum A_{sv} \sin\alpha \ge \dfrac{5(0.7v - v_c)ud}{0.87f_y}$ which effectively reduces the full inclusion of v_c for reduction to find the "residual shear to be taken up by steel" at $v = 1.6v_c$ to zero inclusion at $v = 2.0v_c$.

Worked Example 4.6 – when $1.6v_c < v \le 2.0v_c$

In the previous Example 4.5, if the shear stress $v = 0.85\,\text{N/mm}^2$ which lies between $1.6v_c = 1.6 \times 0.48 = 0.77\,\text{N/mm}^2$ and $2.0v_c = 2 \times 0.48 = 0.96\,\text{N/mm}^2$ $\sum A_{sv} \sin\alpha \ge \dfrac{5(0.7v - v_c)ud}{0.87f_y} = \dfrac{5(0.7 \times 0.85 - 0.48) \times 7228 \times 419}{0.87 \times 460} = 4351\ \text{mm}^2$. If arranged in two rows as in Figure 4.12, use T12 – 225 for both rows : the inner row gives $(12^2/4)\pi \times 3800/225 = 1908\,\text{mm}^2$ > 40% of 4351 mm^2; the outer row gives $(12^2/4)\pi \times 6200/225 = 3114\,\text{mm}^2$. The total area is 1908 + 3114 = 5022mm^2 > 4351 mm^2.

Cl. 6.1.5.7(e) of the Code says, "When $v > 2v_c$ and a reinforcing system is provided to increase the shear resistance, justification should be provided to demonstrate the validity of design." If no sound justification, the structural sizes need be revised.

5.0 Columns

5.1 Slenderness of Columns

Columns are classified as short and slender columns in accordance with their "slenderness". Short columns are those with ratios l_{ex}/h and l_{ey}/b < 15 (braced) and 10 (unbraced) in accordance with Cl. 6.2.1.1(b) of the Code where l_{ex} and l_{ey} are the "effective lengths" of the column about the major and minor axes, b and h are the width and depth of the column.

As defined in Cl. 6.2.1.1 of the Code, a column may be considered braced in a given plane if lateral stability to the structure as a whole is provided by walls or bracing or buttressing designed to resist all lateral forces in that plane. It would otherwise be considered as unbraced.

The effective length is given by (Ceqn 6.46) of the Code as
$l_e = \beta \cdot l_0$ where l_0 is the clear height of the column between restraints and the value β is given by Tables 6.11 and 6.12 of the Code which measures the restraints against rotation and lateral movements at the ends of the column.

Generally slenderness limits for column : $l_0/b \le 60$ as per Cl. 6.2.1.1(f) of the Code. In addition, for cantilever column $l_0 = \dfrac{100b^2}{h} \le 60b$.

<u>Worked Example 5.1</u> : a braced column of clear height $l_0 = 8$ m and sectional dimensions $b = 400$ mm, $h = 550$ mm with its lower end connected monolithically to a thick cap and the upper end connected monolithically to deep transfer beams in the plane perpendicular to the major direction but beam of size 300(W) by 400(D) in the other direction.

By Tables 6.11 and 6.12 of the Code
Lower end condition in both directions : 1
Upper end condition about the major axis : 1
Upper end condition about the minor axis : 2

For bending about the major axis : end conditions 1 – 1, $\beta_x = 0.75$,
$l_{ex} = 0.75 \times 8 = 6$
$l_{ex}/550 = 10.91 < 15$. $\therefore$ a short column.
For bending about the minor axis : end conditions 1 – 2, $\beta_y = 0.8$,

$l_{ey} = 0.8 \times 8 = 6.4$

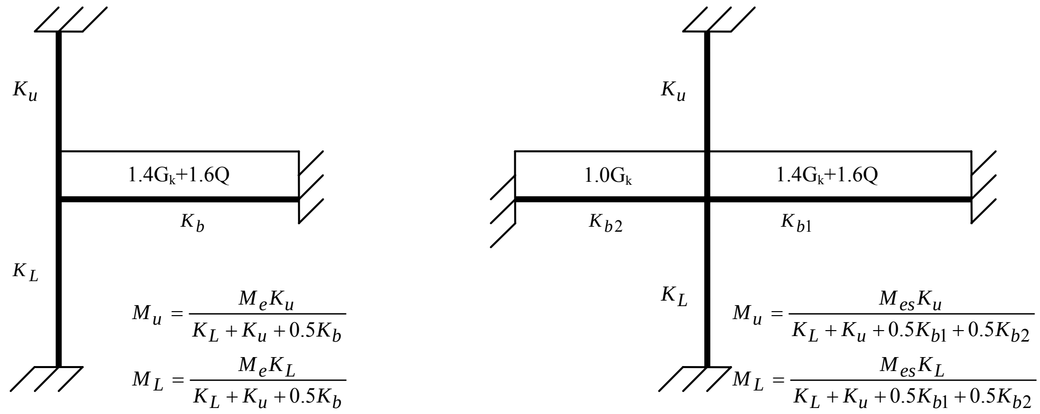
$l_{ey} / 400 = 16 > 15$ $\therefore$ a slender column. $l_{ey} / 400 = 16 < 60$, O.K.

For a slender column, an additional "deflection induced moment" M_{add} will be required to be incorporated in design, as in addition to the working moment.

5.2 Design Moments and Axial Loads on Columns

5.2.1 Determination of Design moments and Axial Loads by sub-frame Analysis

Generally design moments, axial loads and shear forces on columns are that obtained from structural analysis. In the absence of rigorous analysis, (i) design axial load may be obtained by the simple tributary area method with beams considered to be simply supported on the column; and (ii) moment may be obtained by simplified sub-frame analysis as illustrated in Figure 5.1 :



Symbols:

M_e : Beam Fixed End Moment.	K_u : Upper Column Stiffness
M_{es} : Total out of balance Beam Fixed End Moment.	K_L : Upper Column Stiffness
M_u : Upper Column Design Moment	K_{b1} : Beam 1 Stiffness
M_L : Upper Column Design Moment	K_{b2} : Beam 2 Stiffness

Figure 5.1 – Diagrammatic illustration of determination of column design moments by Simplified Sub-frame Analysis

<u>Worked Example 5.2 (Re Column C1 in Plan shown in Figure 5.2)</u>

Design Data :

Slab thickness : 150 mm Finish Load : 1.5 kN/m^2

Live Load : 5 kN/m^2 Beam size : 550(D) × 400(W)

Upper Column height : 3 m Lower Column Height : 4 m

Column size : 400(W) × 600(L)

Column Load from floors above D.L. 443 kN L.L. 129 kN

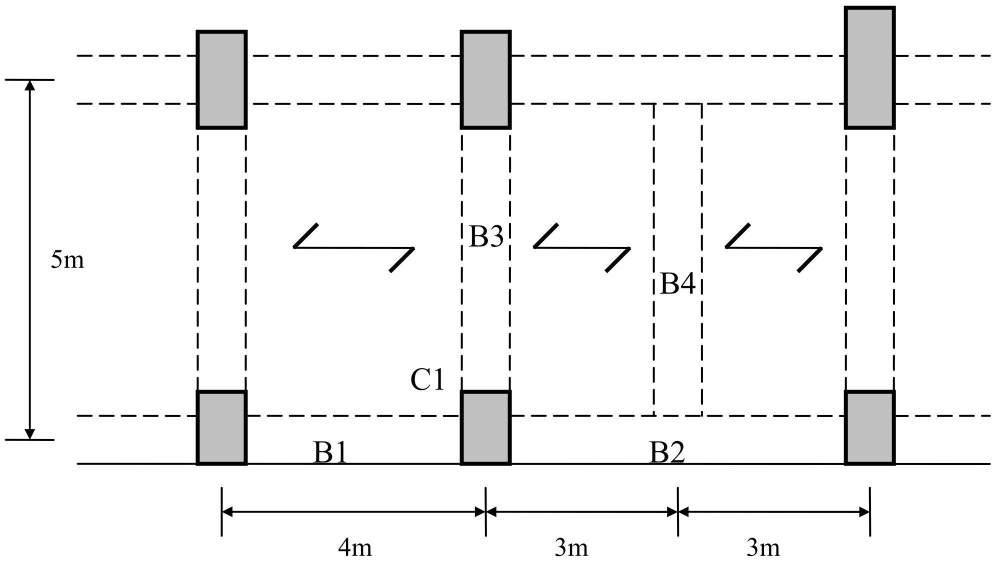


Figure 5.2 – Plan for illustration for determination of design axial load and moment on column by the Simplified Sub-frame Method

Design for Column C1 beneath the floor

Check for slenderness : As per Cl. 6.2.1.1(e) of the Code, the end conditions of the column about the major and minor axes are respectively 2 and 1 at the upper end and 1 at the lower end for both axes (fixed on pile cap). The clear height between restraints is $4000-550=3450$. The effective heights of the column about the major and minor axes are respectively $0.8\times 3.45=2.76$ m and $0.75\times 3.45=2.59$ m. So the slenderness ratios about the major and minor axes are $\frac{2760}{600}=4.6<15$ and $\frac{2590}{400}=6.475<15$. Thus the column is not slender in both directions.

Loads :

Slab: D.L. O.W. $0.15\times 24=3.6$ kN/m^2

Fin. 1.5 kN/m^2

5.1 kN/m^2

L.L. 5.0 kN/m^2

Beam B1 D.L. O.W. $0.4\times 0.55\times 24\times 4=21.12$ kN

End shear of B1 on C1 is D.L. $21.12 \div 2 = 10.56\,\text{kN}$

Beam B3	D.L. O.W.		$0.4 \times (0.55 - 0.15) \times 24 = 3.84\,\text{kN/m}$
	Slab		$5.1 \times 3.5 = 17.85\,\text{kN/m}$
			21.69 kN/m
	L.L. Slab		$5.0 \times 3.5 = 17.5\,\text{kN/m}$
End shear of B3 on C1		D.L.	$21.69 \times 5 \div 2 = 54.23\,\text{kN}$
		L.L.	$17.50 \times 5 \div 2 = 43.75\,\text{kN}$

Beam B4	D.L. O.W.		$0.4 \times (0.55 - 0.15) \times 24 = 3.84\,\text{kN/m}$
	Slab		$5.1 \times 3 = 15.3\,\text{kN/m}$
			19.14 kN/m
	L.L. Slab		$5.0 \times 3 = 15.0\,\text{kN/m}$
End shear of B4 on B2		D.L.	$19.14 \times 5 \div 2 = 47.85\,\text{kN}$
		L.L.	$15.00 \times 5 \div 2 = 37.50\,\text{kN}$

Beam B2	D.L. O.W.		$0.4 \times 0.55 \times 24 \times 6 = 31.68\,\text{kN}$
	B4		47.85 kN
			79.53 kN
	L.L. B4		37.50 kN
End shear of B2 on C1 ,		D.L.	$79.53 \div 2 = 39.77\,\text{kN}$
		L.L.	$37.5 \div 2 = 18.75\,\text{kN}$

Total D.L. on C1	O.W.	$0.4 \times 0.6 \times 24 \times 4 = 23.04\,\text{kN}$
	B1 + B2 + B3	$10.56 + 39.77 + 54.23 = 104.56\,\text{kN}$
	Floor above	443.00 kN
	Sum	570.60 kN

Total L.L. on C1	B1 + B2 +B3	$0 + 18.75 + 43.75 = 62.5\,\text{kN}$
	Floor above	129.00 kN
	Sum	191.50 kN

So the factored axial load on the lower column is
$1.4 \times 570.6 + 1.6 \times 191.5 = 1105.24\,\text{kN}$

Factored fixed end moment bending about the major axis (by Beam B3 alone):

$$M_e = \frac{1}{12} \times (1.4 \times 21.69 + 1.6 \times 17.5) \times 5^2 = 121.6\,\text{kNm}$$

Factored fixed end moment bending about the minor axis by Beam B2:

$$M_{eb2} = 1.4\times\left(\frac{1}{12}\times 31.68+\frac{1}{8}\times 47.85\right)\times 6+1.6\times\left(\frac{1}{8}\times 37.5\right)\times 6 = 95.24\,\text{kNm}$$

Factored fixed end moment bending about the minor axis by Beam B1:

$$M_{eb2} = 1.0\times\left(\frac{1}{12}\times 21.12\right)\times 4 = 7.04\,\text{kNm}$$

So the unbalanced fixed moment bending about the minor axis is

$95.24 - 7.04 = 88.2$ kNm

The moment of inertia of the column section about the major and minor axes are $I_{cx} = \dfrac{0.4\times 0.6^3}{12} = 0.0072\,\text{m}^4$, $I_{cy} = \dfrac{0.6\times 0.4^3}{12} = 0.0032\,\text{m}^4$

The stiffnesses of the upper and lower columns about the major axis are :

$$K_{ux} = \frac{4EI_{cx}}{L_u} = \frac{4E\times 0.0072}{3} = 0.0096E$$

$$K_{Lx} = \frac{4EI_{cx}}{L_L} = \frac{4E\times 0.0072}{4} = 0.0072E$$

The stiffnesses of the upper and lower columns about the minor axis are :

$$K_{uy} = \frac{4EI_{cy}}{L_u} = \frac{4E\times 0.0032}{3} = 0.004267E$$

$$K_{Ly} = \frac{4EI_{cy}}{L_L} = \frac{4E\times 0.0032}{4} = 0.0032E$$

The moment of inertia of the beams B1, B2 and B3 are

$$\frac{0.4\times 0.55^3}{12} = 0.005546\,\text{m}^4$$

The stiffness of the beams B1, B2 and B3 are respectively

$\dfrac{4E\times 0.05546}{4} = 0.005546E$; $\quad\dfrac{4E\times 0.05546}{6} = 0.003697E$; and

$$\frac{4E\times 0.05546}{5} = 0.004437E$$

Distributed moment on the lower column about the major axis is

$$M_{Lx} = \frac{M_{ex}K_{Lx}}{K_{ux}+K_{Lx}+0.5K_{b3}} = \frac{121.6\times 0.0072E}{0.0096E+0.0072E+0.5\times 0.004437E}$$

= 46.03 kNm

Distributed moment on the lower column about the minor axis is

$$M_{Ly} = \frac{M_{ey}K_{Ly}}{K_{uy}+K_{Ly}+0.5(K_{b1}+K_{b2})}$$

$$= \frac{88.2 \times 0.0032E}{0.004267E + 0.0032E + 0.5 \times (0.005546 + 0.003697)E} = 23.35\,\text{kNm}$$

So the lower column should be checked for the factored axial load of 1105.24kN, factored moment of 46.03 kNm about the major axis and factored moment of 23.35 kNm about the minor axis.

5.2.2 Minimum Eccentricity

A column section should be designed for the minimum eccentricity equal to the lesser of 20 mm and 0.05 times the overall dimension of the column in the plane of bending under consideration. Consider Worked Example in 5.2, the minimum eccentricity about the major axis is 20 mm as $0.05 \times 600 = 30 > 20$ mm and that of the minor axis is $0.05 \times 400 = 20$ mm. So the minimum eccentric moments to be designed for about the major and minor axes are both $1105.24 \times 0.02 = 22.1$ kNm. As they are both less than the design moment of 46.03 kNm and 23.35 kNm, they can be ignored.

5.2.3 Check for Slenderness

In addition to the factored load and moment as discussed in 5.2.1, it is required by Cl. 6.2.1.3 of the Code to design for an additional moment M_{add} if the column is found to be slender by Cl. 6.2.1.1. The arrival of M_{add} is an eccentric moment created by the ultimate axial load N multiplied by a pre-determined lateral deflection a_u in the column as indicated by the following equations of the Code.

$$M_{add} = Na_u \qquad \text{(Ceqn 6.52)}$$

$$a_u = \beta_a Kh \qquad \text{(Ceqn 6.48)}$$

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b}\right)^2 \qquad \text{(Ceqn 6.51)}$$

$$K = \frac{N_{uz} - N}{N_{uz} - N_{bal}} \le 1 \ \text{(conservatively taken as 1)} \qquad \text{(Ceqn 6.49)}$$

or by $N_{uz} = 0.45 f_{cu} A_{nc} + 0.87 f_y A_{sc}$ (Ceqn 6.50)

$$N_{bal} = 0.25 f_{cu} bd$$

Final design moment M_t will therefore be the greatest of

(1) M_2, the greater initial end moment due to design ultimate load;

(2) $M_i + M_{add}$ where $M_i = 0.4M_1 + 0.6M_2 \geq 0.4M_2$ (with M_2 as positive and M_1 negative.)

(3) $M_1 + M_{add}/2$
in which M_1 is the smaller initial end moment due to design ultimate load.

(4) $N \times e_{min}$ (discussed in Section 5.2.2 of this Manual)

where the relationship between M_1, M_2, M_{add} and the arrival of the critical combination of design moments due to M_{add} are illustrated in Figure 5.3 reproduced from Figure 6.16 of the Code.

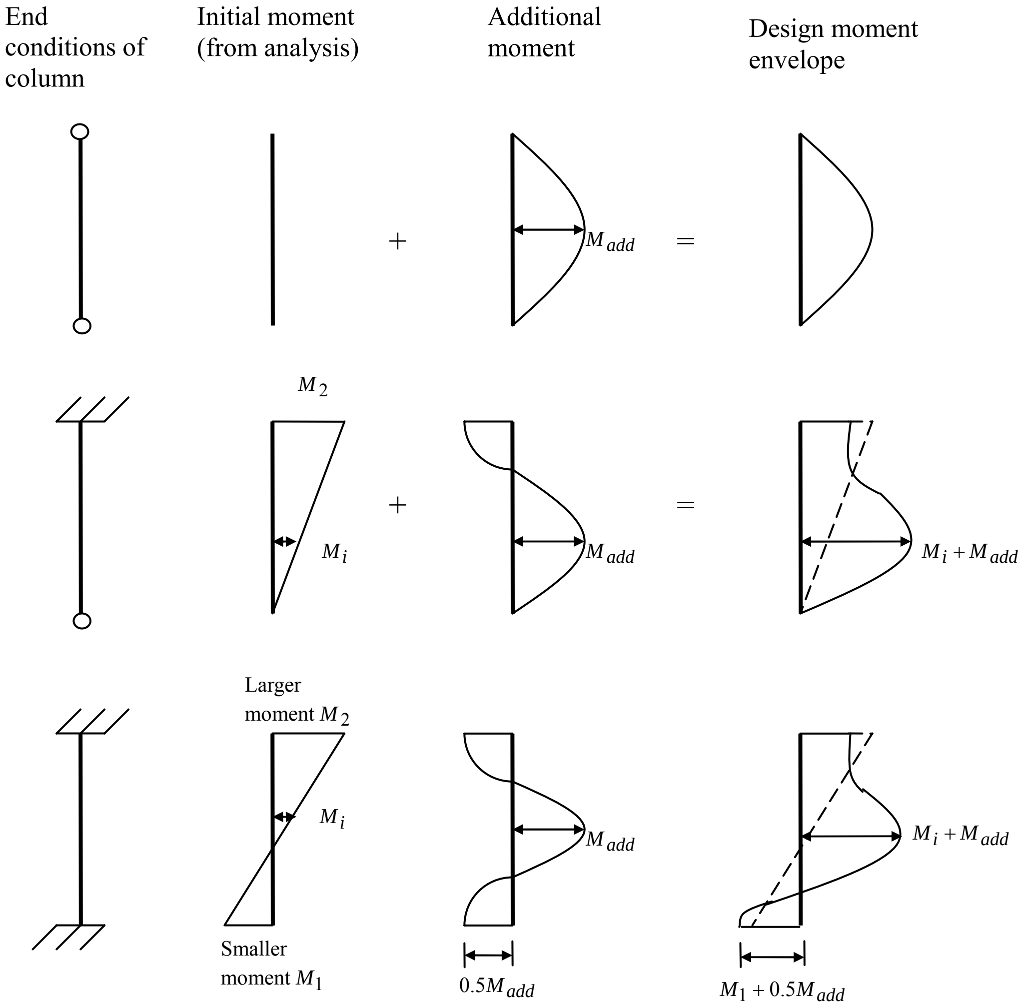


Figure 5.3 – Braced slender columns

In addition to the above, the followings should be observed in the determination of M_t as the enveloping moment of the 4 cases described in the previous paragraph (Re Cl. 6.2.1.3 of the Code) :

(i) In case of biaxial bending (moment significant in two directions), M_t

should be applied separately for the major and minor directions with b in Table 6.13 of the Code be taken as h, the dimension of the column in the plane considered for bending. Re Ceqn 6.48;

(ii) In case of uniaxial bending about the major axis where $l_e / h \le 20$ and $h < 3b$, M_t should be applied only in the major axis;

(iii) In case of uniaxial bending about the major axis only where either $l_e / h \le 20$ or $h < 3b$ is not satisfied, the column should be designed as biaxially bent, with zero M_i in the minor axis;

(iv) In case of uniaxial bending about the minor axis, M_{add} obviously be applied only in the minor axis only.

Worked Example 5.3 :

A slender braced column of grade 35, cross sections
$b = 400$, $h = 500$
$l_{ex} = l_{ey} = 8$ m, $N = 1500$ kN

(i) Moment due to ultimate load about the major axis only, the greater and smaller bending moments due to ultimate load are respectively
$M_{2x} = 153$ kNm and $M_{1x} = 96$ kNm

As $l_{ex} / h = 16 \le 20$; $h = 500 < 3b = 1200$

So needs to check for additional bending in the major axis but with M_{add} based on the minor axis.

Take $K = 1$

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b}\right)^2 = \frac{1}{2000}\left(\frac{8000}{400}\right)^2 = 0.2$$

$a_u = \beta_a Kh = 0.2 \times 1 \times 0.5 = 0.1$
$M_{addx} = Na_u = 1500 \times 0.1 = 150$
$M_{ix} = 0.4M_1 + 0.6M_2 = 0.4(-96) + 0.6 \times 153 = 53.4 < 0.4M_2 = 0.4 \times 153 = 61.2$

The design moment about the major axis will be the greatest of :

(1) $M_{2x} = 153$

(2) $M_{ix} + M_{addx} = 61.2 + 150 = 211.2$

(3) $M_{1x} + M_{addx} / 2 = 96 + 150 / 2 = 171$

(4) $N \times e_{\min} = 1500 \times 0.02 = 30$ as $e_{\min} = 20 < 0.05 \times 500 = 25$

So the greatest design moment is case (2) $M_{ix} + M_{addx} = 211.2$

Thus the section needs only be checked for uniaxial bending with $N = 1500$ kN and $M_x = 211.2$ kNm bending about the major axis.

(ii) Moments due to ultimate loads about the minor axis only, the greater and smaller moments are identical in magnitudes to that in (i), but about the minor axis, repeating the procedure :

$M_{2y} = 143$ kNm and $M_{1y} = 79$ kNm

As $l_{ex}/h = 16 \le 20$; $h = 500 < 3b = 1200$

So needs to checked for additional bending in the major axis.

Take $K = 1$

$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b}\right)^2 = \frac{1}{2000}\left(\frac{8000}{400}\right)^2 = 0.2$

$a_u = \beta_a K h = 0.2 \times 1 \times 0.4 = 0.08$

$M_{addy} = Na_u = 1500 \times 0.08 = 120$

$M_{iy} = 0.4M_{1y} + 0.6M_{2y} = 0.4 \times (-79) + 0.6 \times 143 = 54.2 < 0.4M_{2y} = 0.4 \times 143 = 57.2$

The design moment will be the greatest of :

(1) $M_{2y} = 143$

(2) $M_{iy} + M_{addy} = 57.2 + 120 = 177.2$

(3) $M_{1y} + M_{addy}/2 = 79 + 120/2 = 139$

(4) $N \times e_{\min} = 1500 \times 0.02 = 30$ as $e_{\min} = 20 \le 0.05 \times 400 = 20$

So the greatest design moment is case (2) $M_{iy} + M_{addy} = 177.2$

Thus the section need only be checked for uniaxial bending with $N = 1500$ kN and $M_y = 177.2$ kNm bending about the minor axis.

(iii) Biaxial Bending, there are also moments of $M_{2x} = 153$ kNm and $M_{1x} = 96$ kNm; $M_{2y} = 143$ kNm and $M_{1y} = 79$ kNm. By Cl. 6.1.2.3(f), M_{add} about the major axis will be revised as follows :

Bending about the major axis :

$\beta_a = \frac{1}{2000}\left(\frac{l_e}{h}\right)^2 = \frac{1}{2000}\left(\frac{8000}{500}\right)^2 = 0.128$

$a_u = \beta_a K h = 0.128 \times 1 \times 0.5 = 0.064$

$M_{addx} = Na_u = 1500 \times 0.064 = 96$ kNm.

Thus items (2) and (3) in (i) are revised as

(2) $M_{ix} + M_{addx} = 61.2 + 96 = 157.2$

(3) $M_{1x} + M_{addx}/2 = 96 + 96/2 = 144$

So the moment about major axis for design is 157.2 kNm

Bending about the minor axis :

$M_{2y} = 143\,\text{kNm}$ and $M_{1y} = 79\,\text{kNm}$; same as (ii);

Thus the ultimate design moment about the major axis is 157.2 kNm and that about the minor axis is 177.2 kNm.

Worked Example 5.4 :

A slender braced column of grade 35, cross section
$b = 400$, $h = 1200$
$l_{ex} = l_{ey} = 8\,\text{m}$, $N = 1500\,\text{kN}$, $M_{2x} = 153\,\text{kNm}$ and $M_{1x} = 96\,\text{kNm}$

As $3b = h$, Cl. 6.2.1.3(e) should be used.
Take $K = 1$

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b}\right)^2 = \frac{1}{2000}\left(\frac{8000}{400}\right)^2 = 0.2$$

$$a_u = \beta_a K b = 0.2 \times 1 \times 0.4 = 0.08\,\text{m} > 20\text{ mm}$$

$$M_{addy} = N a_u = 1500 \times 0.08 = 120$$

So the minor axis moment is 120 kNm
As $\dfrac{l_e}{h} = \dfrac{8000}{1200} = 6.67$, the column is not slender about the major axis.
So the major axis moment is simply 153 kNm.

5.3 Sectional Design

Generally the sectional design of column utilizes both the strengths of concrete and steel in the column section in accordance with stress strain relationship of concrete and steel in Figures 3.8 and 3.9 of the Code respectively. Alternatively, the simplified stress block for concrete in Figure 6.1 of the Code can also be used.

5.3.1 Design for Axial Load only

(Ceqn 6.55) of the Code can be used which is

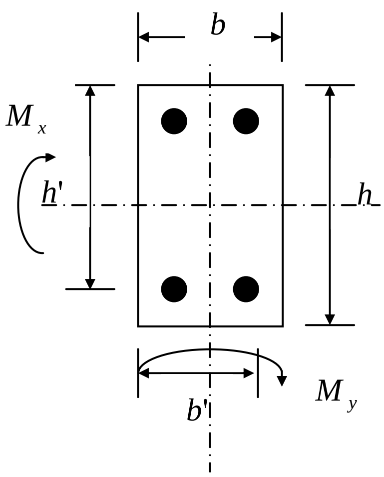
$N = 0.4 f_{cu} A_c + 0.75 A_{sc} f_y$. The equation is particularly useful for a column which cannot be subject to significant moments in such case as the column supporting a rigid structure or very deep beams. There is a reduction of approximately 10% in the axial load carrying capacity as compared with the normal value of $0.45 f_{cu} A_c + 0.87 A_{sc} f_y$ accounting for the eccentricity of $0.05h$.

Furthermore, (Ceqn 6.56) reading $N = 0.35 f_{cu} A_c + 0.67 A_{sc} f_y$ which is applicable to columns supporting an approximately symmetrical arrangement of beams where (i) beams are designed for u.d.l.; and (ii) the beam spans do not differ by more than 15% of the longer. The further reduction is to account for extra moment arising from asymmetrical loading.

5.3.2 Design for Axial Load and Biaxial Bending :

The general section design of a column is accounted for the axial loads and biaxial bending moments acting on the section. Nevertheless, the Code has reduced biaxial bending into uniaxial bending in design. The procedure for determination of the design moment, either M_x' or M_y' bending about the major or minor axes is as follows :

Determine b' and h' as defined by the diagram. In case there are more than one row of bars, b' and h' can be measured to the centre of the group of bars.



(i) Compare $\frac{M_x}{h'}$ and $\frac{M_y}{b'}$.

If $\frac{M_x}{h'} \geq \frac{M_y}{b'}$, use $M_x' = M_x + \beta \frac{h'}{b'} M_y$

If $\frac{M_x}{h'} < \frac{M_y}{b'}$, use $M_y' = M_y + \beta \frac{b'}{h'} M_x$

where β is to be determined from Table 5.1 which is reproduced from Table 6.14 of the Code under the pre-determined $\frac{N}{bhf_{cu}}$;

$N/(bhf_{cu})$	0	0.1	0.2	0.3	0.4	0.5	≥ 0.6
β	1.00	0.88	0.77	0.65	0.53	0.42	0.30

Table 5.1 – Values of the coefficients β

(ii) The M_x' or M_y' will be used for design by treating the section as either (a) resisting axial load N and moment M_x' bending about major axis; or (b) resisting axial load N and moment M_y' bending about minor axis as appropriate.h

5.3.3 Concrete Stress Strain Curve and Design Charts

The stress strain curve for column section design is in accordance with Figure 3.8 of the Code. It should be noted that Amendment No. 1 has revised the Figure by shifting ε_0 to $\dfrac{1.34(f_{cu}/\gamma_m)}{E_c}$. With this revision, the detailed design formulae and design charts have been formulated and enclosed in Appendix F. Apart from the derivation for the normal 4-bar column, the derivation in Appendix F has also included steel reinforcements uniformly distributed along the side of the column idealized as continuum of reinforcements with symbol A_{sh}. The new inclusion has allowed more accurate determination of load carrying capacity of column with many bars along the side as illustrated in Figure 5.4 which is particularly useful for columns of large cross sections. The user can still choose to lump the side reinforcements into the 4 corner bars, with correction to the effective depth as in conventional design by setting $A_{sh} = 0$ in the derived formulae.

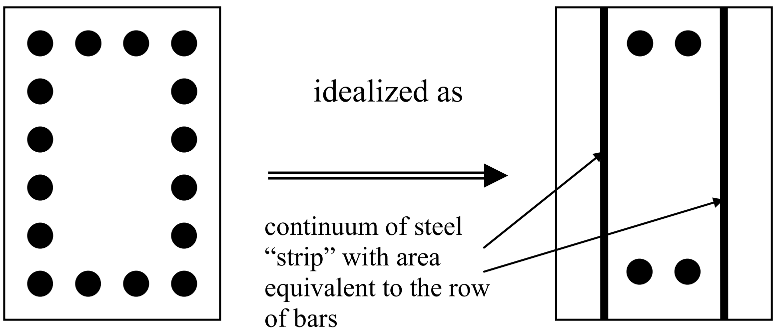


Figure 5.4 – Idealization of steel reinforcements in large column

Figure 5.5 shows the difference between the 2 idealization. It can be seen that the continuum idealization is more economical generally except at the peak moment portion where the 4 bar column idealization shows slight over-design.

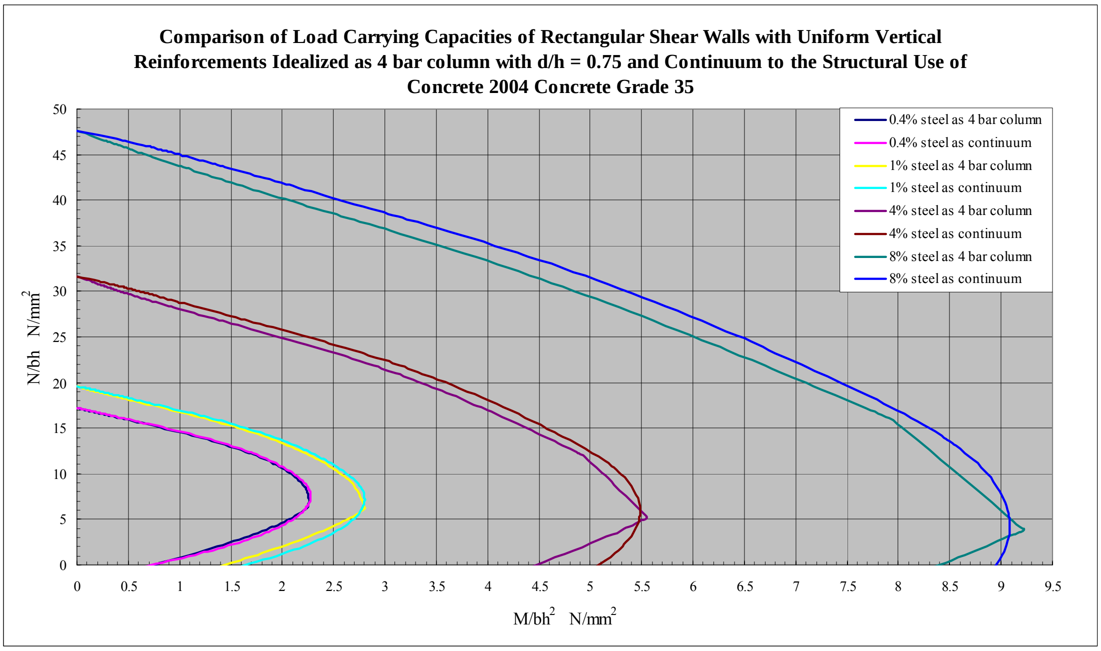


Figure 5.5 – Comparison between Continuum and 4-bar column Idealization

Worked Example 5.5 :

Consider a column of sectional size $b = 400$ mm, $h = 600$ mm, concrete grade 35 and under an axial load and moments of

$$N = 4000\,\text{kN}, \quad M_x = 250\,\text{kNm}, \quad M_y = 150\,\text{kNm},$$

cover to longitudinal reinforcements = 40 mm

Assume a 4-bar column and T40 bars, $h' = 600 - 40 - 20 = 540$ mm;

$b' = 400 - 40 - 20 = 340$ mm;

$$\frac{N}{f_{cu}bh} = \frac{4000000}{35 \times 400 \times 600} = 0.476\,;$$

$\beta = 0.446$ from Table 5.1 or Table 6.14 of the Code;

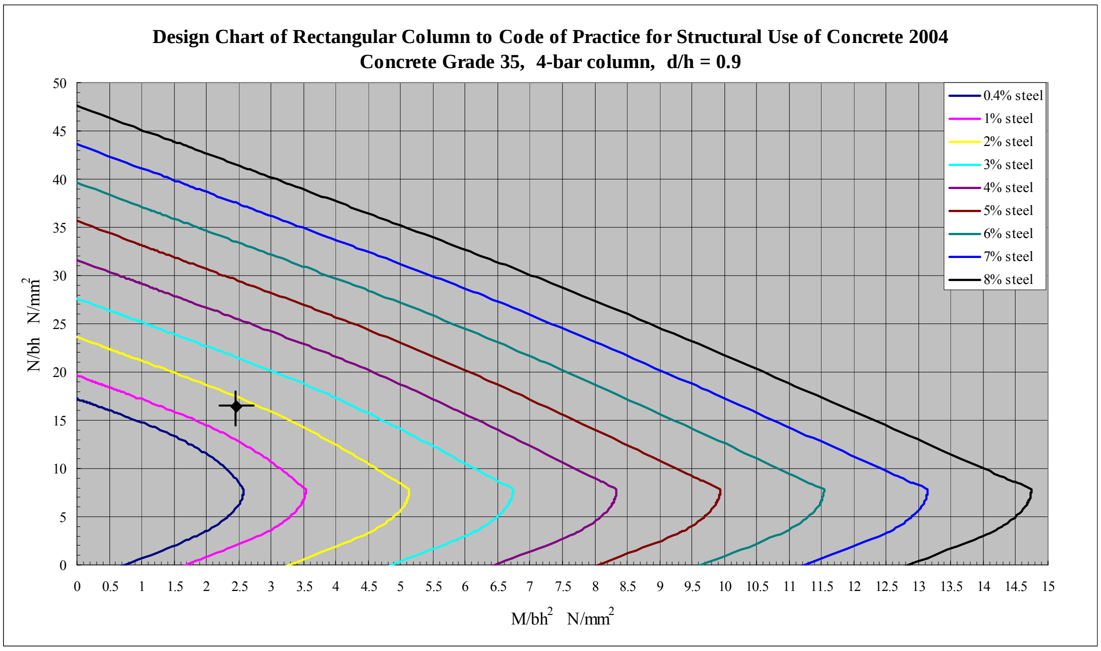
$$\frac{M_x}{h'} = 0.463 > \frac{M_y}{b'} = 0.441\,;$$

$$\therefore M_x' = M_x + \beta \frac{h'}{b'} M_y = 250 + 0.446 \times \frac{540}{340} \times 150 = 356\,\text{kNm}$$

$$\frac{N}{bh} = 16.67\,; \qquad \frac{M}{bh^2} = \frac{356 \times 10^6}{400 \times 600^2} = 2.47\,; \qquad \frac{d}{h} = \frac{540}{600} = 0.9$$

Use Chart F-9 in Appendix F as extracted in Figure 5.6, 1.8% steel is approximated which amounts to $400 \times 600 \times 0.018 = 4320$ mm^2, or 6-T32 (Steel provided is 4826mm^2) The section design is also shown in Figure 5.6,

with two additional T20 bars to avoid wide bar spacing.



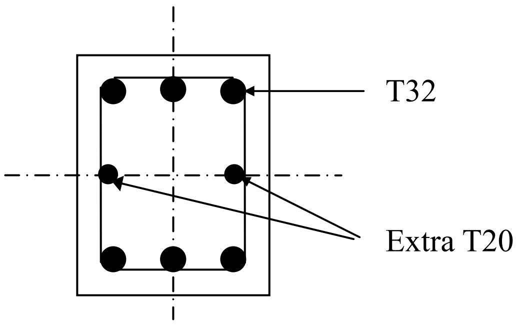


Figure 5.6 – Design Chart and Worked Re-bar details for Worked Example 5.5

Worked Example 5.6 :

Consider a column of sectional size $b = 800$ mm, $h = 1000$ mm, concrete grade 40 and under an axial load and moments

$N = 14400$ kN, $M_x = 2000$ kNm, $M_y = 1500$ kNm,

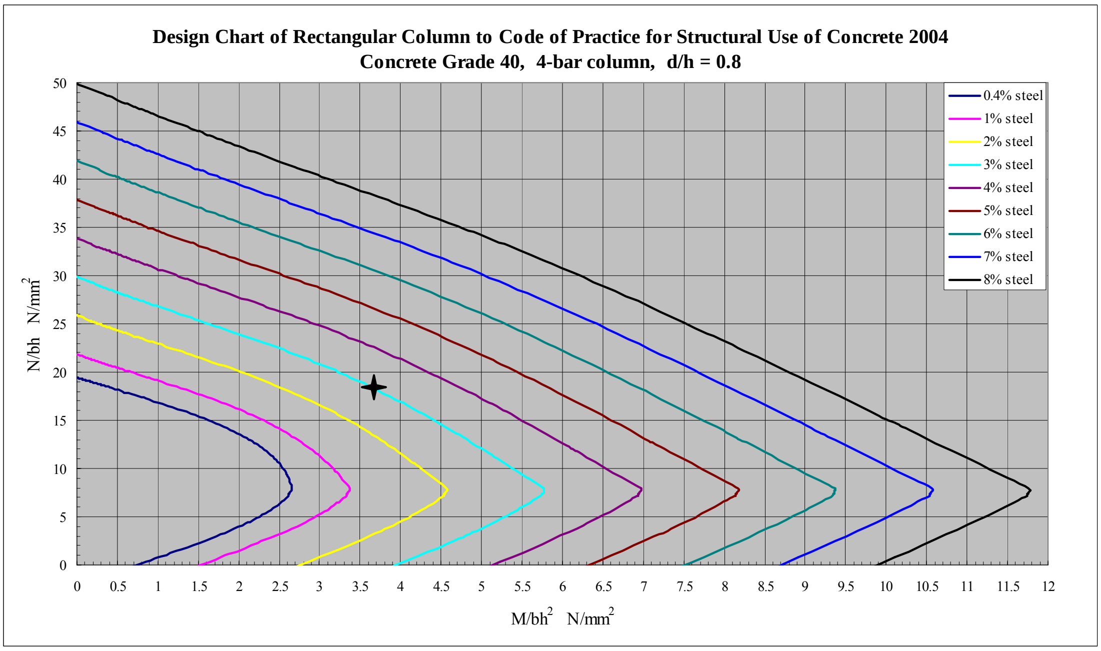
concrete cover to longitudinal reinforcement = 40 mm;
Approximation as a 4-bar column and assume $d/h = 0.8$
$b' = 0.8 \times 800 = 640$ mm; $h' = 0.8 \times 1000 = 800$ mm;
$\frac{N}{f_{cu}bh} = \frac{14400000}{40 \times 800 \times 1000} = 0.45$; $\beta = 0.475$ from Table 5.1 or Table 6.14 of the Code;

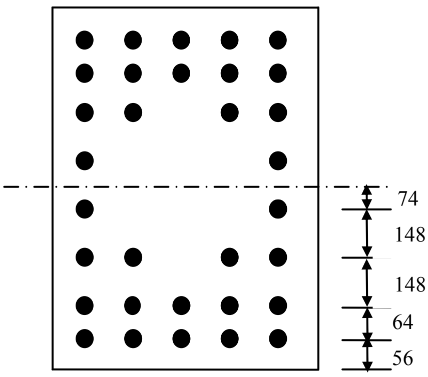
$$\frac{M_x}{h'} = \frac{2000}{800} = 2.5 > \frac{M_y}{b'} = \frac{1500}{640} = 2.34;$$

$$\therefore M_x' = M_x + \beta \frac{h'}{b'} M_y = 2000 + 0.475 \times \frac{800}{640} \times 1500 = 2890.6 \text{ kNm}$$

$$\frac{N}{bh} = 18; \qquad \frac{M}{bh^2} = \frac{2890.6 \times 10^6}{800 \times 1000^2} = 3.61;$$

Use Chart F-12 in Appendix F as extracted in Figure 5.7, 3.0% steel is approximated which amounts to $0.031 \times 800 \times 1000 = 24{,}800 \text{ mm}^2$, or 32-T32 (Steel provided is 25,736mm^2) The arrangement of steel bars is also shown in Figure 5.7. It should be noted that alternate lapping may be required if the column is contributing in lateral load resisting system as the steel percentage exceeds 2.6% as per discussion in 5.4(ii) of this Manual.





Centre of mass of the reinforcements in one half of the section below the centre line is

$$\frac{5 \times 434 + 5 \times 370 + 4 \times 222 + 2 \times 74}{16} = 316$$

So $h' = 316 + 500 = 816$

$$\frac{d}{h} = 0.816 > 0.8$$

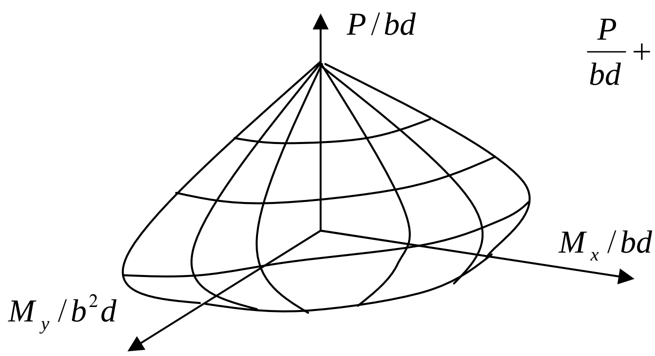
So the original use of $\frac{d}{h} = 0.8$ is OK

Figure 5.7 – Chart and Column Section for Worked Example 5.6

The back-calculation in Figure 5.7 has shown that the $\frac{d}{h}$ ratio is the steel bar arrangement is 0.816 which is greater than the original assumed value of 0.8. So the use of the chart is conservative.

5.3.4 Alternatively, the design of reinforcements can be based on formulae derived in Appendix F. However, as the algebraic manipulations are very complicated (may involve solution of 4^{th} polynomial equations) and cases are many, the approach is practical only by computer methods. Nevertheless, spread sheets have been prepared and 2 samples are enclosed at the end of Appendix F.

5.3.5 The approach by the previous British Code CP110 is based on interaction formula by which the moments of resistance in both directions under the axial loads are determined with the pre-determined reinforcements and the "interaction formula" is checked. The approach is illustrated in Figure 5.8.



$$\frac{P}{bd}+\left(\frac{M_x}{M_{ux}}\right)^{\alpha}+\left(\frac{M_y}{M_{uy}}\right)^{\alpha}\le 1$$

Figure 5.8 – Interaction formula for design of biaxial bending

5.3.6 Direct sectional analysis to Biaxial Bending without the necessity of converting the biaxial bending problem into a uniaxial bending problem :

Though the Code has provisions for converting the biaxial bending problem into a uniaxial bending problem by

(i) searching for the controlling bending axis; and

(ii) aggravate the moments about the controlling bending axis as appropriate to account for the effects of bending in the non-controlling axis;

a designer can actually solve the biaxial bending problem by locating the orientation and the neutral axis depth (which generally does not align with the resultant moment except for circular section) of the column section by balancing axial load and the bending in two directions. Theoretically, by

balancing axial load and the 2 bending moments, 3 equations can be obtained for solution of the neutral axis orientation, neutral axis depth and the required reinforcement. However the solution process, which is often based on trial and error approach, will be very tedious and not possible for irregular section without computer methods. Reinforcements generally need be pre-determined. Figure 5.9 illustrates the method of solution.

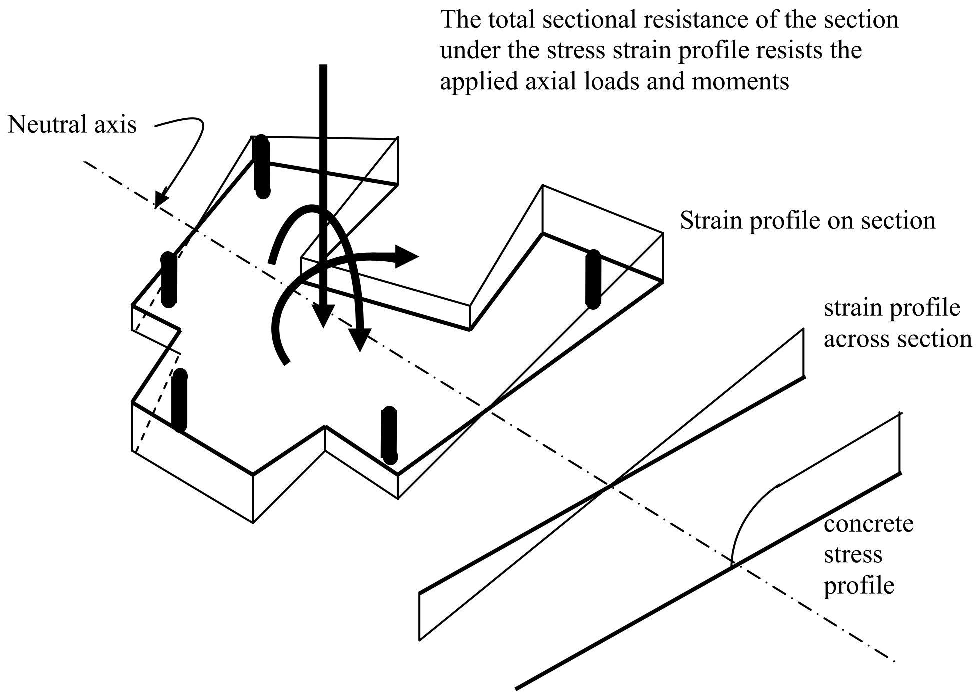


Figure 5.9 – General Biaxial Bending on irregular section

5.4 Detailing requirements for longitudinal bars in columns (generally by Cl. 9.5 and Cl. 9.9.2.1(a) of the Code, the ductility requirements applicable to columns contributing in lateral load resisting system are marked with “D”)

(i) Minimum steel percentage based on gross area of a column is 0.8% (Cl. 9.5.1 of the Code);

(ii) Maximum steel based on gross area of a column is (a) 4% except at lap which can be increased to 5.2% (D) for columns contributing to lateral load resisting system (Cl. 9.9.2.1(a) of the Code); and (b) 6% without laps and 10% at laps for other columns (Cl. 9.5.1 of the Code);

(iii) Bar diameter ≥ 12 mm (Cl. 9.5.1 of the Code);

(iv) The minimum number of bars should be 4 in rectangular columns and 6 in circular columns. In columns having a polygonal cross-section, at least one bar be placed at each corner (Cl. 9.5.1 of the Code);

(v) In any row of longitudinal bars in columns contributing to lateral load

resisting system, the smallest bar diameter used shall not be less than 2/3 of the largest bar diameter used (Cl. 9.9.2.1(a) of the Code). For example, T40 should not be used with T25 and below (D);

(vi) At laps, the sum of reinforcement sizes in a particular layer should not exceed 40% of the breadth at that section (Cl. 9.5.1 of the Code). The requirement is identical to that of beam as illustrated by Figure 3.13;

(vii) For columns contributing to lateral load resisting system, where the longitudinal bars pass through the beams at column beam joints, column bars shall satisfy $\phi \le 3.2h\sqrt{0.8f_{cu}} / f_y$ as per Ceqn 9.7 where h is the beam depth. For grade 35 concrete and based on high yield bar, the limiting bar diameter is simply $\phi \le 0.0368h$, i.e. if beam depth is 600 mm, $\phi \le 22.1$ implying maximum bar size is 20 mm. If the column is not intended to form a plastic hinge, the bar diameter can be increased by 25% (Cl. 9.9.2.1(a) of the Code) (D);

(viii) For columns contributing to lateral load resisting system, where the longitudinal bars terminate in a joint between columns and foundation members with possible formation of a plastic hinge in the column, the anchorage of the column bars into the joint region should commence at 1/2 of the depth of the foundation member or 8 times the bar diameter from the face at which the bars enter the foundation member. Where a plastic hinge adjacent to the foundation face cannot be formed, anchorage can commence at the interface with the foundation (Cl. 9.9.2.1(c) of the Code) as illustrated in Figure 5.10 (D);

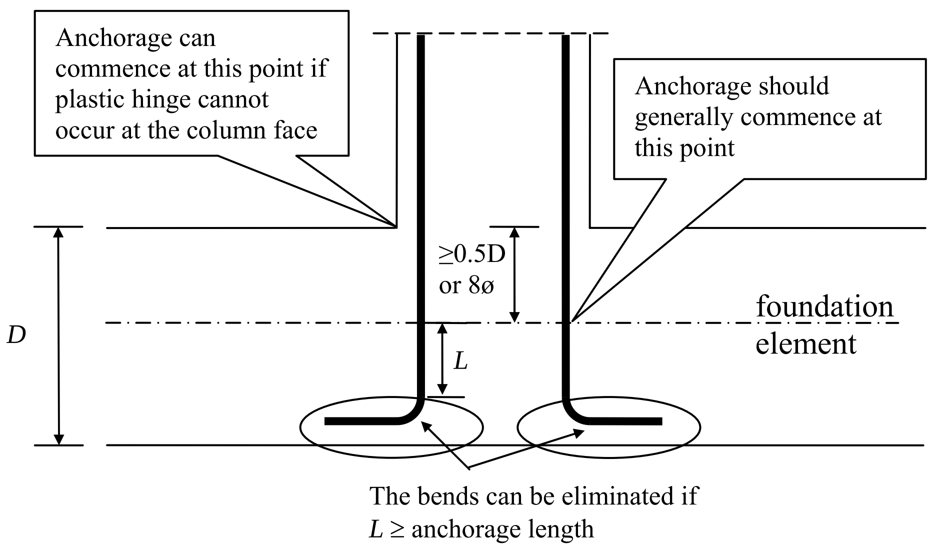


Figure 5.10 – Longitudinal Bar anchorage in foundation for columns contributing to lateral load resisting system

(ix) For columns contributing to lateral load resisting system, where the longitudinal bars anchor into beam (transfer beam or roof beam), in addition to the requirement in (viii), the bars should not be terminated in a joint area without a horizontal 90^o standard hook or an equivalent device as near as practically possible to the far side of the beam and not closer than 3/4 of the depth of the beam to the face of entry. Unless the column is designed to resist only axial load, the direction of bend must always be towards the far face of the column (Cl. 9.9.2.1(c) of the Code) as illustrated in Figures 5.11 and 5.12 (D);

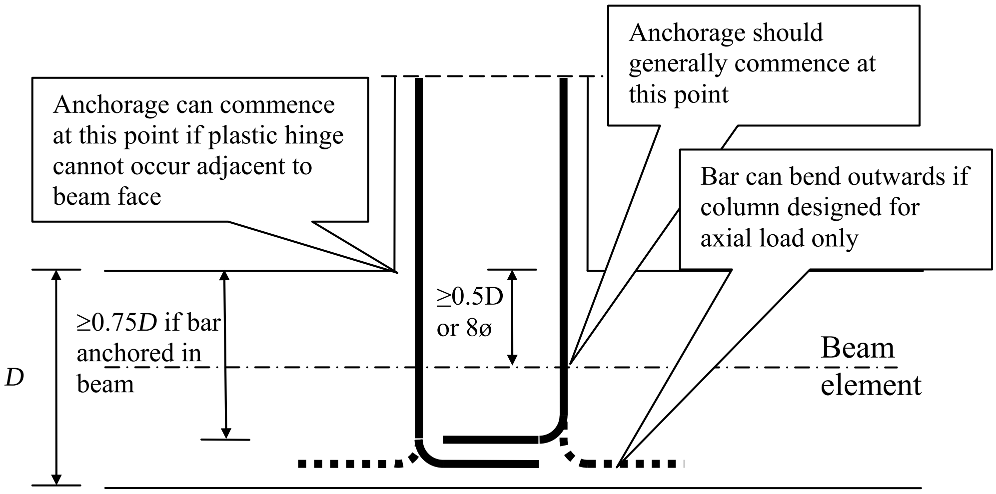


Figure 5.11 – Longitudinal Bar anchorage in Beam (Transfer Beam) for columns contributing to lateral load resisting system

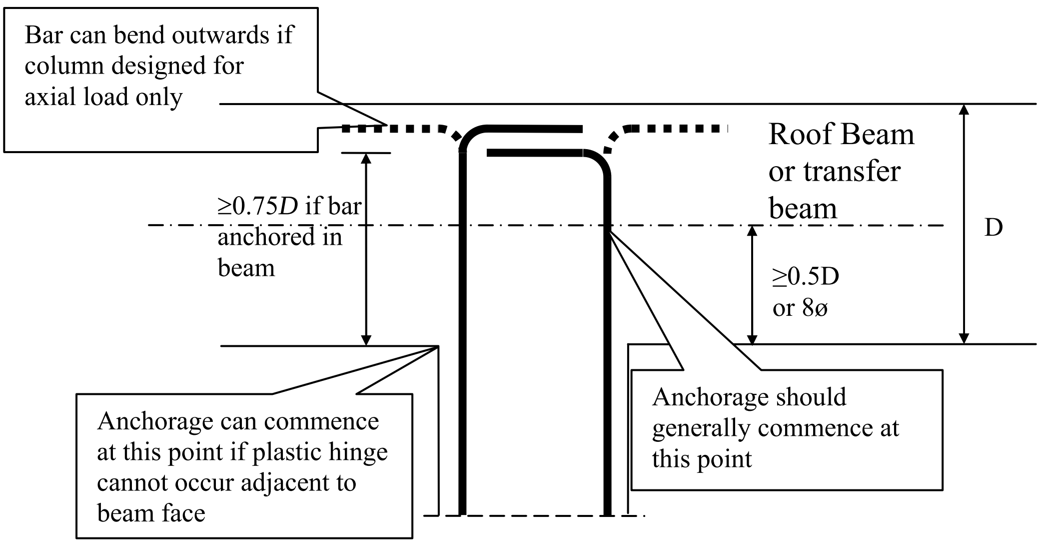


Figure 5.12 – Longitudinal Bar anchorage in Beam (Transfer Beam / Roof Beam) for columns contributing to lateral load resisting system

(x) For laps and mechanical couplers in a column contributing to lateral load resisting system, the centre of the splice must be within the middle quarter of the storey height of the column unless it can be shown that plastic hinges cannot develop in the column adjacent to the beam faces. As per discussion in Section 2.4, such lapping arrangement should be followed in locations such as column joining at pile caps or thick structures. Normal lapping at other floors can usually be followed unless there are very stiff beams, e.g. transfer beams (Cl. 9.9.2.1(d) of the Code). Examples are illustrated in Figure 5.13 (D);

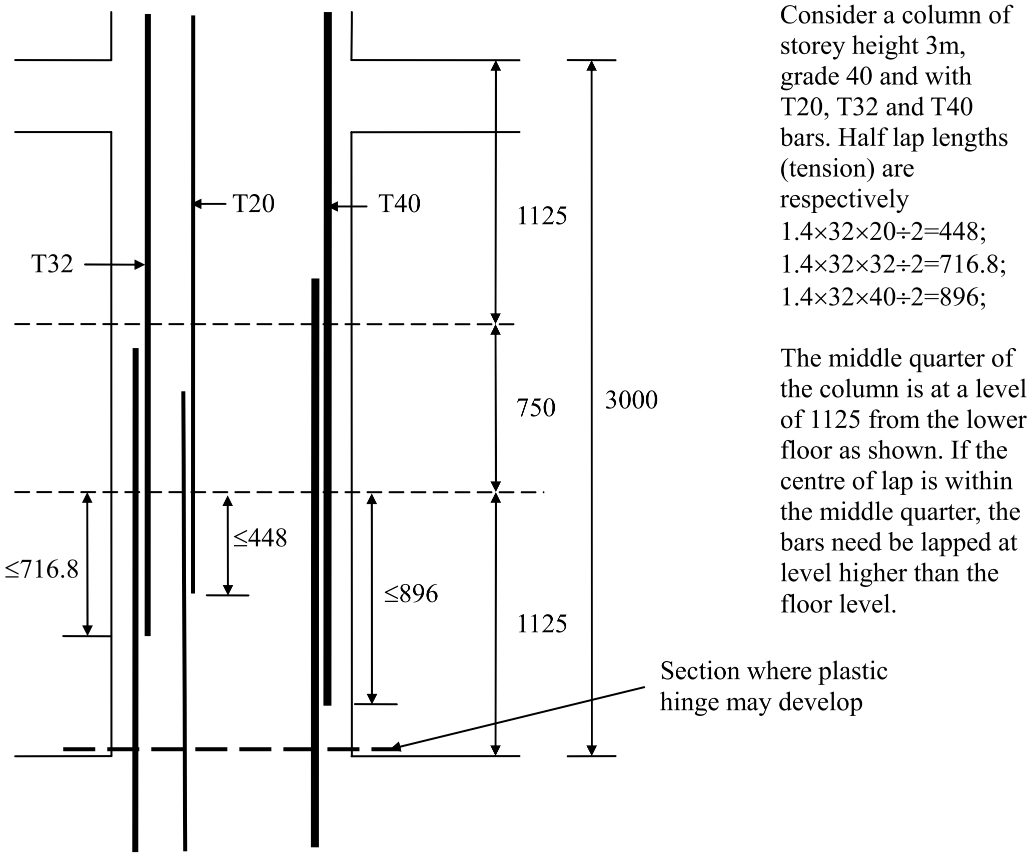


Figure 5.13 – Centre of lapping be within middle quarter of floor height in Column contributing to lateral load resisting system

(xi) Full strength welded splices may be used in any location (Cl. 9.9.2.1(d) of the Code);

(xii) As similar to limitation of lapping of bars in beams as described in Section 3.6(vii), longitudinal bars in columns contributing to lateral load resisting system shall not be lapped in a region where reversing stresses at the ultimate limit state may exceed $0.6 f_y$ in tension or compression

unless each lapped bar is confined by adequate links or ties satisfying (Ceqn 9.6), as explained in Section 3.6(vii) and illustrated by Figure 3.11. Summing up, lapping should be avoided from region with potential plastic hinge and with reversing stresses (Cl. 9.9.2.1(a) of the Code) (D);

(xiii) Minimum clear spacing of bars should be the greatest of (1)bar diameter; (2) 20 mm; and (3) aggregate size + 5 mm (Cl. 8.2 of the Code).

5.5 Detailing Requirements for transverse reinforcements in columns include the general requirements by Cl. 9.5.2 and the ductility requirements in Cl. 9.9.2.2 of the Code (marked with "D") for columns contributing to lateral load resisting system. Items (i) to (iv) below are requirements for columns not within "critical regions". "Critical region" is defined in item (v) and Figure 5.15 for columns contributing to lateral load resisting system:

(i) Diameter of transverse reinforcements $\geq$ the greater of 6 mm and 1/4 of longitudinal bar diameter (Cl. 9.5.2.1 of the Code);

(ii) The spacing of transverse reinforcement shall not exceed 12 times the diameter of the smallest longitudinal bar (Cl. 9.5.2.1 of the Code);

(iii) For rectangular or polygonal columns, every corner bar and each alternate bar (or bundle) shall be laterally supported by a link passing around the bar and having an included angle $\leq 135^{o}$. No bar within a compression zone shall be further than 150 mm from a restrained bar. Links shall be adequately anchored by hooks through angles $\geq 135^{o}$. See Figure 5.14 which is reproduced from Figure 9.5 of the Code (Cl. 9.5.2.2 of the Code);

(iv) For circular columns, loops or spiral reinforcement satisfying (i) to (ii) should be provided. Loops (circular links) should be anchored with a mechanical connection or a welded lap by terminating each end with a 135^{o} hook bent around a longitudinal bar after overlapping the other end of the loop. Spiral should be anchored either by welding to the previous turn or by terminating each end with a 135^{o} hook bent around a longitudinal bar and at not more than 25 mm from the previous turn. Loops and spirals should not be anchored by straight lapping, which causes spalling of the concrete cover (Cl. 9.5.2.2 of the Code). The details are also illustrated in Figure 5.14;

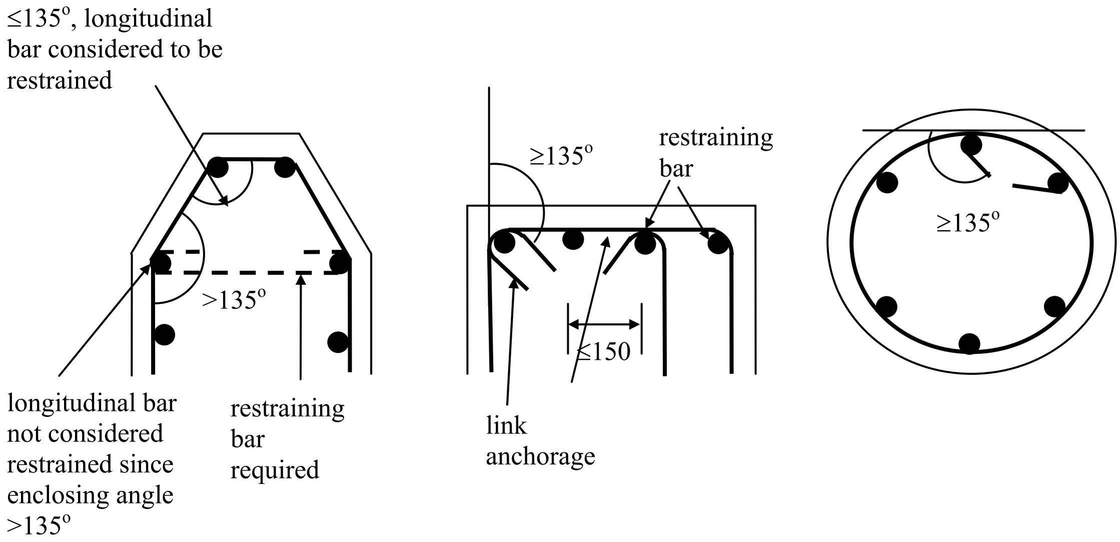


Figure 5.14 – Column transverse reinforcements outside "Critical Regions"

(v) Transverse reinforcements in "critical regions" within columns of limited ductile high strength concrete (contributing to lateral load resisting system) as defined in Figure 5.15 (Re Cl. 9.9.2.2 of the Code) shall have additional requirements as :

(a) For rectangular or polygonal columns, each (not only alternate) longitudinal bar or bundle of bars shall be laterally supported by a link passing around the bar having an included angle of not more than 135°. As such, Figure 5.16 shows the longitudinal bar anchorage requirements in "critical region" (Cl. 9.9.2.2(b) of the Code) (D);

(b) Spacing ≤ 1/4 of the least lateral column dimension in case of rectangular or polygonal column and 1/4 of the diameter in case of a circular column and 6 times the diameter of the longitudinal bar to be restrained (Cl. 9.9.2.2(b) of the Code) (D);

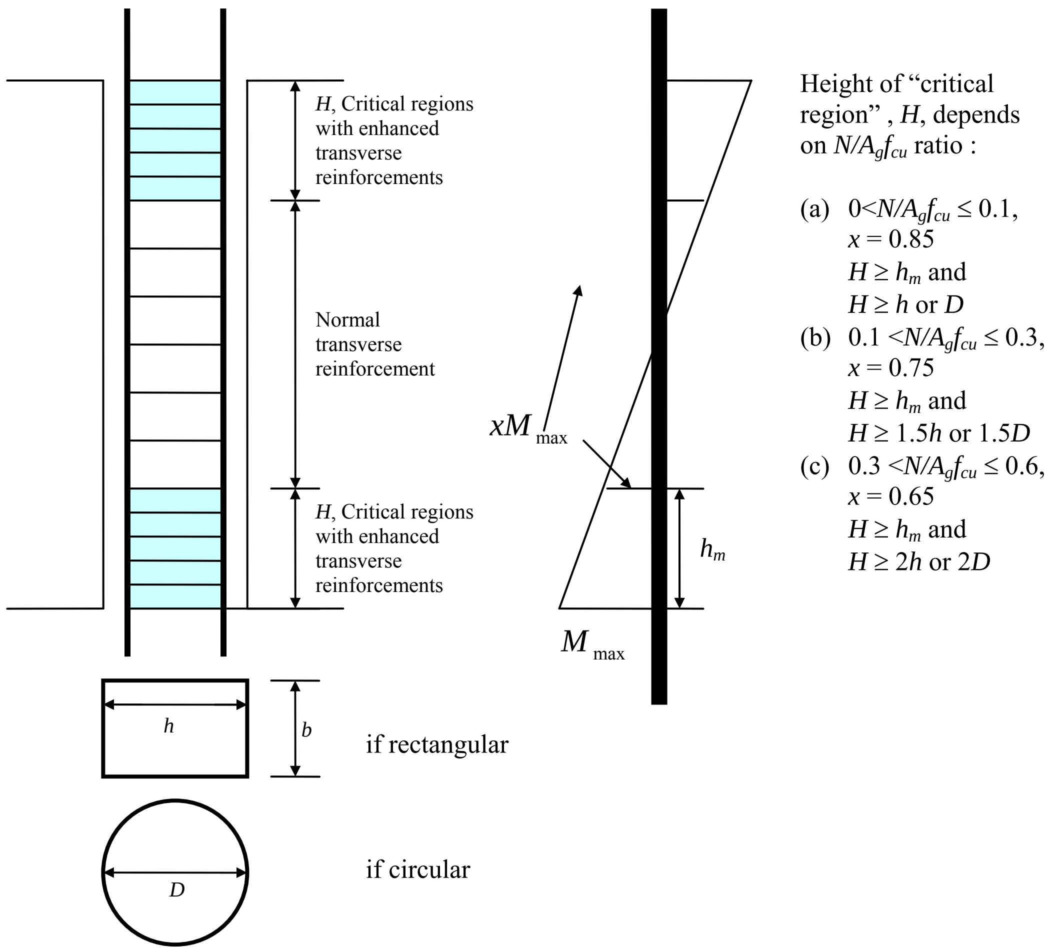


Figure 5.15 – "Critical Regions (Potential Plastic Hinge Regions)" in Columns contributing to lateral load resisting system

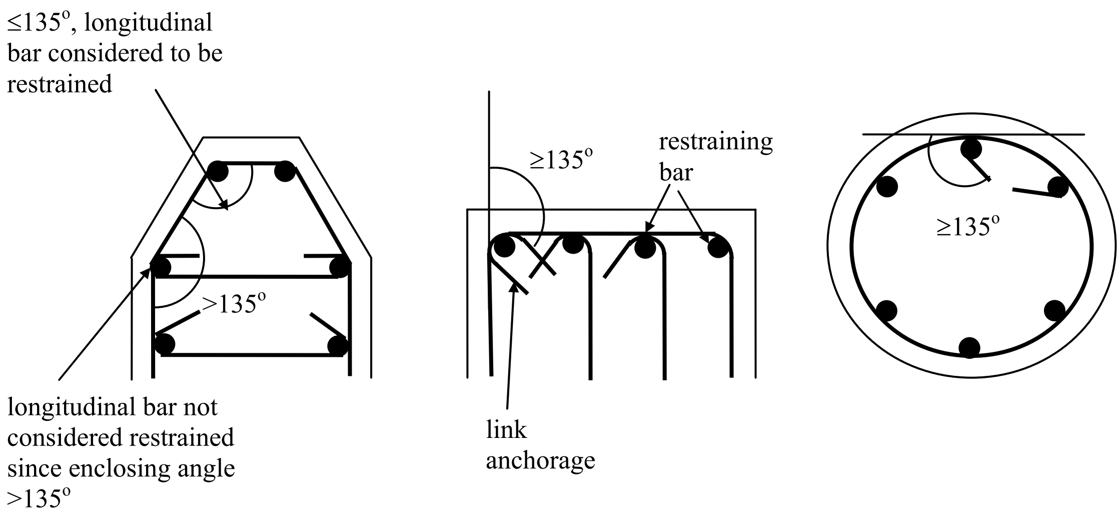


Figure 5.16 – Enhanced transverse reinforcements inside "Critical Regions" in columns contributing to lateral load resisting system

Worked Example 5.7 – for determination of "critical regions" within columns of limited ductile and high strength concrete

Consider a rectangular column of the following details :

Cross section 500×600 mm; height 3 m; grade 65; re-bars : T32

Loads and moments are as follows :

Axial Load 4875 kN

$M_x = 800$ kNm (at top), $M_x = 500$ kNm (at bottom)

$M_y = 450$ kNm (at top) $M_y = 300$ kNm (at bottom)

$$\frac{N}{A_g f_{cu}} = \frac{4875 \times 10^3}{500 \times 600 \times 40} = 0.25 \qquad \therefore x = 0.75 \text{ for determination of critical regions}$$

h_m for bending about X and Y directions are determined as per Figure 5.17.

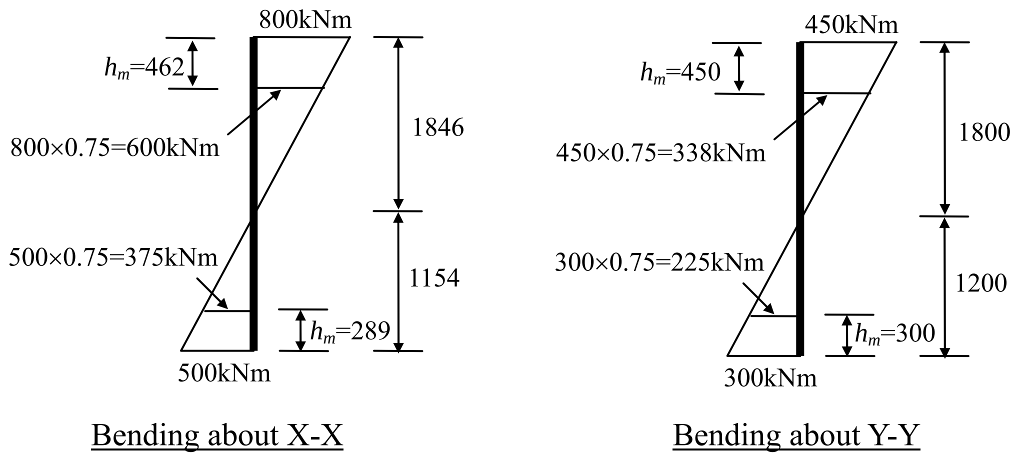
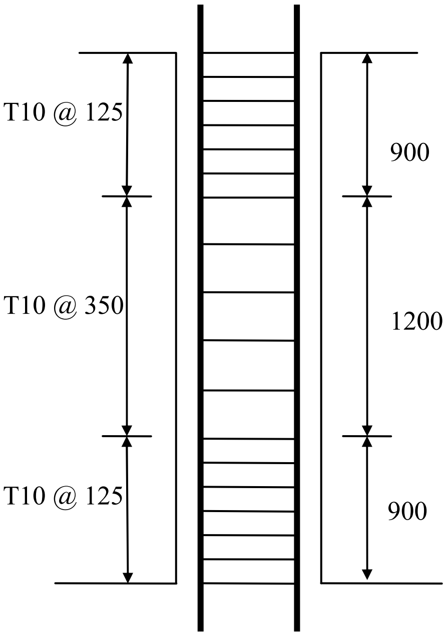


Figure 5.17 – Determination of critical heights in Worked Example 5.7

As the h_m are all less than $1.5h = 1.5 \times 600 = 900$, so the critical regions should then both be 1200 mm from top and bottom and the design of transverse reinforcements is as indicated in Figure 5.18 :



Transverse re-bars

(i) Within critical region (for columns of limited high strength concrete and contributing to lateral load resisting system only) :
Bar size 0.25×32 = 8 mm > 6 mm
Spacing : the lesser of
0.25×500 = 125 mm
6×32 = 192 mm
So spacing is 125 mm

(ii) Within normal region (regardless of whether the column is contributing to lateral load resisting system) :
Bar size 0.25×32 = 8 mm > 6 mm
Spacing 12×32 = 384 mm

Figure 5.18 –Transverse Reinforcement arrangement to Worked Example 5.7

6.0 Column-Beam Joints

6.1 General

The design criteria of a column-beam joint comprise (i) performance not inferior to the adjoining members at serviceability limit state; and (ii) sufficient strength to resist the worst load combination at ultimate limit state. To be specific, the aim of design comprise (a) minimization of the risk of concrete cracking and spalling near the beam-column interface; and (b) checking provisions against diagonal crushing or splitting of the joint and where necessary, providing vertical and horizontal shear links within the joint and confinement to the longitudinal reinforcements of the columns adjacent to the joint.

6.2 The phenomenon of "diagonal splitting" of joint

Diagonal crushing or splitting of column-beam joints is resulted from "shears" and unbalancing moment acting on the joints as illustrated in Figure 6.1(a) and 6.1(b) which indicate typical loadings acting on the joint. Figure 6.1(a) shows a joint with hogging moment on the right and sagging moment on the left, which may be due to a large applied horizontal shear from the right. In contrast, Figure 6.1(b) shows a joint with hogging moment on both sides which is the normal behaviour of a column beam joint under dominant gravity loads. However, it should be noted that the hogging moments on both sides may not balance.

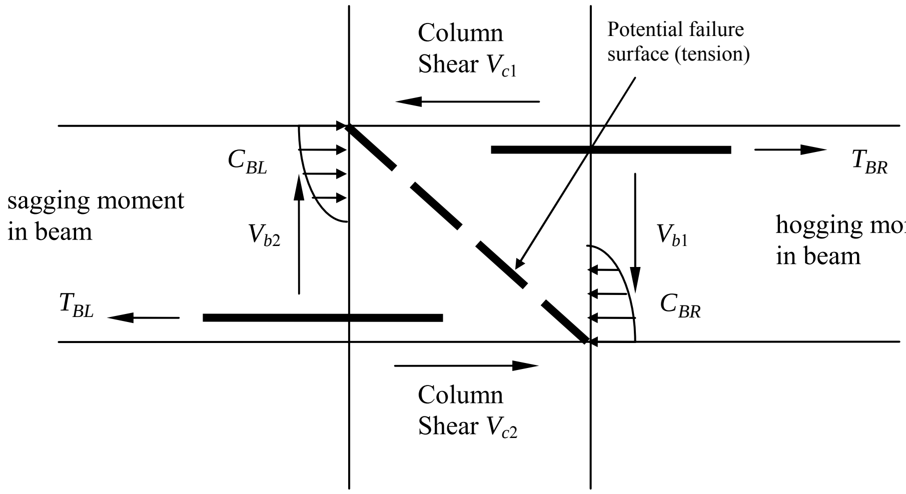


Figure 6.1(a) – Phenomenon of Diagonal Joint Splitting by moments of opposite signs on both sides of joint

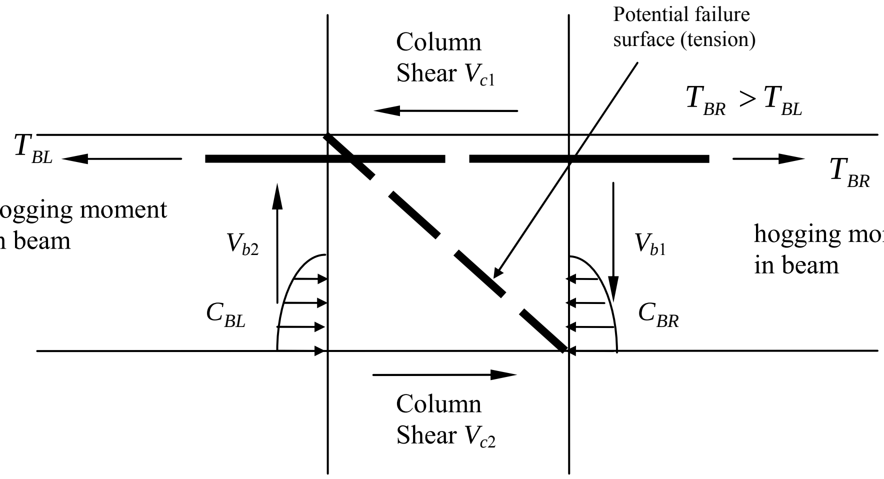


Figure 6.1(b) – Phenomenon of Diagonal Joint Splitting by moments of same sign on both sides of joint

In both cases, the unbalanced forces due to unbalanced flexural stresses by the adjoining beams on both sides of the joint tend to "tear" the joint off with a potential tension failure surface, producing "diagonal splitting". In co-existence with the bending moments, there are shears in the columns which usually tend to act oppositely. The effects by such shears can help to reduce the effects of shears on the column joints created by bending. Reinforcements in form of links may therefore be necessary if the concrete alone is considered inadequate to resist the diagonal splitting.

6.3 Design procedures :

(i) Work out the total nominal horizontal shear force across the joint V_{jh} in X and Y directions generally. V_{jh} should be worked out by considering forces acting on the upper half of the joint as illustrated in Figures 6.2(a) and 6.2(b). Figure 6.2(a) follows the case of Figure 6.1(a) in which the moments in the beams on both sides of the joint are of different signs (i.e. one hogging and one sagging). There is thus a net "shear" of $V_{jh} = T_{BL} + T_{BR} - V_c$ acting on the joint where $T_{BR} = f_y A_{sR}$ and $C_{BL} = T_{BL} = f_y A_{sL}$ are the pull and push forces by the beams in which A_{sR} and A_{sL} are the steel areas of the beams. This approach which originates

from the New Zealand Code NSZ 3103 requires T_{BR} and T_{BL} be increased by 25% under the load capacity concept in which the reinforcing bars in the beam will be assumed to have steel stress equal to 125% yield strength of steel if such assumption will lead to the most adverse conditions. Thus the following equation can be listed :

$$V_{jh} = T_{BL} + T_{BR} - V_c = 1.25 f_y (A_{sL} + A_{sR}) - V_c \qquad \text{(Eqn 6.1)}$$

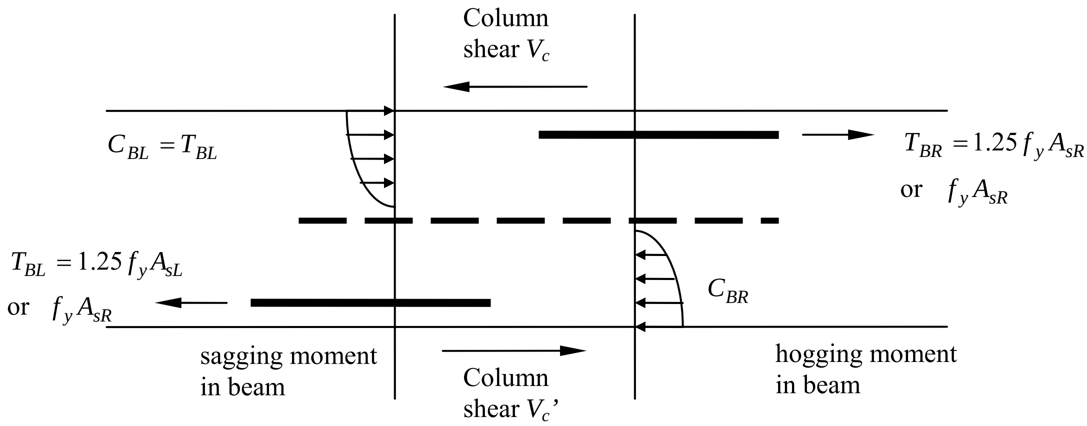


Figure 6.2(a) – Calculation of V_{jh}, opposite sign beam moments on both sides

However, there is a comment that New Zealand is a country of severe seismic activity whilst in Hong Kong the dominant lateral load is wind load. The 25% increase may therefore be dropped and (Eqn 6.1) can be re-written as

$$V_{jh} = T_{BL} + T_{BR} - V_c = f_y (A_{sL} + A_{sR}) - V_c \qquad \text{(Eqn 6.2)}$$

Furthermore, as V_c counteracts the effects of T_{BR} and T_{BL} and V_c is generally small, V_c can be ignored in design. Nevertheless, the inclusion of V_c can help to reduce steel congestion in case of high shear.

Similarly Figure 6.2(b) follows the case of Figure 6.1(b) which may be the case of unbalancing moments due to gravity load without lateral loads or even with the lateral loads, such loads are not high enough to reverse any of the beam moments from hogging to sagging. By similar argument and formulation as for that of Figure 6.2(a), (Eqn 6.3) and (Eqn 6.4) can be formulated for Figure 6.2(b)

$$V_{jh} = T_{BR} - T_{BL} - V_c = 1.25 f_y A_{sR} - \frac{M_L}{z_L} - V_c \qquad \text{(Eqn 6.3)}$$

$$V_{jh} = T_{BR} - T_{BL} - V_c = f_y A_{sR} - \frac{M_L}{z_L} - V_c \qquad \text{(Eqn 6.4)}$$

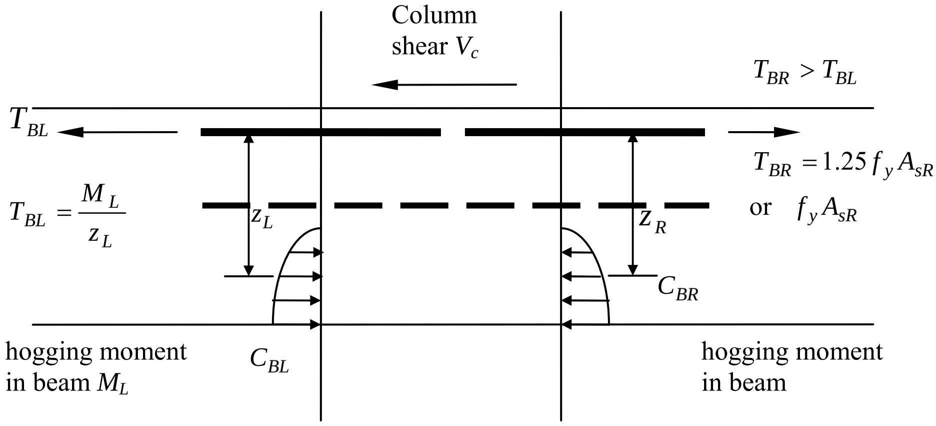


Figure 6.2(b) – Calculation of V_{jh}, same sign beam moments on both sides

Equations (Eqn 6.2) and (Eqn 6.4) will be used in this Manual.

(ii) With the V_{jh} determined, the nominal shear stress is determined by (Ceqn 6.71) in the Code. $v_{jh} = \frac{V_{jh}}{b_j h_c}$

where h_c is the overall depth of the column in the direction of shear
$b_j = b_c$ or $b_j = b_w + 0.5h_c$ whichever is the smaller when $b_c \geq b_w$;
$b_j = b_w$ or $b_j = b_c + 0.5h_c$ whichever is the smaller when $b_c < b_w$;
where b_c is the width of column and b_w is the width of the beam.

Cl. 6.8.1.2 of the Code specifies that "At column of two-way frames, where beams frame into joints from two directions, these forces need be considered in each direction independently." So v_{jh} should be calculated independently for both directions even if they exist simultaneously and both be checked that they do not exceed $0.25 f_{cu}$.

(iii) Horizontal reinforcements based on Ceqn 6.72 reading $A_{jh} = \frac{V_{jh}^*}{0.87 f_{yh}} \left(0.5 - \frac{C_j N^*}{A_g f_{cu}} \right)$ should be worked out in both directions and

be provided in the joint as horizontal links. In Ceqn 6.72, V_{jh}^{*} should be the joint shear in the direction (X or Y) under consideration and N^{*} be the minimum column axial load. If the numerical values arrived at is positive, shear reinforcements of cross sectional areas A_{jh} should be provided. It may be more convenient to use close links which can serve as confinements to concrete and horizontal shear reinforcements in both directions. If the numerical values arrived by (Ceqn 6.72) becomes negative, no horizontal shear reinforcements will be required;

(iv) Similarly vertical reinforcements based on (Ceqn 6.73) reading $A_{jv} = \dfrac{0.4(h_b / h_c)V_{jh}^{*} - C_j N^{*}}{0.87 f_{yv}}$ should be worked out in both directions and be provided in the joint as vertical links or column intermediate bars (not corner bars). Again if the numerical values arrived by (Ceqn 6.73) is negative, no vertical shear reinforcements will be required;

(v) Notwithstanding the provisions arrived at in (iii) for the horizontal reinforcements, confinements in form of closed links within the joint should be provided as per Cl. 6.8.1.7 of the Code as :
   (a) Not less than that in the column shaft as required by Cl. 9.5.2 of the Code, i.e. Section 5.5 (i) to (iv) of this Manual if the joint has a free face in one of its four faces;
   (b) Reduced by half to that provisions required in (a) if the joint is connected to beams in all its 4 faces;
   (c) Link spacing ≤ 10Ø (diameter of smallest column bar) and 200 mm.

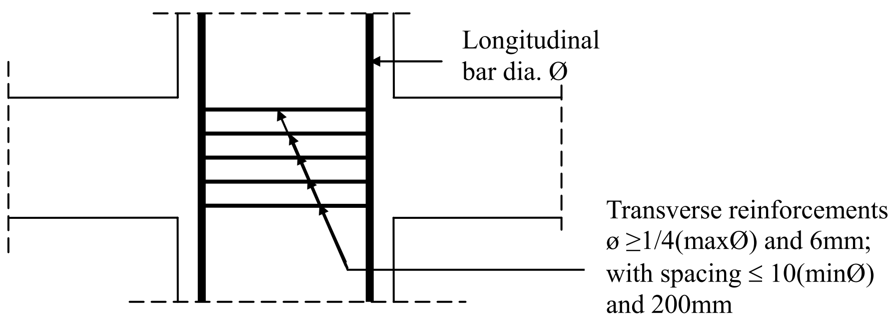


Figure 6.3 – Minimum transverse reinforcements in Column Beam Joint

6.4 Worked Example 6.1:

Consider the column beam joints with columns and beams adjoining as indicated in Figure 6.4 in the X-direction and Y-directions. Concrete grade is 40. All loads, shears and moments are all ultimate values. The design is as follows :

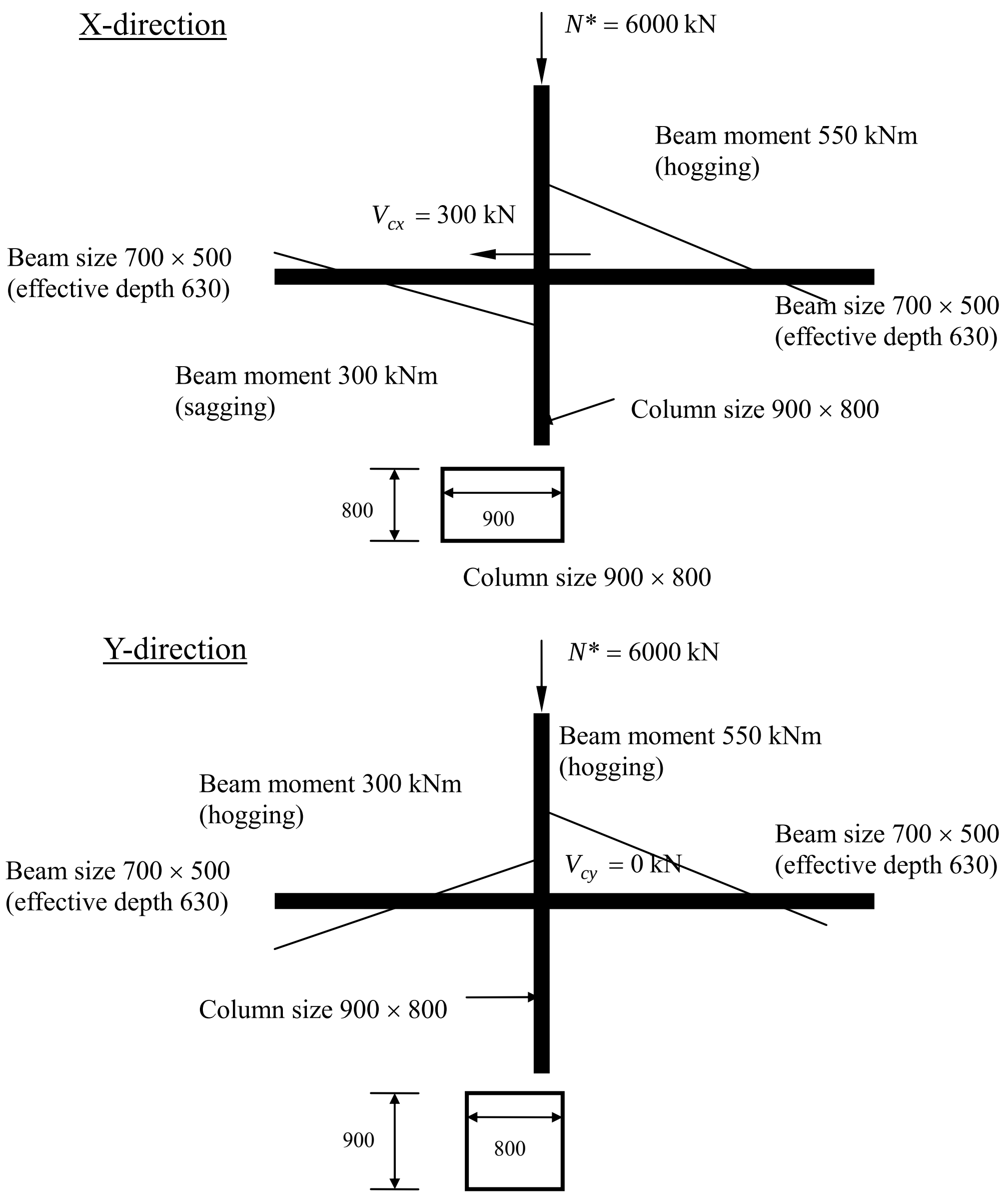
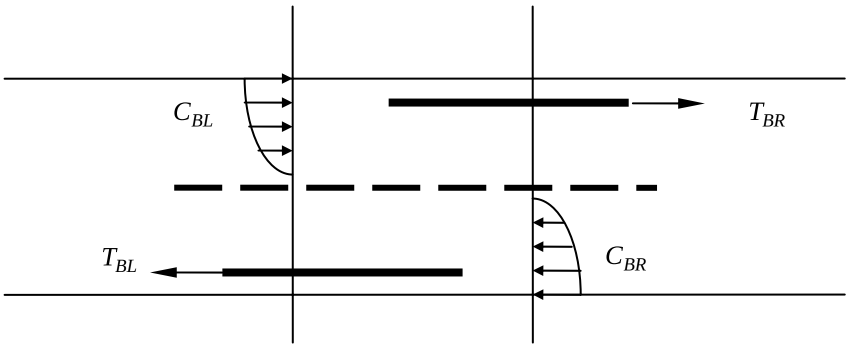


Figure 6.4 – Design Example for Column Beam Joint

(i) Check nominal shear stress :

X-direction

The moments on the left and right beams are of opposite signs. So Figure 6.2(a) is applicable. The top steel provided on the right beam is 3T32, as designed against the ultimate hogging moment of 550kNm with $A_{sR} = 2413\,\text{mm}^2$ whilst the bottom steel provided on the left beam is 4T20 with $A_{sL} = 1257\,\text{mm}^2$, again as designed against the ultimate sagging moment of 300kNm.



$T_{BR} = 460 \times 2413 \times 10^{-3} = 1109.98\,\text{kN};$ $\quad C_{BR} = T_{BR}$

$T_{BL} = 460 \times 1257 \times 10^{-3} = 578.22\,\text{kN};$ $\quad C_{BL} = T_{BL}$

So the total shear is

$$V_{jx} = T_{BL} + T_{BR} - V_{cx} = 1109.98 + 578.22 - 300 = 1388.2\,\text{kN}$$

In the X-direction $\quad h_c = 900$

As $b_c = 800 > b_w = 500$,the effective joint width is the smaller of

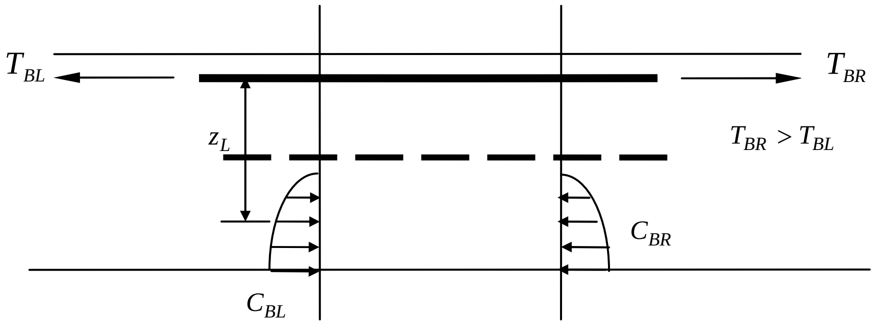
$b_c = 800$ and $b_w + 0.5h_c = 500 + 0.5 \times 900 = 950$, so $b_j = 800$

So, checking against Cl. 6.8.1.3 of the Code,

$$v_{jx} = \frac{V_{jx}}{b_j h_c} = \frac{1388.2 \times 10^3}{800 \times 900} = 1.93\,\text{MPa} < \; 0.25 f_{cu} = 10\,\text{MPa}$$

Y-direction

The moments on the left and right beams are of equal sign, both hogging. So Figure 6.2(b) is applicable. As the moment on the right beam is higher, the potential plastic hinge will be formed on the right beam. Again the top steel provided in the right beam is 3T32 as designed against the ultimate hogging moment of 550 kNm.



$T_{BR} = 460 \times 2413 \times 10^{-3} = 1109.98$ kN; $C_{BR} = T_{BR}$

T_{BL} is to be determined by conventional beam design method for the ultimate hogging moment of 300 kNm

$\dfrac{M}{bd^2} = 1.512$ MPa, $\dfrac{z}{d} = 0.948$, $A_{sL} = 1254.49$ mm^2;

$T_{BL} = 0.87 f_y A_{sL} = 502.05$ kN

As the column shear is zero, by (Eqn 6.3)

$V_{jy} = 1109.98 - 502.05 = 607.93$ kN

In the Y-direction $h_c = 800$, $b_c = 900$

and $b_w + 0.5h_c = 500 + 0.5 \times 800 = 900$, so $b_j = 900$

So, checking against Cl. 6.8.1.3 of the Code,

$$v_{jy} = \frac{V_{jy}}{b_j h_c} = \frac{607.93 \times 10^3}{900 \times 800} = 0.84 \text{ MPa} < 0.25 f_{cu} = 10 \text{ MPa}$$

(ii) To calculate the horizontal joint reinforcement by Ceqn 6.72, reading

$$A_{jh} = \frac{V_{jh}^*}{0.87 f_{yh}} \left(0.5 - \frac{C_j N^*}{A_g f_{cu}} \right)$$

where $C_j = \dfrac{V_{jh}}{V_{jx} + V_{jy}}$

<u>X-direction</u>

$$C_{jx} = \frac{V_{jx}}{V_{jx} + V_{jy}} = \frac{1388.2}{1388.2 + 607.93} = 0.695$$

$$A_{jhx} = \frac{V_{jhx}^*}{0.87 f_{yh}} \left(0.5 - \frac{C_{jx} N^*}{A_g f_{cu}} \right) = \frac{1388.2 \times 10^3}{0.87 \times 460} \left(0.5 - \frac{0.695 \times 6000000}{900 \times 800 \times 40} \right)$$

$= 1232$ mm^2

<u>Y-direction</u>

$$C_{jy} = \frac{V_{jy}}{V_{jx} + V_{jy}} = \frac{607.93}{1388.2 + 607.93} = 0.305$$

$$A_{jhy} = \frac{V_{jhy}^*}{0.87 f_{yh}}\left(0.5 - \frac{C_{jy} N^*}{A_g f_{cu}}\right) = \frac{607.93\times 10^3}{0.87\times 460}\left(0.5 - \frac{0.305\times 6000000}{900\times 800\times 40}\right)$$
$$= 663\ \text{mm}^2$$

Use 6T12 close stirrups (Area provided = 1357 mm²) which can adequately cover shear reinforcements in both directions

(iii) To calculate the vertical joint reinforcement by (Ceqn 6.73 of the Code), reading $A_{jv} = \frac{0.4(h_b / h_c)V_{jh}^* - C_j N^*}{0.87 f_{yh}}$

X-direction

$$A_{jvx} = \frac{0.4(h_b / h_c)V_{jh}^* - C_j N^*}{0.87 f_{yh}} = \frac{0.4(700/900)\times 1388200 - 0.695\times 6000000}{0.87\times 460}$$

$= -9341$. So no vertical shear reinforcement is required.

Y-direction

$$A_{jvy} = \frac{0.4(h_b / h_c)V_{jh}^* - C_j N^*}{0.87 f_{yh}} = \frac{0.4(700/800)\times 607930 - 0.305\times 6000000}{0.87\times 460}$$
$= -4041$. Again no vertical shear reinforcement is required.

(iv) The provision of outermost closed stirrups in the column shaft is T12 at approximately 120mm which is in excess of the required confinement as listed in 6.3(v). So no additional confinement is requirement.

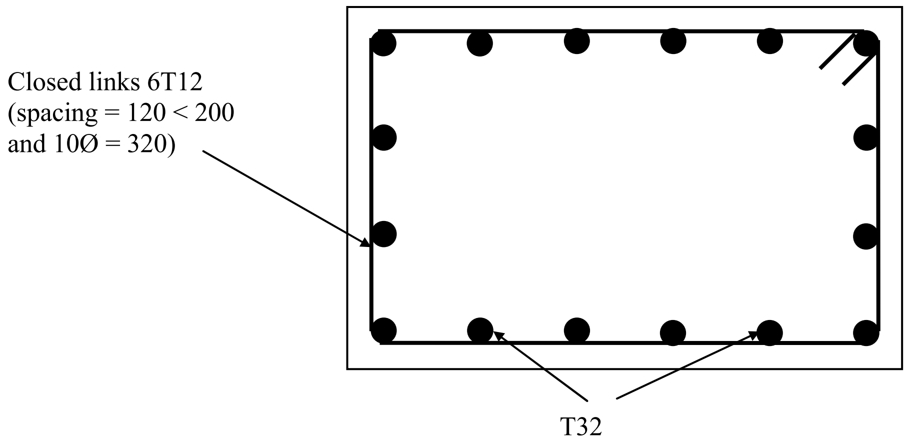


Figure 6.5 – Details of Column Beam Joint Detail for Column Beam Joint (Plan) Design – Other details omitted for clarity

7.0 Walls

7.1 Design Generally

7.1.1 Similar to column by design to resist axial loads and moments.

7.1.2 The design ultimate axial force may be calculated on the assumption that the beams and slabs transmitting force to it are simply supported. (Re Cl. 6.2.2.2(a) and Cl. 6.2.2.3(a) of the Code).

7.1.3 Minimum eccentricity for transverse moment design is the lesser of 20 mm or $h/20$, as similar to columns.

7.2 Categorization of Walls

Walls can be categorized into (i) slender walls; (ii) stocky walls; (iii) reinforced concrete walls; and (iv) plain walls.

7.3 Slender Wall Section Design

7.3.1 Determination of effective height l_e (of minor axis generally which controls) –

(i) in case of monolithic construction, same as that for column; and

(ii) in case of simply supported construction, same as that for plain wall.

7.3.2 Limits of slender ratio (Re Table 6.15 of the Code) –

(i) 40 for braced wall with reinforcements < 1%;

(ii) 45 for braced wall with reinforcements ≥ 1%;

(iii) 30 for unbraced wall.

7.3.3 Other than 7.3.1 and 7.3.2, reinforced concrete design is similar to that of columns.

7.4 Stocky Wall

7.4.1 As similar to column, stocky walls are walls with slenderness ratio < 15 for braced walls and slenderness ratio < 10 for unbraced walls;

7.4.2 Stocky reinforced wall may be designed for axial load n_w only by (Ceqn 6.59) of the Code provided that the walls support approximately symmetrical arrangement of slabs with uniformly distributed loads and the spans on either side do not differ by more than 15%;

$$n_w \le 0.35 f_{cu} A_c + 0.67 f_y A_{sc}$$

7.4.3 Other than 7.4.2 and the design for deflection induced moment M_{add}, design of stocky wall is similar to slender walls.

7.5 Reinforced Concrete Walls design is similar to that of columns with categorization into slender walls and stocky walls.

7.6 Plain Wall – Plain wall are walls the design of which is without consideration of the presence of the reinforcements.

7.6.1 Effective height of unbraced plain wall, where l_0 is the clear height of the wall between support, is determined by :

(a) $l_e = 1.5 l_0$ when it is supporting a floor slab spanning at right angles to it;
(b) $l_e = 2.0 l_0$ for other cases.

Effective height ratio for braced plain wall is determined by

(a) $l_e = 0.75 l_0$ when the two end supports restraint movements and rotations;
(b) $l_e = 2.0 l_0$ when one end support restraint movements and rotations and the other is free;
(c) $l_e = l_0'$ when the two end supports restraint movements only;
(b) $l_e = 2.5 l_0'$ when one end support restraint movements only and the other is free; where l_0' in (c) and (d) are heights between centres of supports.

7.6.2 For detailed design criteria including check for concentrated load, shear, load carrying capacities etc, refer to Cl. 6.2.2.3 of the Code.

7.7 Sectional Design

The sectional design of wall section is similar to that of column by utilizing stress strain relationship of concrete and steel as indicated in Figure 3.8 and 3.9 of the Code. Alternatively, the simplified stress block of concrete as indicated in Figure 6.1 can also be used. Nevertheless, the Code has additional

requirements in case both in-plane and transverse moments are "significant" and such requirements are not identical for stocky wall and slender wall.

7.7.1 Wall with axial load and in-plane moment

Conventionally, walls with uniformly distributed reinforcements along its length can be treated as if the steel bars on each side of the centroidal axis are lumped into two bars each carrying half of the steel areas as shown in Figure 7.1 and design is carried out as if it is a 4 bar column. Nevertheless, it is suggested in this Manual that the reinforcements can be idealized as a continuum (also as shown in Figure 7.1) which is considered as a more realistic idealization. Derivation of the formulae for the design with reinforcements idealized as continuum is contained in Appendix G, together with design charts also enclosed in the same Appendix.

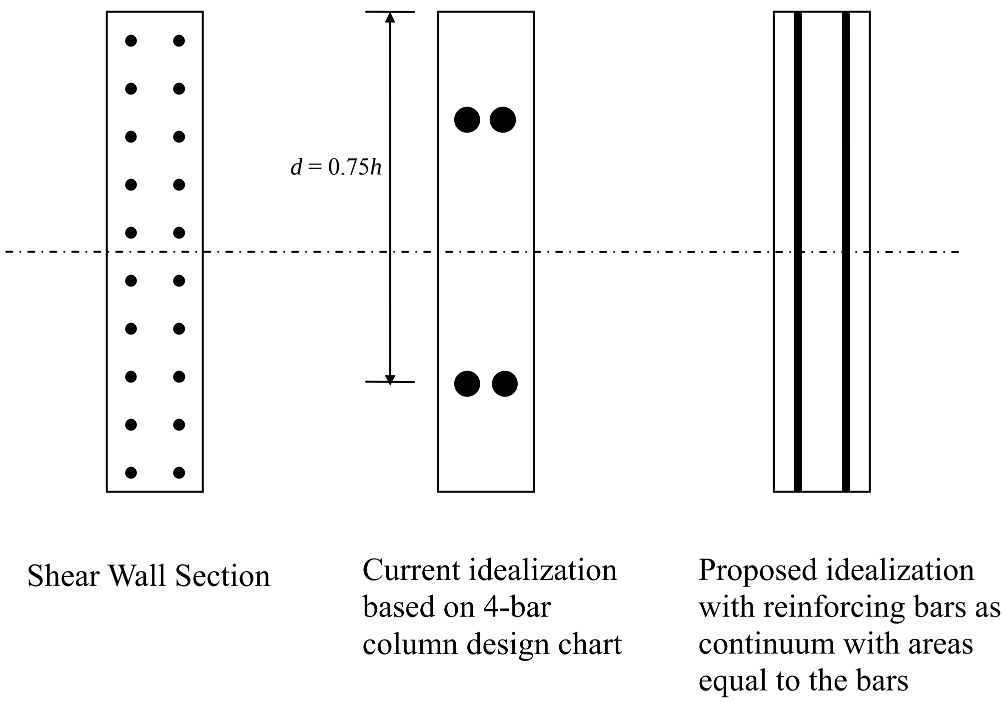


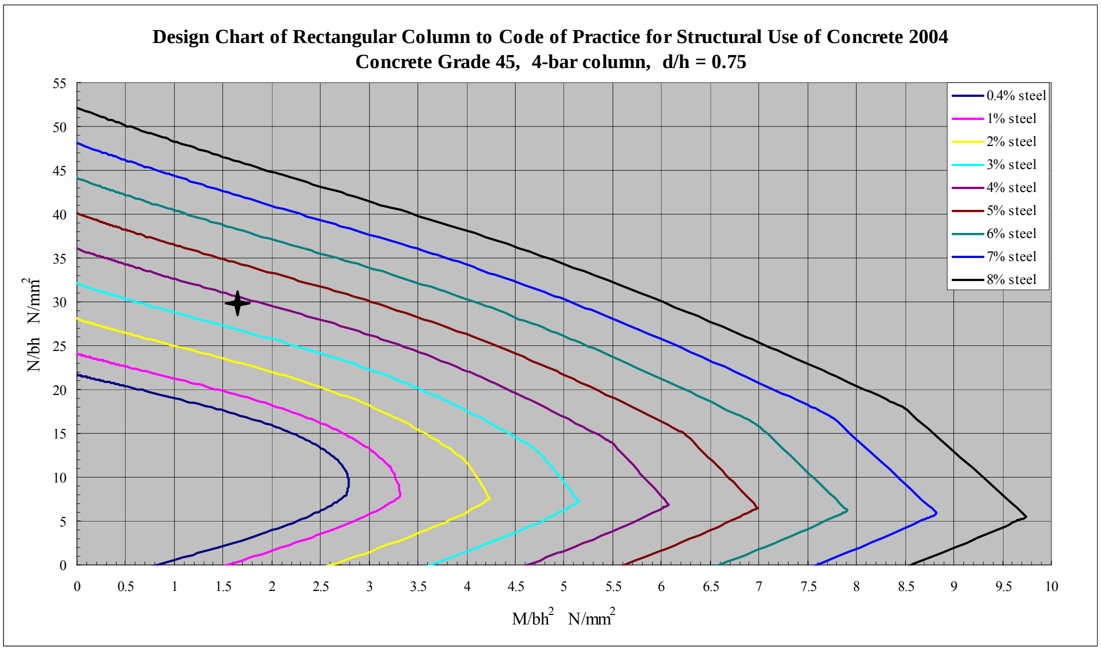
Figure 7.1 – Idealization of Reinforcing bars in shear wall

Worked Example 7.1

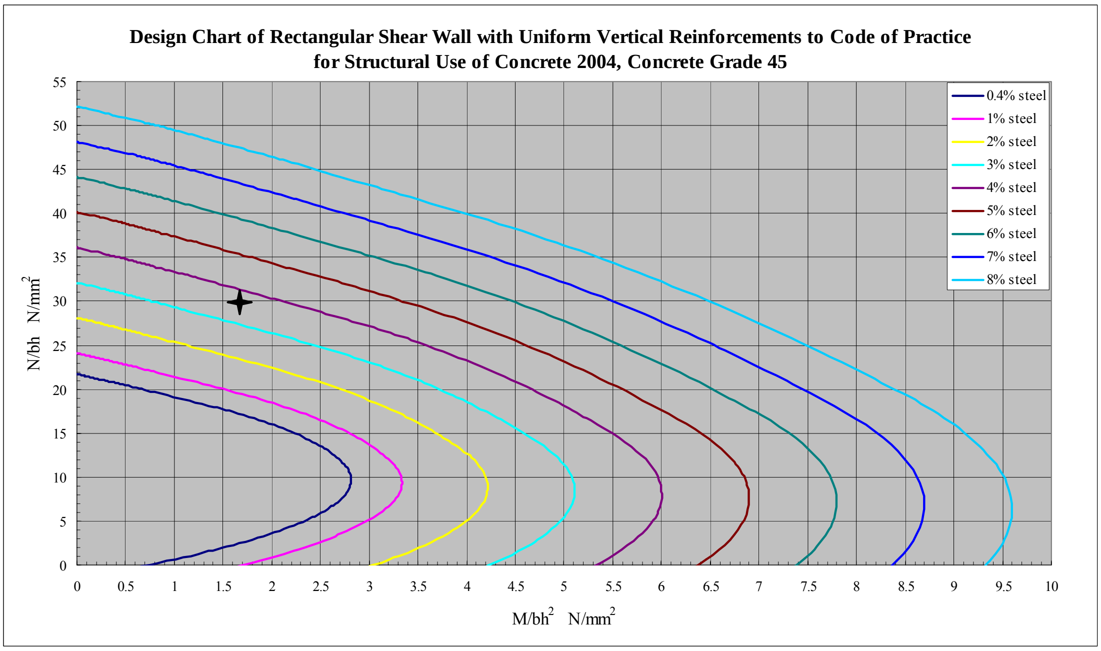
Consider a wall of thickness 300 mm, plan length 3000 mm and under an axial load $P = 27000$ kN and in-plane moment $M_x = 4500$ kNm. Concrete grade is 45. The problem is an uniaxial bending problem. Then

$$\frac{P}{bh} = \frac{27000\times10^3}{300\times3000} = 30 \text{ and } \frac{M}{bh^2} = \frac{4500\times10^6}{300\times3000^2} = 1.67.$$

If based on the 4-bar column chart with $d/h = 0.75$, $p = 3.8\%$, requiring T32 – 140 (B.F.)



If use chart based on continuum of bars, the reinforcement ratio can be slightly reduced to 3.7%.



By superimposing the two design charts as in Figure 7.2, it can be seen that the idealization of steel re-bars as continuum is generally more conservative.

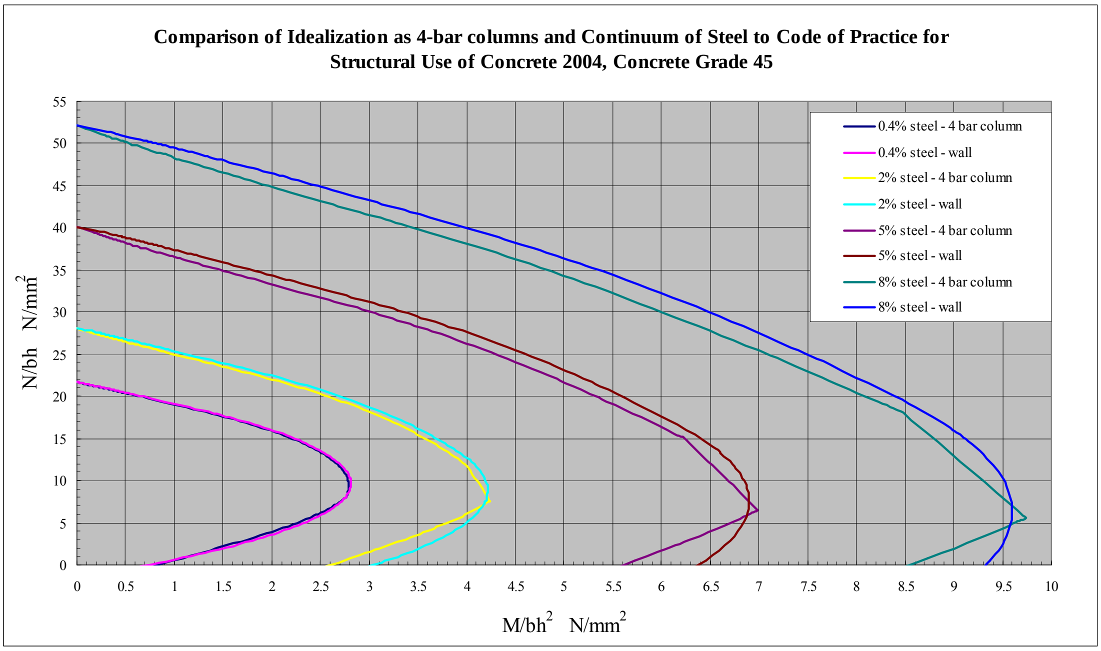


Figure 7.2 – Comparison of design curve between idealization of steel bars as 4 bar column and continuum

7.7.2 Wall with axial load and transverse moment

The design will also be similar to that of column with the two layers of longitudinal bars represented by the bars in the 4-bar column charts as shown in Figure 7.3

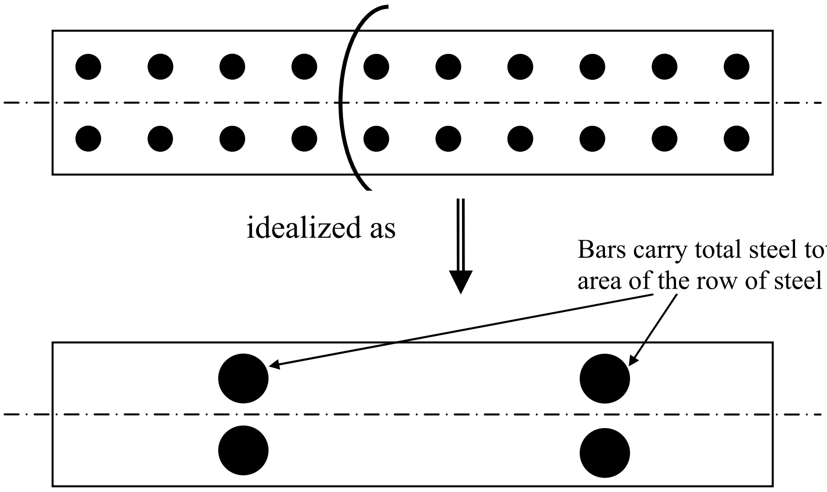


Figure 7.3 – Sectional design for column with axial load and transverse moment

7.7.3 Wall with significant in-plane and transverse moments

The Code has not defined the extent of being "significant". Nevertheless, if significant in-plane and transverse moments exist, the Code effectively requires the wall section be examined at various points (for stocky wall) and unit lengths (for slender wall) along the length of the wall at the splitting up of the axial load and in-plane moment as demonstrated in Figure 7.4.

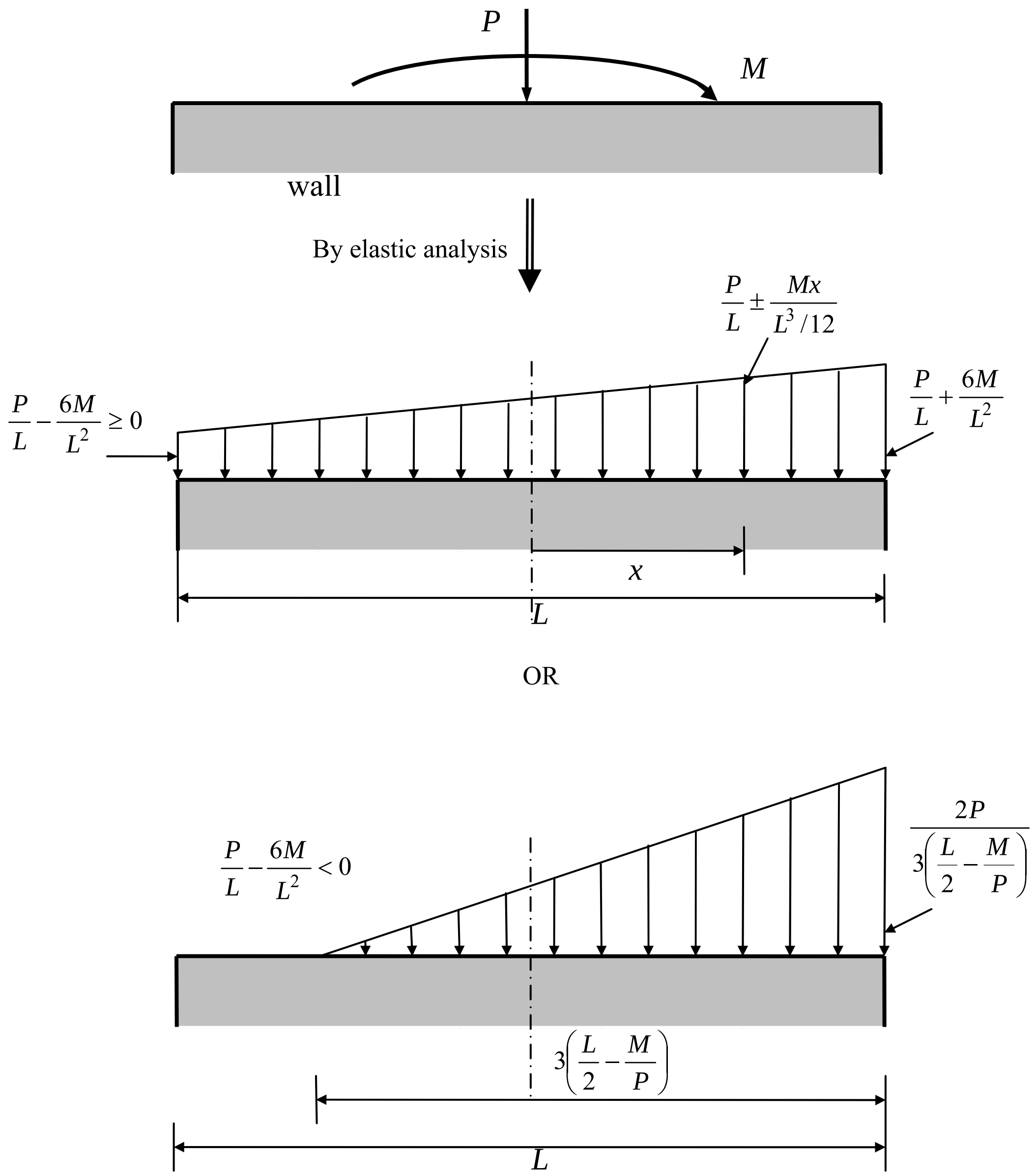


Figure 7.4 – conversion of axial load (kN) and in-plane moment (kNm) into linear varying load (kN/m) along wall section

Worked Example 7.2

Consider a grade 45 wall of thickness 300 mm, plan length 3000 mm and

under an axial load $P = 27000$ kN and in-plane moment $M_x = 4500$ kNm and transverse moment $M_y = 300$ kNm as shown in Figure 7.5. By elastic analysis, the load intensities at the 4 points as resolution of P and M_x are :

A : $\dfrac{27000}{3} + \dfrac{6 \times 4500}{3^2} = 12000$ kN/m;

B : $\dfrac{27000}{3} + \dfrac{4500 \times 0.5}{3^3/12} = 10000$ kN/m

C : $\dfrac{27000}{3} - \dfrac{4500 \times 0.5}{3^3/12} = 8000$ kN/m;

D : $\dfrac{27000}{3} - \dfrac{6 \times 4500}{3^2} = 6000$ kN/m

The varying load intensities are as indicated in Figure 7.5.

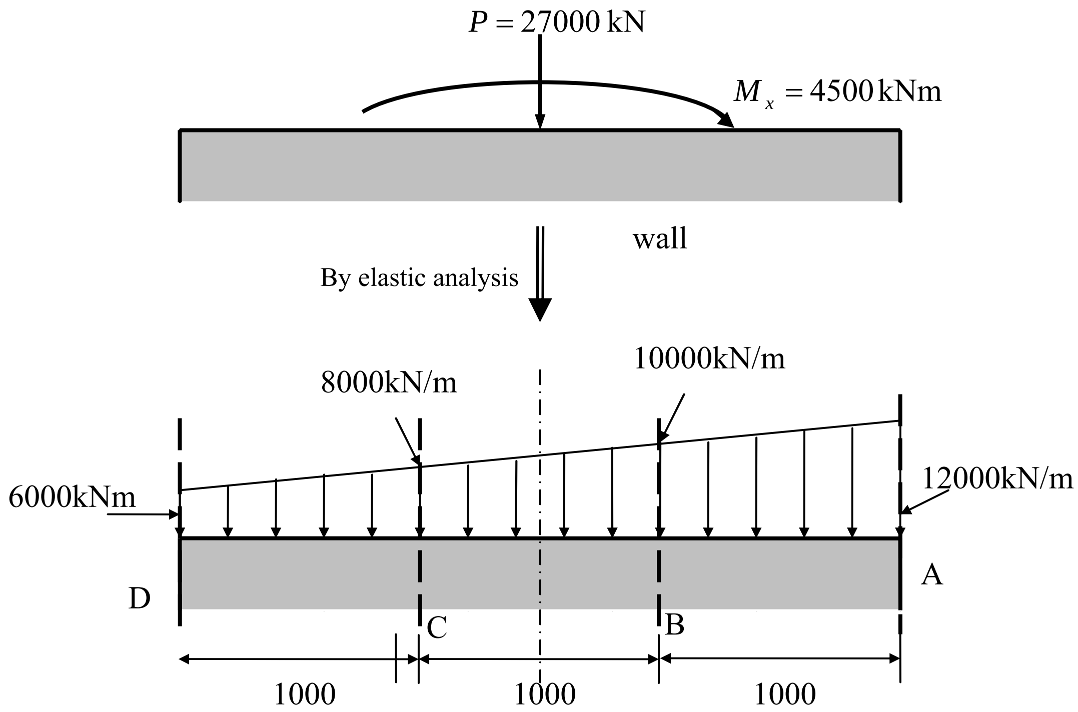


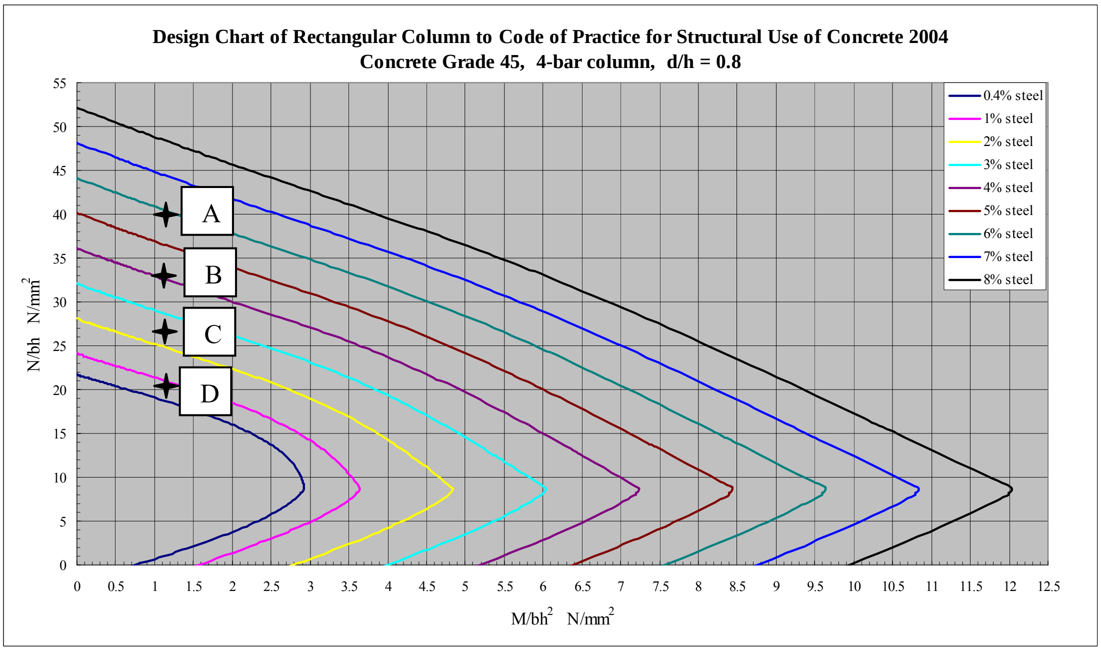
Figure 7.5 – Conversion of axial load (kN) and in-plane moment (kNm) into linear varying load (kN/m) along wall section for Worked Example 7.2

(i) If the wall is considered stocky, each of the points with load intensities as determined shall be designed for the load intensities as derived from the elastic analysis and a transverse moment of $300 \div 3 = 100$ kNm/m by Clause 6.2.2.2(f)(iv) of the Code. Consider one metre length for each point, the 4 points shall be designed for the following loads with section 1000 mm by 300 mm as tabulated in

Table 7.1, i.e. all the points are undergoing uniaxial bending and the sectional design are done in the same Table in accordance with the chart extracted from Appendix F:

Point	A	B	C	D
Axial Load	12000	10000	8000	6000
In-plane Mt	0	0	0	0
Transverse Mt	100	100	100	100
N / bh	40	33.33	26.67	20
M / bh^2	1.11	1.11	1.11	1.11
p (%)	5.9	4.1	2.4	0.6
Re-bars (BF)	T40 – 140	T40 – 200	T32 – 225	T20 – 300

Table 7.1 – Summary of Design for Worked Example 7.2 as a stocky Wall



The Code is not clear in the assignment of reinforcements at various segments of the section based on reinforcements worked out at various points. The assignment can be based on the tributary length principle, i.e. the reinforcement derived from A shall be extended from A to mid-way between A and B; the reinforcement derived from B be extended from mid-way between A and B to mid-way between B and C etc. As such, the average reinforcement ratio is 3.25%. Nevertheless, as a more conservative approach, the assignment of reinforcement design between A and B should be based on A and that

of B and C be based on B etc. As such the reinforcement ratio of the whole section will be increased to 4.13% and the reinforcement ratio at D is not used.

(ii) If the wall is slender, by Cl. 6.2.2.2(g)(i) of the Code, the wall should be divided into "unit lengths" with summing up of loads. Consider the three units AB, BC and CD. The loads and in-plane moments summed from the trapezoidal distribution of loads are as follows, with the assumption that the transverse moment of 300 kNm has incorporated effects due to slenderness :

For Unit Length AB :

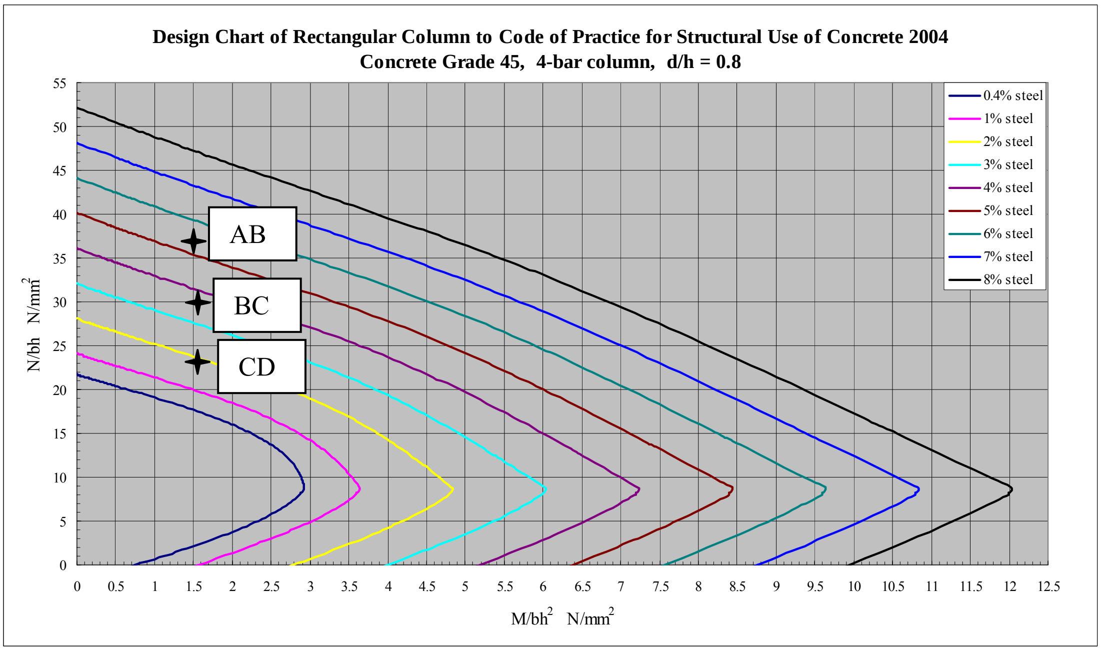
Summed axial load = $\dfrac{12000+10000}{2}\times 1 = 11000\,\text{kN}$

Summed in-plane moment $\dfrac{12000-10000}{2}\times 1\times\left(\dfrac{2}{3}-\dfrac{1}{2}\right)\times 1 = 167$ kNm.

The summed axial loads and moments on the unit lengths BC and CD are similarly determined and design is summarized in Table 7.2, with reference to the design chart extracted from Appendix F. In the computation of M_x/h' and M_y/b', h' and b' are taken as 750 and 225 respectively.

Unit Length	AB	BC	CD
Axial Load	11000	9000	7000
In-plane Mt (M_x)	167	167	167
Transverse Mt (M_y)	100	100	100
M_x/h'	0.227	0.227	0.227
M_y/b'	0.444	0.444	0.444
$N/f_{cu}bh$	0.272	0.222	0.172
β	0.684	0.744	0.801
$M_y' = M_y + \beta(b'/h')M_x$	134.2	137.2	140.1
N/bh	36.67	30	23.33
M_y'/hb^2	1.49	1.524	1.556
p (%)	5.4	3.7	1.9
Re-bars (BF)	T40 – 155	T32 – 145	T25 – 175

Table 7.2 – Design of Wall for Worked Example 7.2 as a slender wall



The average steel percentage is 3.67%.

So the reinforcement worked out by Clause 6.2.2.2(g)(i) of the Code for a slender wall is between the results of the two methods of reinforcement ratios assignments as described in sub-section (i) based on Clause 6.2.2.2(f)(iv) of the Code.

(iii) Summary of the reinforcements design of the three approaches

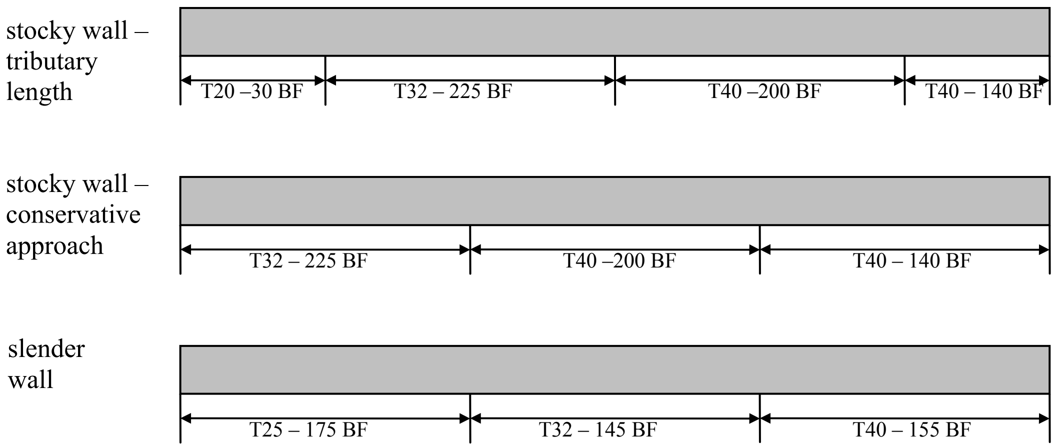


Figure 7.6 – Summary of reinforcement details of Worked Example 7.2

(iv) The approach recommended in the Code appears to be reasonable and probably economical as higher reinforcement ratios will be in region of high stresses. However, it should be noted that if moment arises from wind loads where the direction can reverse, design for the

reversed direction may result in almost same provisions of reinforcements at the other end.

As the division of segments or points as recommended by the Code for design of wall with significant transverse and in-plane moments is due to the inaccurate account by the biaxial bending formula used for design of column, more accurate analysis can be done by true biaxial bending analysis as discussed in Section 5.3.5 and Figure 5.8 of this Manual, so long the "plane remain plane" assumption is valid, though the design can only be conveniently done by computer methods. The sections with reinforcement ratios arrived at in (i) and (ii) have been checked against by the software ADSEC, the section in (ii) has yielded an applied moment / moment capacity ratio of 0.8 showing there is room for slight economy. Nevertheless, the first reinforcement ratio in (i) is inadequate as checked by ADSEC whilst the second one yielded an over design with applied moment / moment capacity ratio up to 0.68.

7.8 The following Worked Example 7.3 serves to demonstrate the determination of design moment for a slender wall section, taking into account of additional moment due to slenderness.

<u>Worked Example 7.3</u>

Wall Section : thickness : 200 mm, plan length : 2000 mm;
Wall Height : 3.6 m,
Concrete grade : 35
Connection conditions at both ends of the wall : connected monolithically with floor structures shallower than the wall thickness.

<u>Check for slenderness</u>

Generally only necessary about the minor axis.
End conditions are 2 for both ends, $\beta = 0.85$ (by Table 6.11 of the Code);
$l_e = 0.85 \times 3.6 = 3.06$ m

Axial Load : $N = 7200$ kN, $M_x = 1800$ kNm at top and 1200 kNm at bottom , $M_y = 25$ kNm at top and 24 kNm at bottom.

Determination of final design moment M_t about the major and minor axes is similar to (i)

For bending about the major axis, $l_e / h = 3060/2000 = 1.53 < 15$, so $M_{add} = 0$, M_x will be the greatest of

(1) $M_2 = 1800$;

(2) $M_i + M_{add} = 0.4\times(-1200)+0.6\times1800+0 = 600$;
$< 0.4\times1800 = 720$

(3) $M_1 + M_{add}/2 = 1200+0 = 1200$; and

(4) $N\times e_{\min} = 7200\times0.02 = 144$.

So $M_x = 1800$ kNm for design.

For bending about the minor axis, $l_e / b = 3060/200 = 15.3 > 15$,

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b}\right)^2 = \frac{1}{2000}\left(\frac{3060}{200}\right)^2 = 0.117$$

$$a_u = \beta_a Kh = 0.117\times1\times0.2 = 0.0234$$

$M_{add} = Na_u = 7200\times0.0234 = 168.48$ kNm, M_y will be the greatest of

(1) $M_2 = 25$;

(2) $M_i + M_{add} = 0.4\times25+168.48 = 178.48$;
as $0.4\times(-24)+0.6\times25 = 5.4 < 0.4\times25 = 10$

(3) $M_1 + M_{add}/2 = 24+168.48/2 = 108.3$; and

(4) $N\times e_{\min} = 7200\times0.02 = 144$. So $M_y = 178.48$ kNm for design.

So the factored axial load and moments for design are

$N = 7200$ kN; $M_x = 1800$ kNm; $M_y = 178.48$ kNm

Design can be performed in accordance with Cl. 6.2.2.2(g) of the Code as demonstrated in Worked Examples 7.2 and by calculations with the formulae derived in Appendices F and G. However, the calculations are too tedious and cases to try are too many without the use of computer methods. Spread sheets have been devised to solve the problem with a sample enclosed in Appendix G.

7.9 Detailing Requirements

There are no ductility requirements in the Code for walls. The detailing requirements are summarized from Cl. 9.6 of the Code :

Vertical reinforcements for reinforced concrete walls :

(i) Minimum steel percentage : 0.4%. When this reinforcement controls the design, half of the steel area be on each side;

(ii) Maximum steel percentage : 4%;

(iii) All vertical compression reinforcements should be enclosed by a link as shown in Figure 7.7;

(iv) Maximum distance between bars : the lesser of 3 times the wall thickness and 400 mm as shown in Figure 7.7.

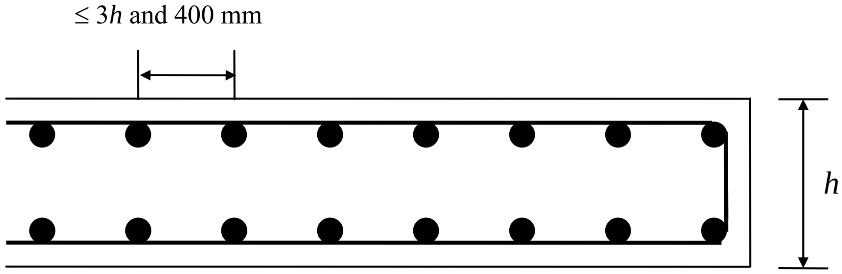


Figure 7.7 – Vertical reinforcements for walls

Horizontal and transverse reinforcements for reinforced concrete walls

(i) If the required vertical reinforcement does not exceed 2%, horizontal reinforcements be provided as follows and in accordance with Figure 7.8 :

(a) Minimum percentage is 0.25% for $f_y = 460$ MPa and 0.3% for $f_y = 250$ MPa;

(b) bar diameter ≥ 6 mm and 1/4 of vertical bar size;

(c) spacing ≤ 400 mm.

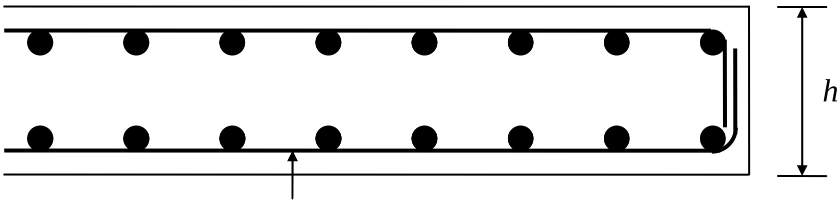


Figure 7.8 – Horizontal reinforcements for walls with vertical reinforcement ≤ 2%

(ii) If the required vertical reinforcement > 2%, links be provided as follows as shown in Figure 7.9 :
- (a) to enclose every vertical compression longitudinal bar;
- (b) no bar be at a distance further than 200 mm from a restrained bar at which a link passes round at included angle ≤ 90°;
- (c) minimum diameter : the greater of 6 mm and 1/4 of the largest compression bar;
- (d) maximum spacing : twice the wall thickness in both the horizontal and vertical directions. In addition, maximum spacing not to exceed 16 times the vertical bar diameter in the vertical direction.

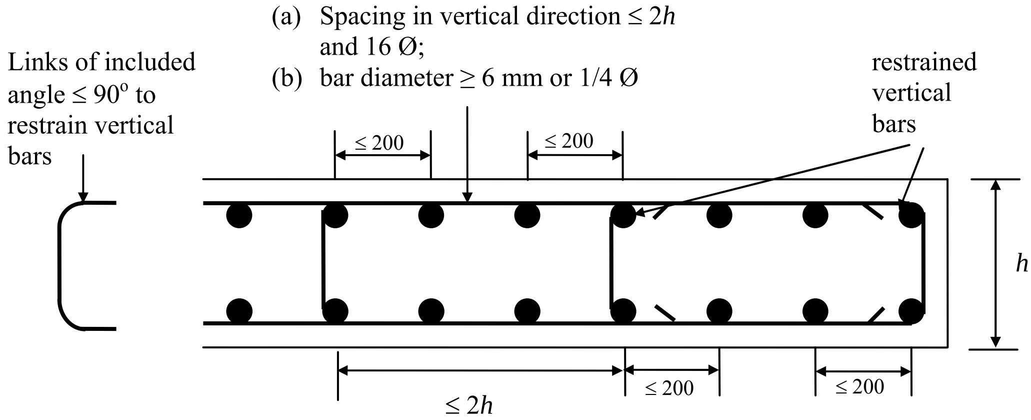


Figure 7.9 – Anchorage by links on vertical reinforcements of more than 2%

<u>Plain walls</u>

If provided, minimum reinforcements : 0.25% for $f_y = 460\,\text{MPa}$ and 0.3% for $f_y = 250\,\text{MPa}$ in both directions generally.

8.0 Corbels

8.1 General – A corbel is a short cantilever projection supporting a load-bearing member with dimensions as shown :

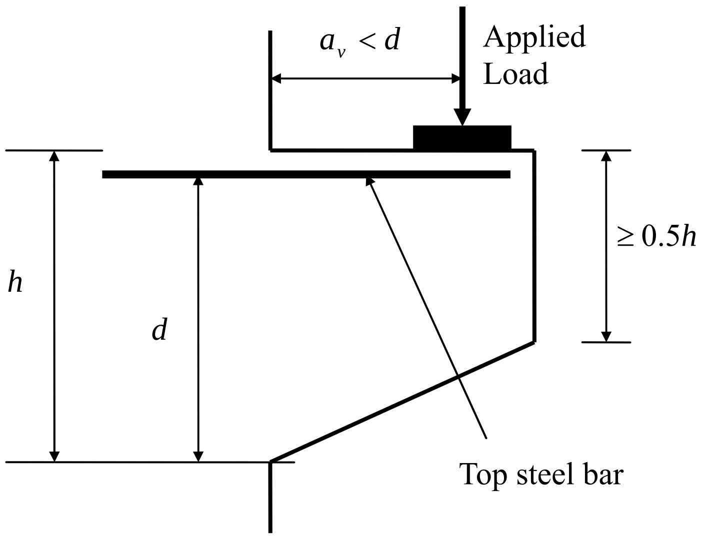


Figure 8.1 – Dimension requirement for a Corbel

8.2 Basis of Design (Cl. 6.5.2 of the Code)

8.2.1 According to Cl. 6.5.2.1 of the Code, the basis of design method of a corbel is that it behaves as a "Strut-and-Tie" model as illustrated in Figure 8.2. The strut action (compressive) is carried out by concrete and the tensile force at top is carried by the top steel.

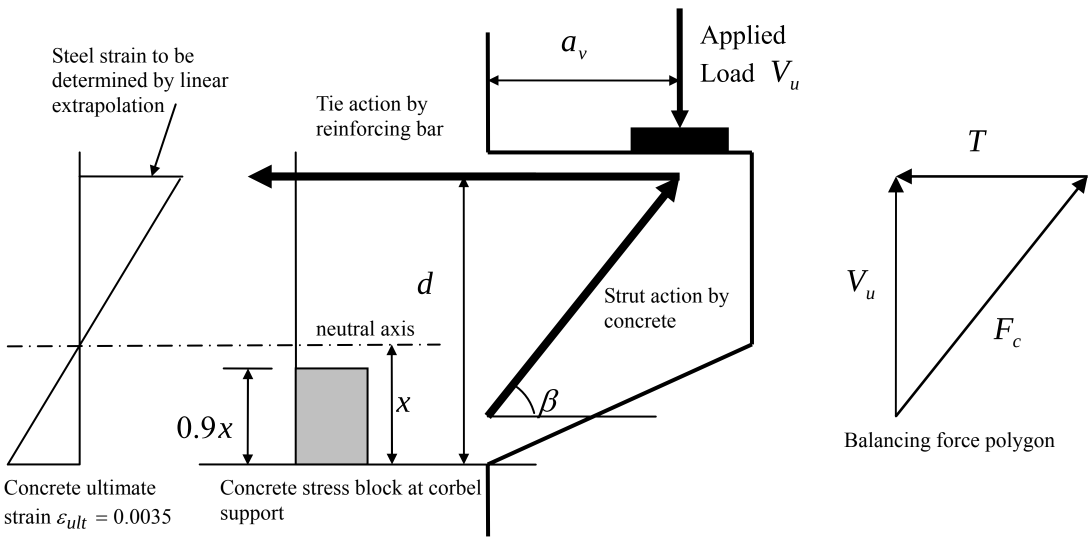


Figure 8.2 – Strut-and-Tie Action of a Corbel

8.2.2 Magnitude of resistance provided to the horizontal force should be not less than one half of the design vertical load, thus limiting the value of the angle β in Figure 8.2 or in turn, that the value of a_v cannot be too small.

8.2.3 Strain compatibility be ensured.

8.2.4 In addition to the strut-and tie model for the determination of the top steel bars, shear reinforcements should be provided in form of horizontal links in the upper two thirds of the effective depth of the corbel. The horizontal links should not be less than one half of the steel area of the top steel.

8.2.5 Bearing pressure from the bearing pad on the corbel should be checked and properly designed in accordance with "Code of Practice for Precast Concrete Construction 2003" Cl. 2.7.9. In short, the design ultimate bearing pressure to ultimate loads should not exceed

(i) $0.4 f_{cu}$ for dry bearing;
(ii) $0.6 f_{cu}$ for bedded bearing on concrete;
(iii) $0.8 f_{cu}$ for contact face of a steel bearing plate cast on the corbel with each of the bearing width and length not exceeding 40% of the width and length of the corbel.

The net bearing width is obtained by

$$\frac{ultimate \;\; load}{effective \;\; bearing \;\; length \times ultimate \;\; bearing \;\; stress}$$

The Precast Concrete Code 2003 (in Cl. 2.7.9.3 of the Precast Concrete Code) has specified that the effective bearing length of a bearing be the least of :

(i) physical bearing length;
(ii) one half of the physical bearing length plus 100 mm;
(iii) 600 mm.

8.3 Design Formulae for the upper steel tie

The capacity of concrete in providing lateral force as per Figure 8.2 is $0.45 f_{cu} \times b \times 0.9x = 0.405 f_{cu} bx$ where b is the length of the corbel.

The force in the compressive strut is therefore $F_c = 0.405 f_{cu} bx \cos\beta$.

By the force polygon, $F_c \sin\beta = V_u \Rightarrow 0.405 f_{cu} bx \sin\beta \cos\beta = V_u$

As $\tan\beta = \dfrac{d - 0.45x}{a_v}$; $\cos\beta = \dfrac{a_v}{\sqrt{a_v^{\,2} + (d - 0.45x)^2}}$

$$\sin\beta = \frac{(d - 0.45x)}{\sqrt{a_v^{\,2} + (d - 0.45x)^2}}$$

So $0.405 f_{cu} bx \dfrac{a_v (d - 0.45x)}{a_v^{\,2} + (d - 0.45x)^2} = V_u \Rightarrow V_u = \dfrac{0.405 f_{cu} bxa_v (d - 0.45x)}{a_v^{\,2} + (d - 0.45x)^2}$

Expanding and re-arranging

$$(0.2025V_u + 0.18225 f_{cu} ba_v)x^2 - 0.9d(V_u + 0.45 f_{cu} ba_v)x + V_u\left(a_v^{\,2} + d^2\right) = 0$$

Putting $A = 0.2025V_u + 0.18225 f_{cu} ba_v$; $B = -0.9d(V_u + 0.45 f_{cu} ba_v)$

$C = V_u\left(a_v^{\,2} + d^2\right)$

$$x = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \qquad \text{(Eqn 8-1)}$$

By the equilibrium of force, the top steel force is $T = V_u \cot\beta = \dfrac{V_u a_v}{d - 0.45x}$

(Eqn 8-2)

The strain at the steel level is, by extrapolation of the strain diagram in Figure 8.2 is $\varepsilon_s = \dfrac{d - x}{x}\varepsilon_{ult} = \dfrac{d - x}{x} \times 0.0035$ (Eqn 8-3)

8.4 Design Procedure :

(i) Based on the design ultimate load and a_v, estimate the size of the corbel and check that the estimated dimensions comply with Figure 8.1;

(ii) Check bearing pressures;

(iii) Solve the neutral axis depth x by the equation (Eqn 8-1).

(iv) By the assumption plane remains plane and that the linear strain at the base of the corbel is the ultimate strain of concrete $\varepsilon_{ult} = 0.0035$, work out the strain at the top steel level as ε_s;

(v) Obtain the steel stress as $\sigma_s = E_s \varepsilon_s$ where $E_s = 200 \times 10^6$ kPa. However, the stress should be limited to $0.87 f_y$ even $\varepsilon_s \ge 0.002$;

(vi) Obtain the force in the top steel bar T by (Eqn 8-2)

(vii) Check that $T \ge 0.5V_u$;

(viii) Obtain the required steel area of the top steel bars A_{st} by $A_{st} = \dfrac{T}{\sigma_s}$

(ix) Check the shear stress by $v = \frac{V_u}{bd}$. If $v > v_c$ (after enhancement as applicable), provide shear reinforcements by $\frac{A_{sv}}{s_v} = \frac{b(v - v_c)}{0.87 f_y}$ over the upper $\frac{2}{3}d$ where A_{sv} is the cross sectional area of each link and s_v is the link spacing.

(x) Check that the total shear area provided which is $\frac{A_{sv}}{s_v}d$ is not less than half of the top steel area, i.e. $A_{sv} \times \frac{d}{s_v} \geq \frac{1}{2} A_{st}$ even if $v < v_r$.

8.5 Detailing Requirements

(i) By Cl. 6.5.2.2 of the Code, anchorage of the top reinforcing bar should either

(a) be welded to a transverse bar of equivalent strength or diameter. The bearing area of the load should stop short of the transverse bar by a distance equal to the cover of the tie reinforcement as shown in Figure 8.3(a); or

(b) bent back to form a closed loop. The bearing area of the load should not project beyond the straight portion of the bars forming the tension reinforcements as shown in Figure 8.3(b).

(ii) By Cl. 6.5.2.3 of the Code, shear reinforcements be provided in the upper two thirds of the effective depth and total area not less than half of the top bars as shown in Figure 8.3(a) and 8.3(b).

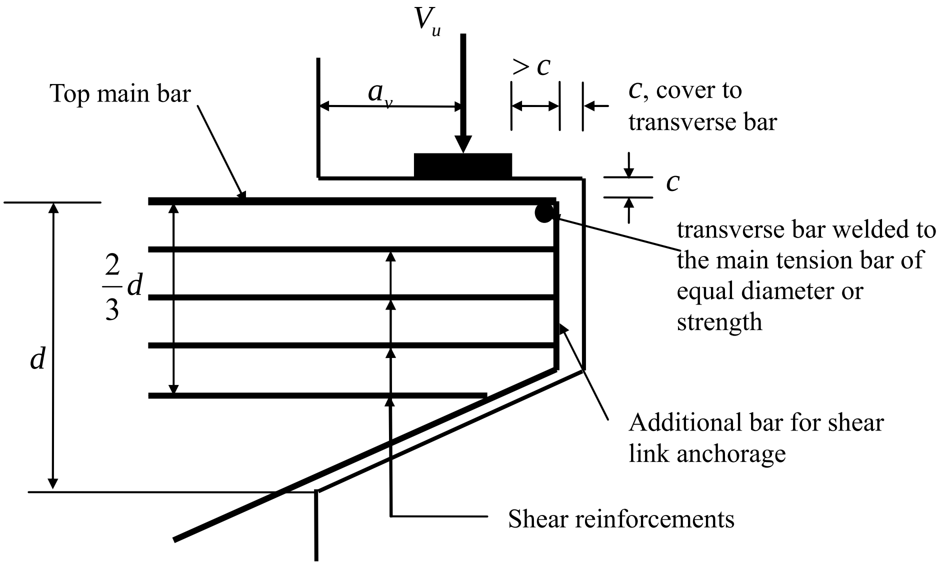


Figure 8.3(a) – Typical Detailing of a Corbel

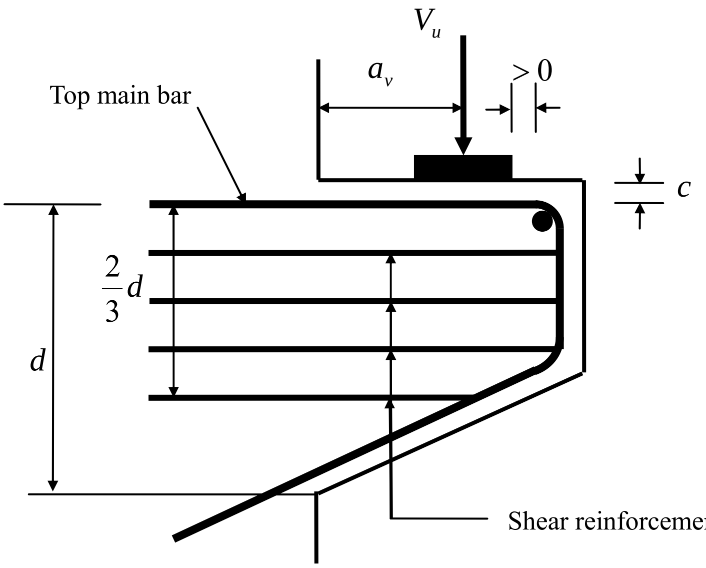


Figure 8.3(b) – Typical Detailing of a Corbel

8.6 Worked Example 8.1

Design a corbel to support an ultimate load of 600 kN at a distance 200 mm from a wall support, i.e. $V_u = 600\,\text{kN}$, $a_v = 200\,\text{mm}$. The load is transmitted from a bearing pad of length 300 mm. Concrete grade is 40.

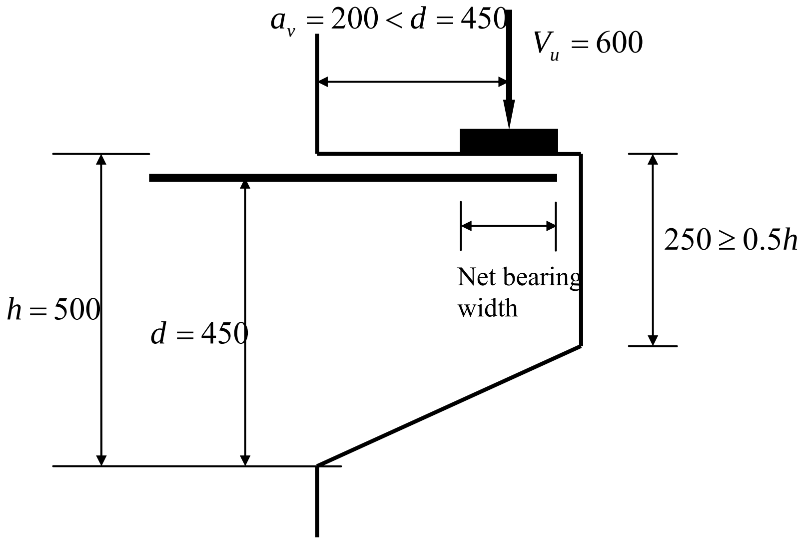


Figure 8.4 – Worked Example. 8.1

1. The dimensions of the corbel are detailed as shown which comply with the requirement of Cl. 6.5.1 of the Code with length of the corbel $b = 300$ mm;
2. Check bearing stress :
   Design ultimate bearing stress is $0.8 f_{cu} = 0.8 \times 40 = 32$ MPa
   Net bearing width is $\dfrac{600 \times 10^3}{300 \times 32} = 62.5$ mm.
   So use net bearing width of bearing pad 70 mm.
3. With the following parameters :
   $V_u = 600$ kN; $f_{cu} = 40$ MPa; $b = 300$ mm;
   $a_v = 200$ mm; $d = 450$ mm
   substituted into (Eqn 8-1)
   $$(0.2025V_u + 0.18225 f_{cu} b a_v)x^2 - 0.9d(V_u + 0.45 f_{cu} b a_v)x + V_u(a_v^2 + d^2) = 0$$
   Solving $x = 276.77$ mm.
4. The strain at steel level,
   $$\varepsilon_s = \frac{d - x}{x}\varepsilon_{ult} = \frac{450 - 276.77}{276.77} \times 0.0035 = 0.00219 > 0.002$$
5. The stress in the top steel is $0.87 f_y$ as $\varepsilon_s > 0.002$;
   (if $\varepsilon_s \leq 0.002$, $f_s = E_s \times \varepsilon_s$ where $E_s = 200$ GPa)
6. The force in the top steel is

$$T=\frac{V_u a_v}{d-0.45x}=\frac{600\times200}{450-0.45\times276.77}=368.71\,\text{kN}>\ 0.5\times600=300\,\text{kN};$$

7. Steel area required is $\dfrac{368710}{0.87\times460}=921.32\,\text{mm}^2$, provide 3T20 (0.7%);

8. $v_c=0.556\times(40/25)^{1/3}=0.65$ MPa without enhancement. With enhancement, it becomes $\dfrac{2d}{a_v}\times0.65=2.925\,\text{MPa}$.

9. Check shear stress

   $\dfrac{600000}{450\times300}=4.444\,\text{MPa}>\ v_c=2.925\,\text{MPa}$.

   So shear reinforcement $\dfrac{A_{sv}}{s_v}=\dfrac{b(v-v_c)}{0.87f_y}=\dfrac{300(4.444-2.925)}{0.87\times460}=1.14\,\text{mm}$;

   $A_{sv}=1.14\times450=513\,\text{mm}^2$. So use <u>3T12 closed links</u> over the top 300 mm.

10. Area of 3T12 closed link is 678 mm^2 > half of area of tensile top steel = 0.5×3×314 = 471 mm^2.

The details of the Corbel is finally as shown in Figure 8.5.

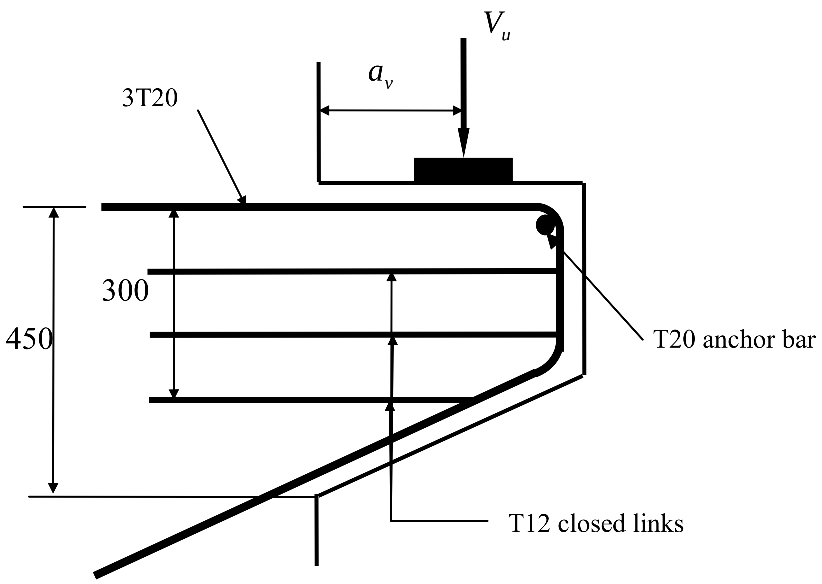


Figure 8.5 – Detailing of Worked Example 8.1

8.7 Resistance to horizontal forces

Cl. 9.8.4 requires additional reinforcement connected to the supported member

to transmit external horizontal force exerted to the corbel in its entirety. However, it should be on the conservative side if strain compatibility is also considered in designing the corbel to resist also this horizontal force N_c as in addition to the vertical load V_u. This is in consistency with the Code requirement. The force polygon as modified from Figure 8.2 will becomes

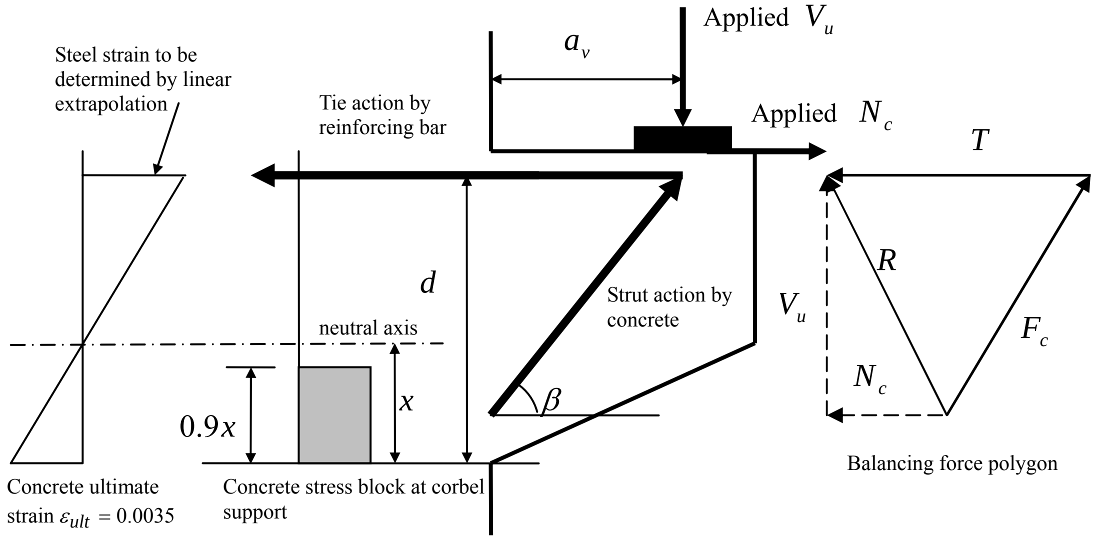


Figure 8.6 – Strut-and-Tie Action of a Corbel with inclusion of horizontal force

From Figure 8.6 and formulae derivation in Section 8.3 of this Manual, it can be seen that the determination of the neutral axis depth x and subsequently the strain profile of the root of the corbel is independent of N_c. Thus the steps (i) to (v) in Section 8.4 of this Manual can be followed in calculation of x, ε_s and σ_s as if N_c does not exist.

However, the tension in the top bar will be $T = N_c + \dfrac{V_u a_v}{d - 0.45x}$ (Eqn 8-4)

And the steel area of the top bar can be worked out as $A_{st} = \dfrac{T}{\sigma_s}$

8.8 Worked Example 8.2

If an additional horizontal force of 200kN is exerted on the corbel in Example 8.1, tending to pull away from the root of the corbel, the total tensile force to be resisted by the top bars will be $T = 368.71 + 200 = 568.71$ kN and the top bar area required is $\dfrac{568.71 \times 10^3}{0.87 \times 460} = 1421.06$ mm^2, as the strain at the steel level has exceed 0.002. The top bar has to be increased from 3T25.

9.0 Cantilever Structures

9.1 Cl. 1.4 of the Code defines "Cantilever Projection" as "a structural element that cantilevers from the main structure, for example, canopies, balconies, bay windows, air conditioning platforms." In addition, PNAP 173 which refers to cantilevered reinforced concrete structures in general indicates more clearly design and construction criteria to be complied with.

9.2 Design Considerations

Design considerations for a cantilevered structure from the Code (Table 7.3, Cl. 9.4 etc. of the Code) and PNAP 173 are summarized as follows :

Slabs and Beams in General

(i) The span to overall depth of cantilever beams or slabs should not be greater than 7;
(ii) For cantilever span more than 1000 mm, a beam-and-slab type of arrangement should be used instead of pure slab cantilever where practicable (PNAP173 App. A 1(a));
(iii) The minimum percentage of top tension longitudinal reinforcement based on the gross cross-sectional concrete area should be 0.25% for all reinforcement grades generally (PNAP173 App. A 6(c)). However, if the cantilever structure is a flanged beam where the flange is in tension, the minimum steel percentage is 0.26% for T-section and 0.2% for L-section but based on the gross area of the rectangular portion of width of the web times the structural depth as per Table 9.1 of the Code. The more stringent requirement shall prevail;
(iv) Diameter of the longitudinal reinforcement ≥ 10 mm as illustrated in Figure 9.1 (PNAP173 App. A 6(c));
(v) The centre-to-centre spacing of the top tension longitudinal bars ≤ 150mm as illustrated in Figure 9.1 (PNAP173 App. A 6(c));
(vi) For cantilevered structure exposed to weathering, cover to all reinforcement ≥ 40 mm (PNAP173 App. A 8(a));
(vii) Anchorage of tension reinforcement shall be based on steel stress of $0.87f_y$ and (a) full anchorage length should be provided with location of commencement in accordance with Cl. 9.4.3 of the Code as illustrated in

Figures 9.1 and 9.2; and (b) minimum anchorage length of 45 times the longitudinal bar diameter in accordance with PNAP 173 App. A 6(d). The different commencement points of anchorage lengths as indicated by PNAP 173 Appendices B and C are not adopted in this Manual. However, requirements for the lengths of curtailment of tension reinforcement bars PNAP173 and Cl. 9.2.1.6 of the Code in relation to curtailment of tension reinforcements are amalgamated. They are shown in Figures 9.1 and 9.2.

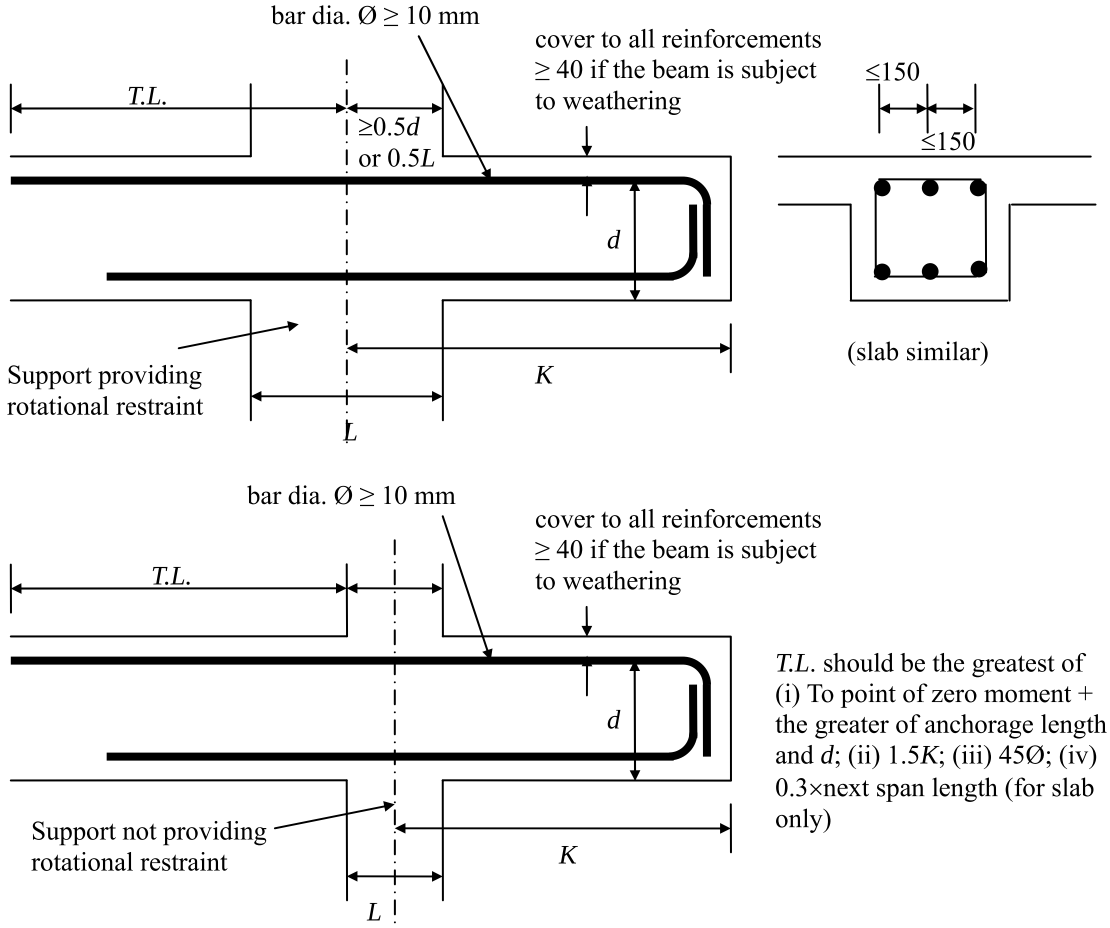


Figure 9.1 – Anchorage and maximum longitudinal bar spacing in Cantilevers as required by the Code and PNAP 173

Beam in particular

(viii) The overall depth at support should be at least 300 mm as shown in Figure 9.2;

(ix) For cantilever beam connected with continuous beams, requirements for curtailment of longitudinal bars into the next continuous span are similar to slab except that half of the bars can be curtailed at 0.75K + L/2 as

shown in Figure 9.2;

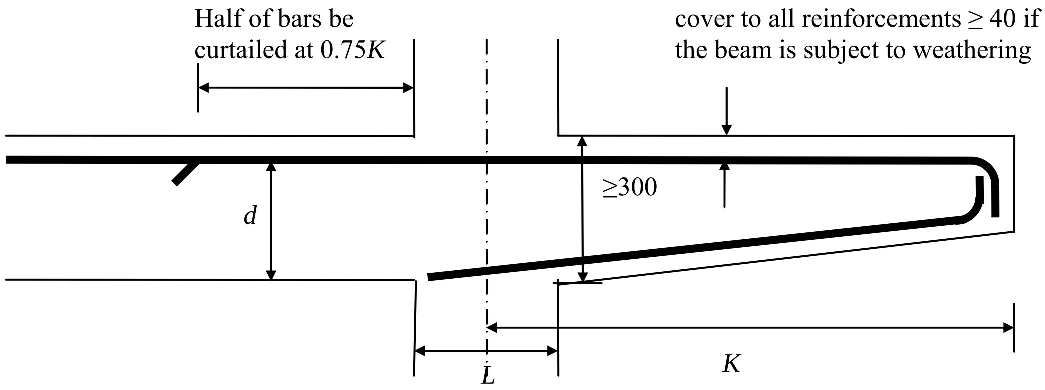


Figure 9.2 – Particular requirements for cantilever beams as required by the Code and PNAP 173

Slab in particular

(x) Minimum overall slab thickness (PNAP173 App. A 6(a)):
   (a) 100 mm for span ≤ 500mm;
   (b) 125 mm for 500 mm < span ≤ 750mm;
   (c) 150 mm for span > 750 mm;

(xi) Reinforcements be high yield bars in both faces and in both directions (PNAP173 App.A 6(c));

(xii) Particular attentions to loads as shown in Figure 9.3 should be given :

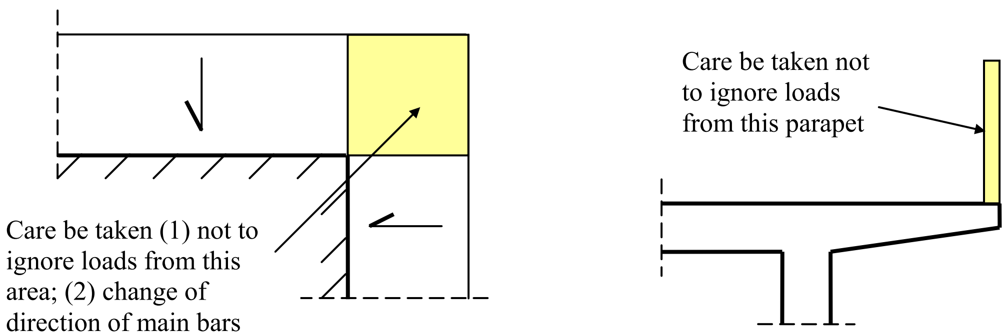


Figure 9.3 – Loads on cantilever slab (PNAP173 App.A 6(e))

(xiii) For a cantilever slab with a drop at the supporting end, top reinforcement bars ≤ 16 mm in diameter should be used in order that an effective and proper anchorage into the supporting beam and internal slab can be

developed as illustrated in Figure 9.4. (PNAP173 App. A 6(d))

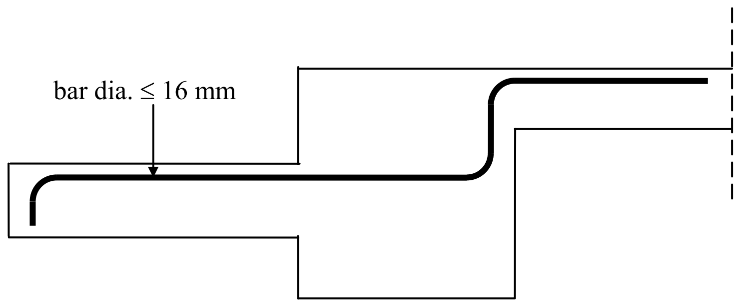


Figure 9.4 – Cantilever slab with drop at supporting end

(xiv) Cantilevered slabs exposed to weathering should satisfy :

(1) maximum crack width at the tension face ≤ 0.1 mm under serviceability check OR stress of deformed high yield steel bar ≤ 100 MPa when checking the flexural tension under working load condition (PNAP173 App. A 8(a));

(2) Cover to all reinforcement at the exposed surface ≥ 40 mm. (PNAP173 App. A 8(a)).

9.3 Worked Example 9.1

R.C. design of a cantilevered slab as shown in Figure 9.5 is subject to weathering. Concrete grade is 35.

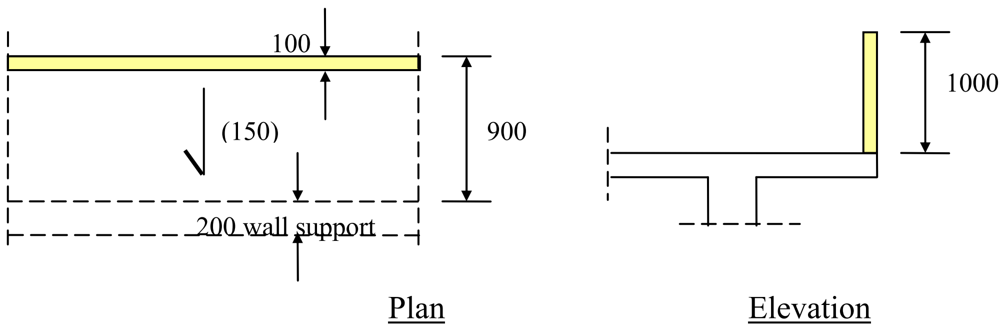


Figure 9.5 – Cantilever slab in Worked Example 9.1

Loading D.L. O.W. $0.15 \times 24 = 3.6\,\text{kN/m}^2$

Fin $2.0\ \text{kN/m}^2$

$5.6\ \text{kN/m}^2$

Para. $0.1 \times 1.0 \times 24 = 2.4\,\text{kN/m}$

L.L. 1.5 kN/m^2

Effective span is taken to be $900 + 0.5 \times 150 = 975$

Moment $= (1.4 \times 5.6 + 1.6 \times 1.5) \times 0.975 \times 0.975 / 2 + 1.4 \times 2.4 \times 0.925$

$= 7.975$ kNm/m

Design for ultimate state,

$d = 150 - 40 - 5 = 105$

$$\frac{M}{bd^2} = \frac{7.975 \times 10^6}{1000 \times 105^2} = 0.723$$

$$A_{st} = \frac{7.975 \times 10^6}{0.87 \times 460 \times 0.95 \times 105} = 200 \text{ mm}^2\text{/m.}$$

Use T10 – 150 (Area provided is 523 mm^2/m)

If the slab is subject to weathering, check the service stress by equation in item (2) in Table 7.4 of the Code reading

$$f_s = \frac{2 f_y A_{st,req}}{3 A_{st,prov}} \times \frac{1}{\beta_b}$$

Note : $\beta_b = 1$ as no moment redistribution in cantilever.

If f_s is to be limited to 100 N/mm^2, $A_{st,req} = 200$ mm^2/m

$$A_{st,prov} = \frac{2 f_y A_{st,req}}{3 f_s} \times \frac{1}{\beta_b} = \frac{2 \times 460 \times 200}{3 \times 100} \times \frac{1}{1} = 613 \text{ mm}^2$$

Use T10 – 100 (area provided is 785 mm^2/m or 0.52%)

Alternatively, crack width is checked by (Ceqn 7.1) and (Ceqn 7.2)

To calculate crack width, it is first necessary to assess the neutral axis depth x by the elastic theory in accordance with the cracked section of Figure 7.1 of the Code on the basis of a cracked section.

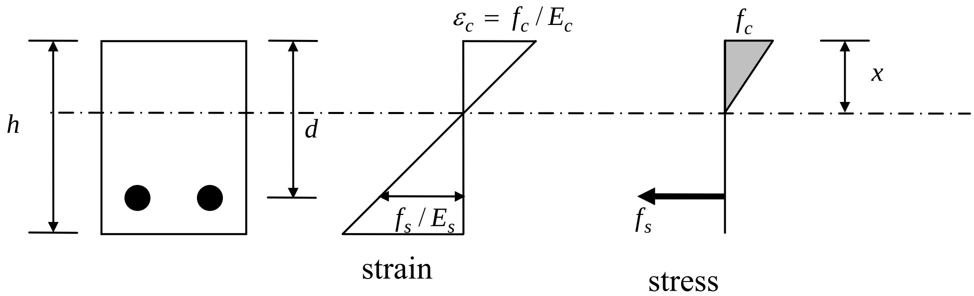


Figure 9.6 – Stress/strain relation of a cracked R.C. section

E_c is the long term value which, by Cl. 7.2.3 of the Code is taken as half of the instantaneous value which is $23.7 \div 2 = 11.85$ GPa

$E_s = 200$ kN/mm^2

Consider equilibrium of the section in Figure 9.6.

$$\frac{1}{2} f_c bx = f_s A_{st} \Rightarrow \frac{1}{2} E_c \varepsilon_c bx = E_s \frac{\varepsilon_c (d - x)}{x} A_{st}$$

$$\Rightarrow \frac{1}{2} E_c bx^2 + E_s A_{st} x - E_s dA_{st} = 0 \qquad \text{(Eqn 9.1)}$$

Consider 1 m width of the section in Worked Example 9.1, $b = 1000$ (Eqn 9.1) becomes

$$\frac{1}{2} \times 11.85 \times 1000 x^2 + 200 \times 785 x - 200 \times 105 \times 785 = 0$$

Solving $x = 41.14$ mm

Taking moment about the centroid of the triangular concrete stress block (the moment should be the unfactored moment which is 5.817kNm/m as it is a checking on serviceability limit state), the steel tensile stress can be worked out as

$$M = f_s A_{st} \left(d - \frac{x}{3} \right) \Rightarrow f_s = \frac{M}{A_{st} \left(d - \frac{x}{3} \right)} = \frac{5.817 \times 10^6}{785 \left(105 - \frac{41.14}{3} \right)} \qquad \text{(Eqn 9.2)}$$

$= 81.17$ N/mm^2

So the strain of the steel is

$$\varepsilon_s = \frac{81.17}{200 \times 10^3} = 0.000406 < \frac{0.8 f_y}{E_s} = 0.0184 .$$

So checking of crack width by (Ceqn 7.1) is applicable.

At the extreme fibre of the concrete at the tension side, the strain is

$$\varepsilon_1 = \varepsilon_s \frac{(h - x)}{(d - x)} = 0.000406 \times \frac{150 - 41.14}{105 - 41.14} = 0.000692$$

By (CEqn 7.2), to include the stiffening effect of cracked concrete,

$$\varepsilon_m = \varepsilon_1 - \frac{b_t (h - x)(a' - x)}{3 E_s A_s (d - x)} = 0.000692 - \frac{1000(150 - 41.14)(150 - 41.14)}{3 \times 200 \times 10^3 \times 785 \times (105 - 41.14)}$$

$= 0.000298$

The expected shrinkage strain, in accordance with Cl. 3.1.8 of the Code is $\varepsilon_{cs} = c_s K_L K_c K_e K_j K_s$ where

$c_s = 3.0$;

$K_L = 275 \times 10^{-6}$ for normal air according to Figure 3.6 of the Code;

$K_c = 1.17$ according to Figure 3.3 of the Code for cement content 434 kg/m^3 and water cement ratio 0.47 for grade 35;

$K_e = 0.91$ according to Figure 3.7 of the Code for $h_e = 150$;

$K_j = 1$ according to Figure 3.5 at time at infinity.

$$K_s = \frac{1}{1+\rho\alpha_e} = \frac{1}{1+0.0052\times\frac{200}{11.85}} = 0.919$$ according to (Ceqn 3.3)

So the expected shrinkage strain is

$$\varepsilon_{cs} = 3.0\times 275\times 10^{-6}\times 1.17\times 0.91\times 1.0\times 0.919 = 0.000807 > 0.0006.$$

Thus it is subjected to "abnormally high shrinkage" according to the Code and half of the expected strain be added to ε_m.

$$\therefore \varepsilon_m = 0.000298 + 0.000807\times 0.5 = 0.000702$$

The cracked width should be the greatest at the concrete surface mid-way between steel bars as illustrated in Figure 9.7;

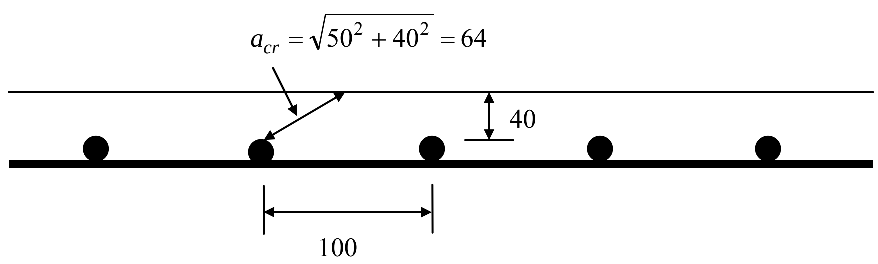


Figure 9.7 – Illustration of a_{cr} in Worked Example 9.1

By (CEqn 7.1) the cracked width is

$$\omega = \frac{3a_{cr}\varepsilon_m}{1+2\left(\frac{a_{cr}-c_{\min}}{h-x}\right)} = \frac{3\times 64\times 0.000702}{1+2\left(\frac{64-40}{150-41.14}\right)} = 0.0935\text{ mm} \le 0.1\text{ mm}$$ as required by PNAP 173. So O.K.

As PNAP 173 requires either checking of working stress below 100 MPa or crack width ≤ 0.1 mm, it should be adequate if any one of the conditions is satisfied. Apparently it would be simpler to check only

the former.

Summing up, reinforcement details is as shown :

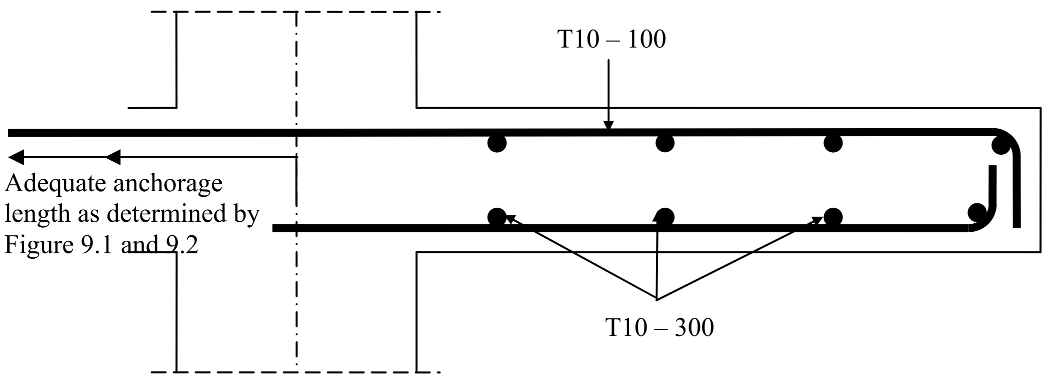


Figure 9.8 – Reinforcement Details for Worked Example 9.1

9.4 R.C. Detailing

Apart from the requirements stipulated in the preceding sections, reference can also be made to the drawings attached at the Appendices B and C of PNAP 173, especially for the locations of anchorage length commencement. However, it should be noted that not all sketches in PNAP 173 indicate locations of anchorage length commence from mid-support widths.

10.0 Transfer Structures

10.1 According to Cl. 5.5 of the Code, transfer structures are horizontal elements which redistribute vertical loads where there is a discontinuity between the vertical structural elements above and below.

10.2 In the analysis of transfer structures, consideration should be given to the followings as per Cl. 5.5 of the Code :

(i) Construction and pouring sequence – the effects of construction sequence can be important in design of transfer structures due to the comparatively large stiffness of the transfer structure and sequential built up of stiffness of structures above the transfer structure as illustrated in Figure 10.1;

(ii) Temporary and permanent loading conditions – especially important when it is planned to cast the transfer structures in two shifts and use the lower shift to support the upper shift as temporary conditions, thus creating locked-in stresses;

(iii) Varying axial shortening of elements supporting the transfer structures – which leads to redistribution of loads. The phenomenon is more serious as the transfer structure usually possesses large flexural stiffness in comparison with the supporting structural members, behaving somewhat between (a) flexible floor structures on hard columns; and (b) rigid structures (like rigid cap) on flexible columns;

(iv) Local effects of shear walls on transfer structures – shear walls will stiffen up transfer structures considerably and the effects should be taken into account in more accurate assessment of structural behaviour;

(v) Deflection of the transfer structures – will lead to redistribution of loads of the superstructure. Care should be taken if the structural model above the transfer structure is analyzed separately with the assumption that the supports offered by the transfer structures are rigid. Re-examination of the load redistribution should be carried out if the deflections of the transfer structures are found to be significant;

(vi) Lateral shear forces on the transfer structures – though the shear is lateral, it will nevertheless create out-of-plane loads in the transfer structures which needs be taken into account;

(vii) Sidesway of the transfer structures under lateral loads and unbalanced gravity loads should also be taken into account. The effects should be considered if the transfer structure is analyzed as a 2-D model.

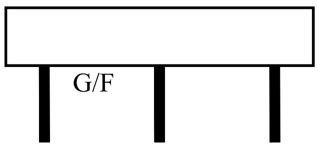
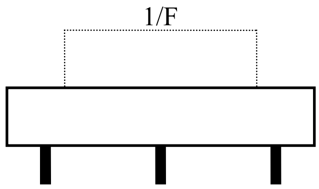
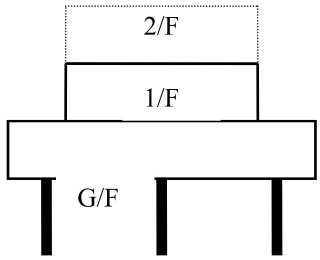
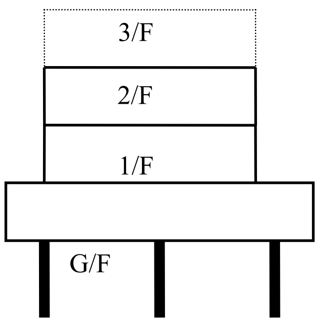
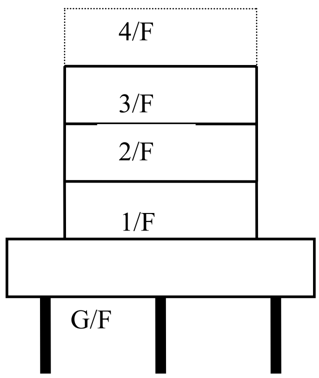
Stage (1) : Transfer Structure (T.S.) just hardened  Stress/force in T.S. : $\{F_T\}$ due to own weight of T.S. Stiffness : the T.S only	Stage (2) : Wet concrete of 1/F just poured  Stress/force in T.S. : $\{F_T\}$ + $\{F_1\}$, $\{F_1\}$ being force induced in transfer structure due to weight of 1/F structure. Stiffness : the T.S. only.	Stage (3) : 1/F hardened and 2/F wet concrete just poured  Stress/force in T.S. being due to $\{F_T\}$ + $\{F_1\}$ + $\{F_2\}$, $\{F_2\}$ being force induced in transfer structure due to weight of 2/F structure. Stiffness : the T.S. + 1/F.
Stage (4) : 2/F hardened and 3/F wet concrete just poured  Stress/force in T.S. : $\{F_T\}$ + $\{F_1\}$ + $\{F_2\}$ + $\{F_3\}$, $\{F_3\}$ being force induced in T.S. due to weight of 3/F structure. Stiffness : T.S. + 1/F + 2/F	Stage (5) : 3/F hardened and 4/F wet concrete just poured  Stress/force in T.S. : $\{F_T\}$ + $\{F_1\}$ + $\{F_2\}$ + $\{F_3\}$ + $\{F_4\}$, $\{F_4\}$ being force induced in T.S. due to weight of 4/F structure. Stiffness : T.S. + 1/F + 2/F + 3/F	Stage (6) and onwards Structure above transfer structure continues to be built. Final force induced on T.S. becomes $\{F_n\}$ + $\{F_{n-1}\}$ + $\{F_{n-2}\}$ + ... + $\{F_2\}$ + $\{F_1\}$ + $\{F_T\}$.

Figure 10.1 – Diagrammatic illustration of the Effects of Construction Sequence of loads induced on transfer structure

10.3 Mathematical modeling of transfer structures as 2-D model (by SAFE) :

The general comments in mathematical modeling of transfer structures as 2-D model to be analyzed by computer methods are listed :

(i) The 2-D model can only be analyzed against out-of-plane loads, i.e. vertical loads and out-of-plane moments. Lateral loads have to be analyzed separately;

(ii) It is a basic requirement that the transfer structure must be adequately stiff so that detrimental effects due to settlements of the columns and walls being supported on the transfer structure are tolerable. In view of the relatively large spans by comparing with pile cap, such settlements should be checked. Effects of construction sequence may be taken into account in checking;

(iii) The vertical settlement support stiffness should take the length of the column/wall support down to a level of adequate restraint against further settlement such as pile cap level. Reference can be made to Appendix H discussing the method of "Compounding" of vertical stiffness and the underlying assumption;

(iv) Care should be taken in assigning support stiffness to the transfer structures. It should be noted that the conventional use of either $4EI/L$ or $3EI/L$ have taken the basic assumption of no lateral movements at the transfer structure level. Correction to allow for sidesway effects is necessary, especially under unbalanced applied moments such as wind moment. Fuller discussion and means to assess such effects are discussed in Appendix H;

(v) Walls which are constructed monolithically with the supporting transfer structures may help to stiffen up the transfer structures considerably. However, care should be taken to incorporate such stiffening effect in the mathematical modeling of the transfer structures which is usually done by adding a stiff beam in the mathematical model. It is not advisable to take the full height of the wall in the estimation of the stiffening effect if it is of many storeys as the stiffness can only be gradually built up in the storey by storey construction so that the full stiffness can only be effected in supporting the upper floors. Four or five storeys of walls may be used for multi-storey buildings. Furthermore, loads induced in these stiffening structures (the stiff beams) have to be properly catered for which should be resisted by the wall forming the stiff beams;

10.4 Modeling of the transfer structure as a 3-dimensional mathematical model can eliminate most of the shortcomings of 2-dimensional analysis discussed in section 10.3, including the effects of construction sequence if the software has provisions for such effects. However, as most of these softwares may not have the sub-routines for detailed design, the designer may need to "transport" the 3-D model into the 2-D model for detailed design. For such "transportation", two approaches can be adopted :

(i) Transport the structure with the calculated displacements by the 3-D software (after omission of the in-plane displacements) into the 2-D software for re-analysis and design. Only the displacements of the nodes with external loads (applied loads and reactions) should be transported. A 2-D structure will be re-formulated in the 2-D software for re-analysis by which the structure is re-analyzed by forced displacements (the transported displacements) with recovery of the external loads (out-of-plane components only) and subsequently recovery of the internal forces in the structure. Theoretically results of the two models should be identical if the finite element meshing and the shape functions adopted in the 2 models are identical. However, as the finite element meshing of the 2-D model is usually finer than that of the 3-D one, there are differences incurred between the 2 models, as indicated by the differences in recovery of nodal forces in the 2-D model. The designer should check consistencies in reactions acting on the 2 models. If large differences occur, especially when lesser loads are revealed in the 2-D model, the designer should review his approach;

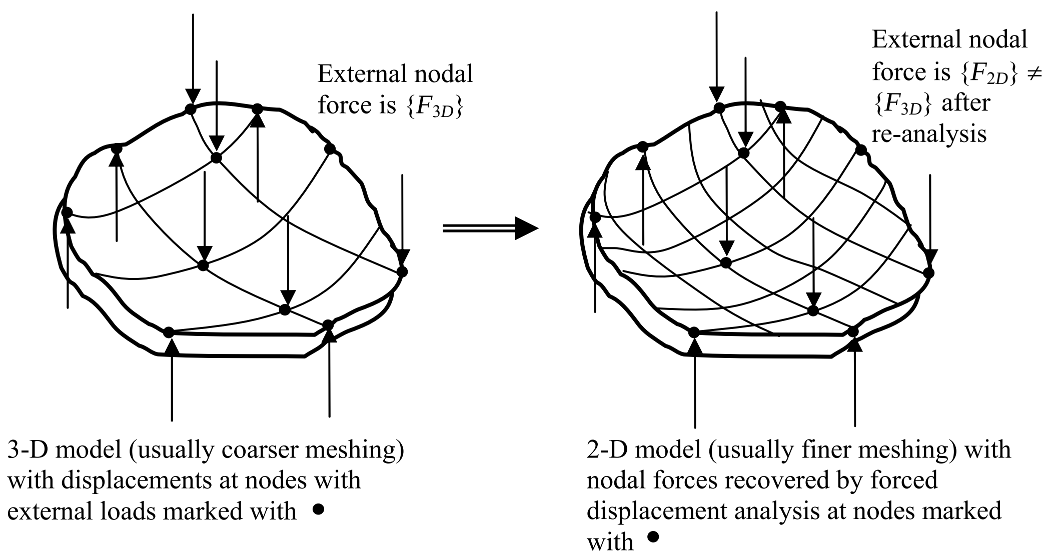


Figure 10.2 – 3-D model to 2-D with transportation of nodal displacements

(ii) Transport the out-of-plane components of the external loads (applied loads and reactions) acting on the 3-D model to the 2-D model for further analysis. This type of transportation is simpler and more reliable as full recovery of loads acting on the structure is ensured. However, in the re-analysis of the 2-D structure, a fixed support has to be added on any point of the structure for analysis as without which the structure will be unstable. Nevertheless, no effects due to this support will be incurred by this support because the support reactions should be zero as the transported loads from the 3-D model are in equilibrium.

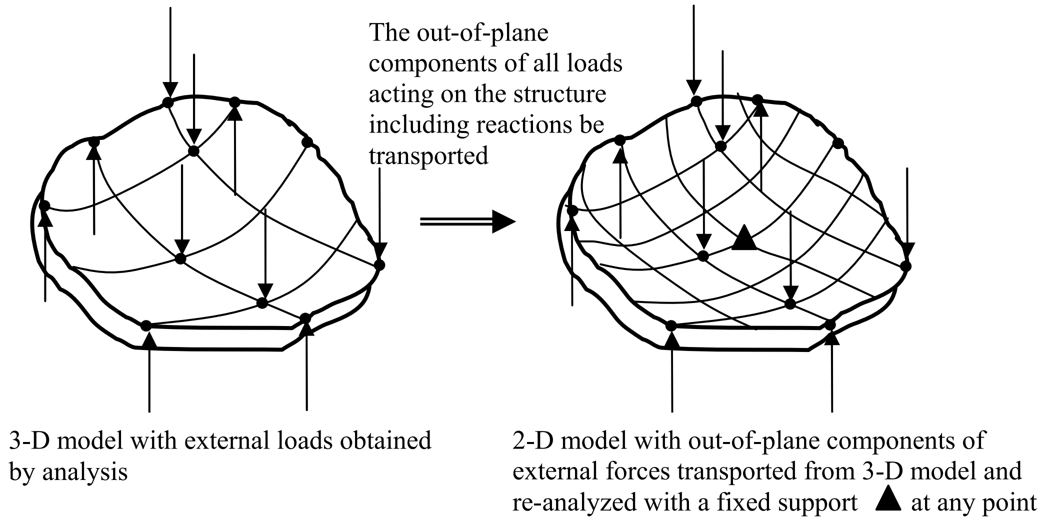


Figure 10.3 – 3-D model to 2-D with transportation of nodal forces

10.5 Structural Sectional Design and r.c. detailing

The structural sectional design and r.c. detailing of a transfer structure member should be in accordance with the structural element it simulates, i.e. it should be designed and detailed as a beam if simulated as a beam and be designed and detailed as a plate structure if simulated as a plate structure. Though not so common in Hong Kong, if simulation as a "strut-and-tie" model is employed, the sectional design and r.c. detailing should accordingly be based on the tie and strut forces so analyzed.

The commonest structural simulation of a transfer plate structure is as an assembly of plate bending elements analyzed by the finite element method. As such, the analytical results comprising bending, twisting moments and out-of-plane shears should be designed for. Reference to Appendix D can be made for the principles and design approach of the plate bending elements.

11.0 Footings

11.1 Analysis and Design of Footing based on the assumption of rigid footing

Cl. 6.7.1 of the Code allows a footing be analyzed as a "rigid footing" provided it is of sufficient rigidity with uniform or linearly varying pressures beneath. As suggested by the Code, the critical section for design is at column or wall face as marked in Figure 11.1, though in case of circular columns, the critical section may need be shifted into 0.2 times the diameter of the column, as in consistency with Cl. 5.2.1.2(b) of the Code.

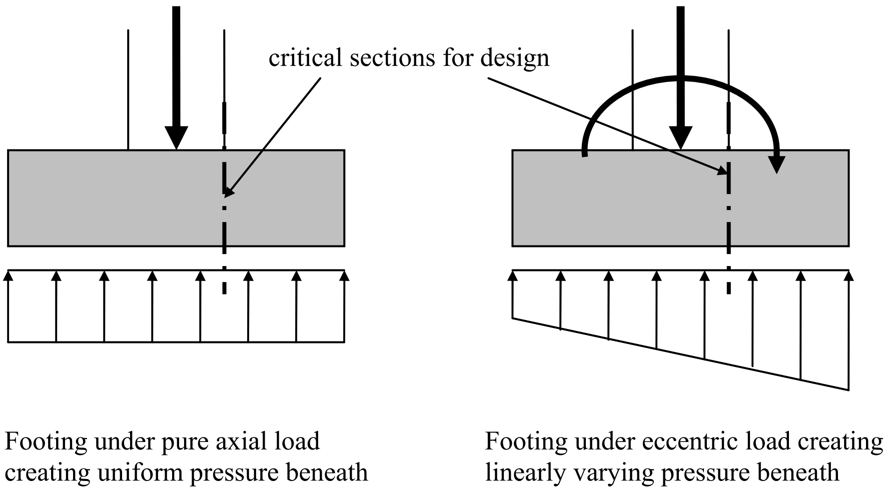


Figure 11.1 – Assumed Reaction Pressure on Rigid Footing

As it is a usual practice of treating the rigid footing as a beam in the analysis of its internal forces, Cl. 6.7.2.2 of the Code requires concentration of steel bars in areas with high stress concentrations as illustrated in Figure 11.2.

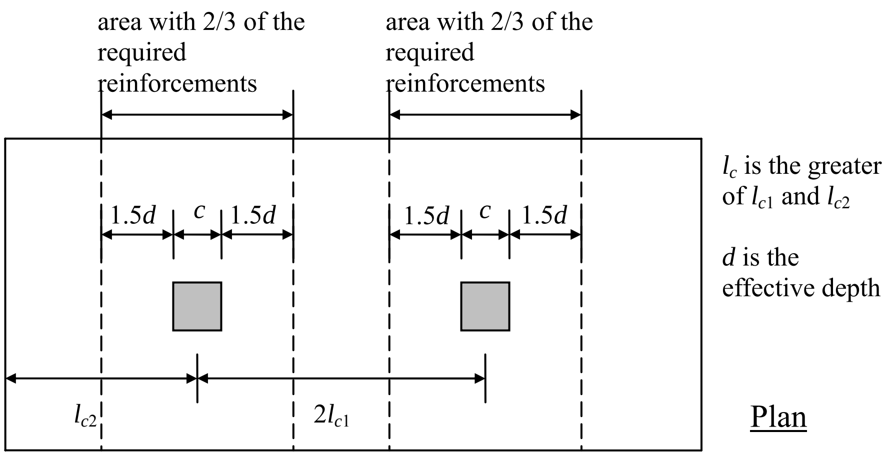


Figure 11.2 – Distribution of reinforcing bars when $l_c > (3c/4 + 9d/4)$

Cl. 6.7.2.4 of the Code requires checking of shear be based on (i) section through the whole width of the footing (as a beam); and (ii) local punching shear check as if it is a flat slab. (Re Worked Example 4.5 in Section 4).

11.2 Worked Example 11.1

Consider a raft footing under two column loads as shown in Figure 11.3. Design data are as follows :

Column Loads (for each): Axial Load: D.L. 800 kN L.L. 200 kN

Moment D.L. 100kNm L.L. 20 kNm

Overburden soil : 1.5 m deep

Footing dimensions : plan dimensions as shown, structural depth 400 mm, cover = 75 mm; Concrete grade of footing : grade 35

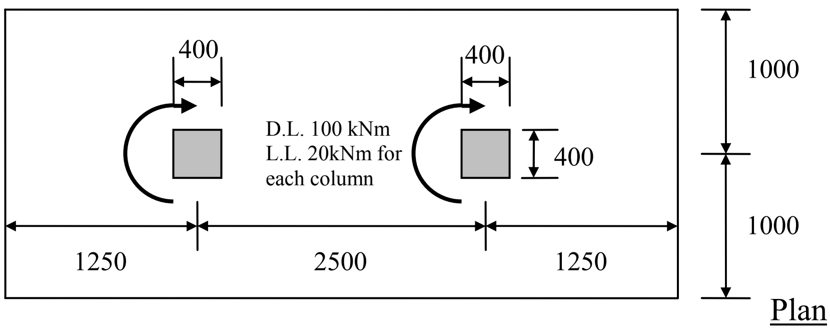


Figure 11.3 – Footing layout for Worked Example 11.1

(i) Loading Summary :

D.L. Column: $2\times 800 =$ 1600 kN;

O.W. $5.0\times 2.0\times 0.4\times 24 = 96$ kN

Overburden Soil $5.0\times 2\times 1.5\times 20 =$ 300 kN

Total 1996 kN

Moment (bending upwards as shown in Figure 11.3)

$2\times 100 = 200\,\text{kNm}$

L.L. Column $2\times 200 = 400\,\text{kN}$.

Moment (bending upwards as shown in Figure 11.3)

$2\times 20 = 40\,\text{kNm}$

Factored load : Axial load $1.4\times 1996 + 1.6\times 400 = 3434.4\,\text{kN}$

Moment $1.4\times 200 + 1.6\times 40 = 344\,\text{kNm}$

(ii) The pressure beneath the footing is first worked out as :

At the upper end : $\dfrac{3434.4}{5\times 2}+\dfrac{6\times 344}{5\times 2^2}=343.44+103.2=446.64\,\text{kN/m}^2$

At the lower end : $\dfrac{3434.4}{5\times 2}-\dfrac{6\times 344}{5\times 2^2}=343.44-103.2=240.24\,\text{kN/m}^2$

Critical section $\dfrac{3434.4}{5\times 2}+\dfrac{344\times 0.2}{5\times 2^3/12}=343.44+20.64=364.08\,\text{kN/m}^2$

The pressures are indicated in Figure 11.3(a)

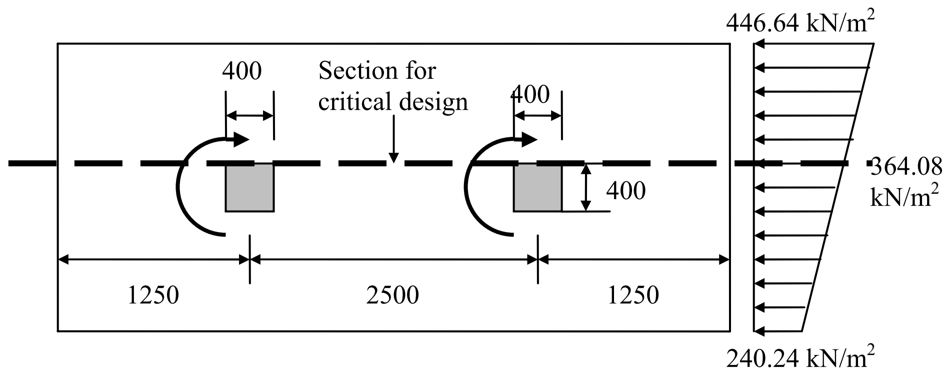


Figure 11.3(a) – Bearing Pressure for Worked Example 10.1

(iv) At the critical section for design as marked in Figure 11.3(a), the total shear is due to the upward ground pressure minus the weight of the footing and overburden soil ($1.4(0.4\times 24+1.5\times 20)=55.44$ kN/m^2) which is $\left(\dfrac{446.64+364.08}{2}\right)\times 0.8\times 5-55.44\times 0.8\times 5=1399.68\,\text{kN}$

The total bending moment is

$$(364.08-55.44)\times\frac{0.8^2}{2}\times 5+\left(\frac{446.64-364.08}{2}\right)\times 0.8^2\times\frac{2}{3}\times 5$$

$=581.89\,\text{kNm}$

(v) Design for bending : Moment per m width is :

$\dfrac{581.89}{5}=116.38\,\text{kNm/m};$

$d=400-75-8=317$ mm, assume T16 bars

$K=\dfrac{M}{bd^2}=\dfrac{116.38\times 10^6}{1000\times 317^2}=1.158,$

By the formulae in Section 3 for Rigorous Stress Approach,

$p_0=0.306\,\%$; $A_{st}=969\,\text{mm}^2\text{/m}$

As $l_c=1250>3c/4+9d/4=3\times 400/4+9\times 317/4=1013$, two thirds

of the reinforcements have to be distributed within a zone of $c+1.5\times 2d$ from the centre and on both sides of the column, i.e. a total width of $400+1.5\times 317\times 2=1.351$ m about the centre line of the columns.
Total flexural reinforcements over the entire width is
$969\times 5=4845$ mm^2, 2/3 of which in $1.351\times 2=2.702$ m.
So $4845\times 2/3/2.702=1195$ mm^2/m within the critical zone.
So provide T16 – 150.
Other than the critical zone, reinforcements per metre width is
$4845/3/(5-2.702)=703$ mm^2/m. Provide T16 – 275.

Design for Strip Shear : Total shear along the critical section is 1399.86 kN, thus shear stress is

$$v=\frac{1399.68\times 10^3}{5000\times 317}=0.883\ \text{N/mm}^2$$

$> \quad v_c=0.79\times 0.306^{1/3}\left(\frac{400}{317}\right)^{1/4}\frac{1}{1.25}\times\left(\frac{35}{25}\right)^{1/3}=0.505$ N/mm^2 as per Table 6.3 of the Code.
So shear reinforcement required is

$$\frac{A_{sv}}{s_v}=\frac{b(v-v_c)}{0.87f_{yv}}=\frac{5000(0.883-0.505)}{0.87\times 460}<\frac{5000\times 0.4}{0.87\times 460}=4.998\ \text{mm}^2/\text{mm}$$

Within the two-thirds (of total width 2.675 m) with heavier shear reinforcement :

$4.998\times\frac{2}{3}\div 2.702=1.233$ mm^2/m. Use T10 – 175 s.w. and – 300 l.w.

In the rest of the footing,

$4.998\times\frac{1}{3}\div 2.298=0.724$ mm/m. Use T10 – 300 BWs.

(vi) Check punching shear along perimeter of column
Factored load by a column is $1.4\times 800+1.6\times 200=1440$ kN. By Cl. 6.1.5.6(d), along the column perimeter,

$$\frac{V_{eff}}{ud}=\frac{1440\times 10^3}{4\times 400\times 317}=2.84<0.8\sqrt{f_{cu}}=4.7\ \text{MPa. O.K.}$$

Locate the next critical perimeter for punching shear checking as shown in Figure 11.3(b) which is at 1.5d from the column face.
Weight of overburden soil and weight of footing is
$1.351^2\times 55.44-1.4\times 0.4^2\times 1.5\times 20=94.47$ kN

Upthrust by ground pressure is $\frac{3434.4}{5\times 2}\times 1.351^2=627.03$ kN

Net load along the critical perimeter is
$1440 + 94.47 - 627.03 = 907.44\,\text{kN}$

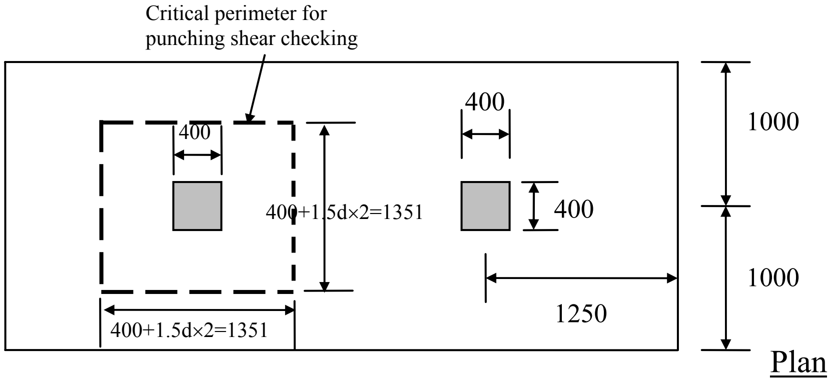


Figure 11.3(b) – checking punching shear for Worked Example 10.1

By (Ceqn 6.40)

$$V_{eff} = V_t\left(1 + \frac{1.5M_t}{V_t x_{sp}}\right) = 907.44\left(1 + \frac{1.5 \times 172}{907.44 \times 1.351}\right) = 1098.41\,\text{kN}$$

Punching shear stress is $v = \dfrac{1098.41 \times 10^3}{1351 \times 4 \times 317} = 0.641\,\text{N/mm}^2$

As $v < 1.6v_c = 0.808$, use (Ceqn 6.44) in determining punching shear reinforcement,

$$\frac{(v - v_c)ud}{0.87 f_{yv}} = \frac{(0.641 - 0.489) \times 1351 \times 4 \times 317}{0.87 \times 460} < \frac{0.4 \times 1351 \times 4 \times 317}{0.87 \times 460}$$

$=1712\text{m}^2$. The reinforcement should be distributed in the manner as that of flat slab, i.e. with 40%, 685mm^2 (i.e. 9 nos. of T10) at $0.5d$ (158.5mm) and others 1027 mm^2 (i.e. 13 nos. of T10)at $1.25d$ (396.25mm) away from the surface of the column as per the advice in Figure 6.13 of the Code.

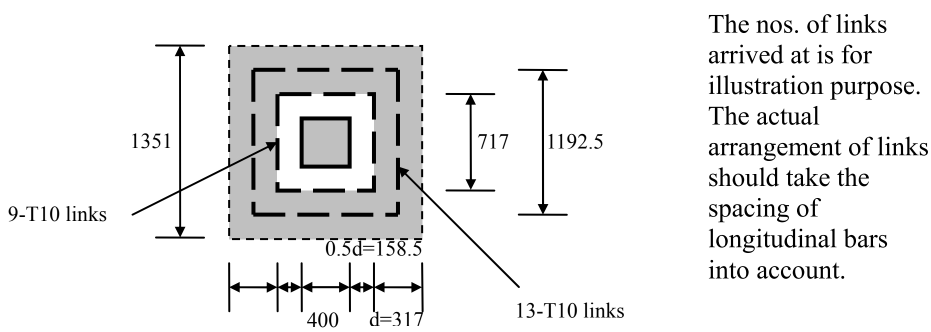


Figure 11.3(c) – Area for punching shear reinforcement

So the provision by the strip shear obtained in (v) which is greater is adopted as per Cl. 6.7.2.4 of the Code which requires the more "severe" provision for checking of strip and punching shears.

(vii) Checking of bending and shear in the direction parallel to the line joining the columns can be carried out similarly. However, it should be noted that there is a net "torsion" acting on any section perpendicular to the line joining the two columns due to linearly varying ground pressure. To be on the conservative side, shear arising due to this torsion should be checked and designed accordingly as a beam as necessary. Nevertheless, one can raise a comment that the design has to some extent be duplicated as checking of bending has been carried out in the perpendicular direction. Furthermore, for full torsion to be developed for design in accordance with (Ceqn 6.65) to (Ceqn 6.68) of the Code, the "beam" should have a free length of beam stirrup width + depth to develop the torsion (as illustrated in Figure 3.31 in Section 3) which is generally not possible for footing of considerable width. As unlike vertical shear where enhancement can be adopted with "shear span" less than $2d$ or $1.5d$, no similar strength enhancement is allowed in Code, though by the same phenomenon there should be some shear strength enhancement. So full design for bending in both ways together with torsion will likely result in over-design.

(viii) The flexural and shear reinforcements provisions for the direction perpendicular to the line joining the columns is

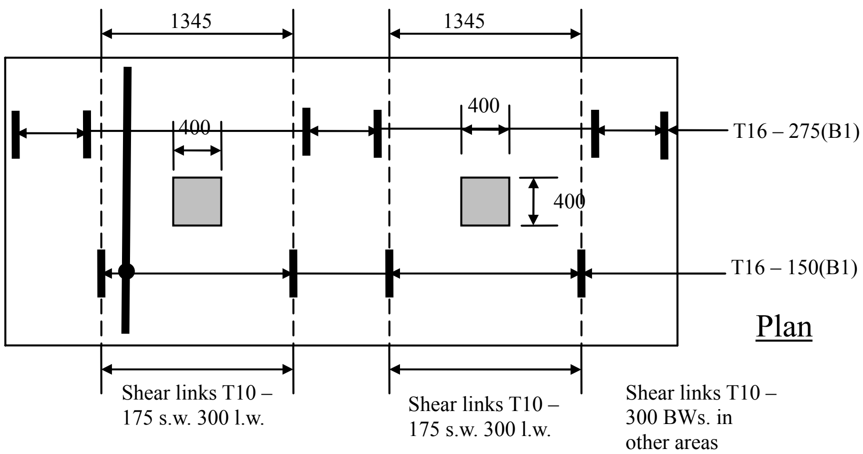


Figure 11.3(c) – Reinforcement Details for Worked Example 11.1 (in the direction perpendicular to the line joining the two columns only)

11.3 Flexible Footing Analysis and Design

As contrast to the footing analyzed under the rigid footing assumption, the analysis of footing under the assumption of its being a flexible structure will take the stiffness of the structure and the supporting ground into account by which the deformations of the structure itself will be analyzed. The deformations will affect the distribution of the internal forces of the structure and the reactions which are generally significantly different from that by rigid footing analysis. Though it is comparatively easy to model the cap structure, it is difficult to model the surface supports provided by the ground because :

(i) the stiffness of the ground with respect to the hardness of the subgrade and geometry of the footing are difficult to assess;

(ii) the supports are interacting with one another instead of being independent "Winkler springs" supports. However, we are currently lacking computer softwares to solve the problem. Use of constant "Winkler springs" thus becomes a common approach.

As the out-of-plane deformations and forces are most important in footing analysis and design, flexible footings are often modeled as plate bending elements analyzed by the finite element method as will be discussed in 11.4 in more details.

11.4 Analysis and Design by Computer Method

The followings are highlighted for design of footing modeled as 2-D model (idealized as assembly of plate bending elements) on surface supports:

(i) The analytical results comprise bending, twisting moments and out-of-plane shears for consideration in design;

(ii) As local "stresses" within the footing are revealed in details, the rules governing distribution of reinforcements in footing analyzed as a beam need not be applied. The design at any location in the footing can be based on the calculated stresses directly. However, if "peak stresses" (high stresses dropping off rapidly within short distance) occur at certain locations as illustrated in Figure 11.4 which are often results of finite element analysis at points with heavy loads or point supports, it would be reasonable to "spread" the stresses over certain width for design.

Nevertheless, care must be taken not to adopt widths too wide for "spreading" as local effects may not be well captured.

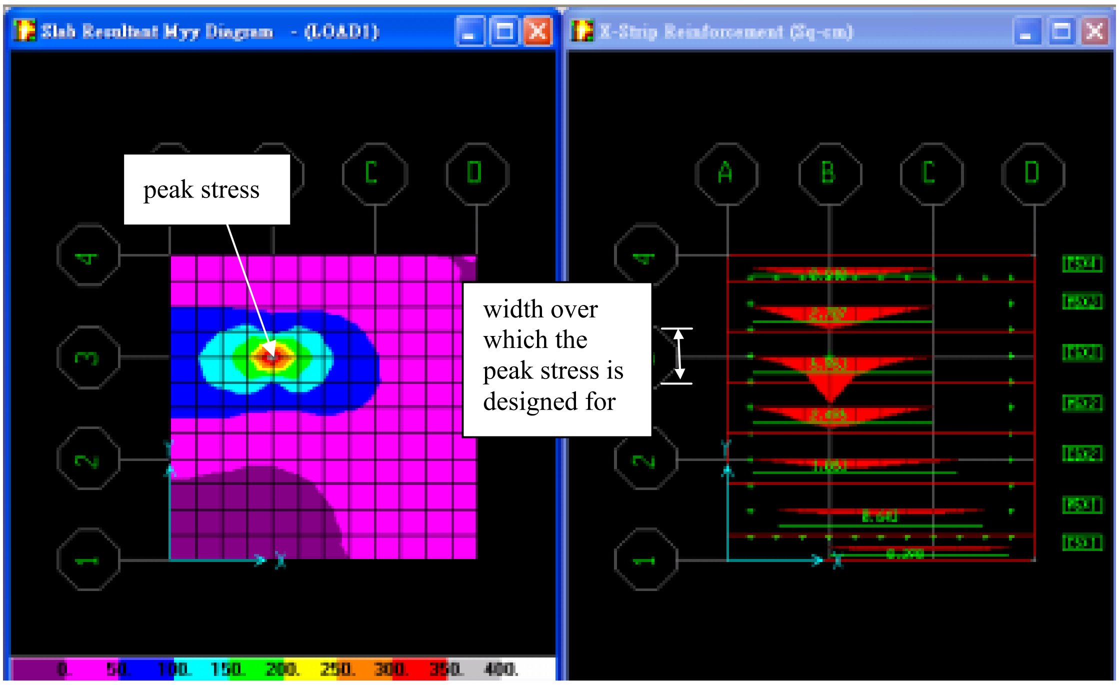


Figure 11.4 – Spreading of peak stress over certain width for design

(iii) The design against flexure should be done by the "Wood Armer Equations" listed in Appendix D, together with discussion of its underlying principles. As the finite element mesh of the mathematical model is often very fine, it is a practice of "lumping" the design reinforcements of a number of nodes over certain widths and evenly distributing the total reinforcements over the widths, as is done by the popular software "SAFE". Again, care must be taken in not taking widths too wide for "lumping" as local effects may not be well captured. The design of reinforcements by SAFE is illustrated on the right portion of Figure 11.4;

(iv) The principle together with a worked example for design against shear is included in Appendix D, as illustrated in Figure D5a to D5c. It should be noted that as the finite element analysis give detailed distribution of shear stresses on the structure, it is not necessary to carry out shear distribution into column and mid-strips as is done for flat slab under empirical analysis in accordance with the Code. The checking of shear and design of shear reinforcements can be based directly on the shear stresses revealed by the finite element analysis.

12.0 Pile Caps

12.1 Rigid Cap analysis

Cl. 6.7.3 of the Code allows a pile cap be analyzed and designed as a “rigid cap” by which the cap is considered as a perfectly rigid structure so that the supporting piles deform in a co-planar manner at their junctions with the cap. As the deformations of the piles are governed, the reactions by the piles can be found with their assigned (or assumed) stiffnesses. If it is assumed that the piles are identical (in stiffnesses), the reactions of the piles follow a linearly varying pattern. Appendix I contains derivation of formulae for solution of pile loads under rigid cap assumption.

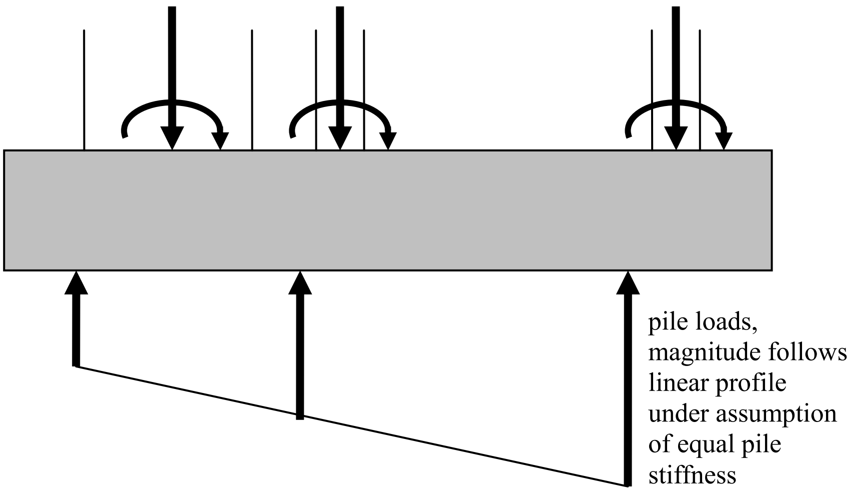


Figure 12.1 – Pile load profile under rigid cap assumption

Upon solution of the pile loads, the internal forces of the pile cap structure can be obtained with the applied loads and reactions acting on it as a free body. The conventional assumption is to consider the cap as a beam structure spanning in two directions and perform analysis and design separately. It is also a requirement under certain circumstances that some net torsions acting on the cap structure (being idealized as a beam) need be checked. As the designer can only obtain a total moment and shear force in any section of full cap width, there may be under-design against heavy local effects in areas having heavy point loads or pile reactions. The Code (Cl. 6.7.3.3) therefore imposes a condition that shear enhancement of concrete due to closeness of applied load and support cannot be applied.

Cl. 6.7.3.5 of the Code requires checking of torsion based on rigid body

theory which is similar to discussion in Section 11.2 (vii).

12.2 Worked Example 12.1 (Rigid Cap Design)

The small cap as shown in Figure 12.2 is analyzed by the rigid cap assumption and will then undergo conventional design as a beam spanning in two directions.

Design data : Pile cap plan dimensions : as shown
Pile cap structural depth : 2 m
Pile diameter : 2 m
Concrete grade of Cap : 35
Cover to main reinforcements : 75 mm
Column dimension : 2 m square
Factored Load from the central column :
$P = 50000$ kN
$M_x = 2000$ kNm (along X-axis)
$M_y = 1000$ kNm (along Y-axis)

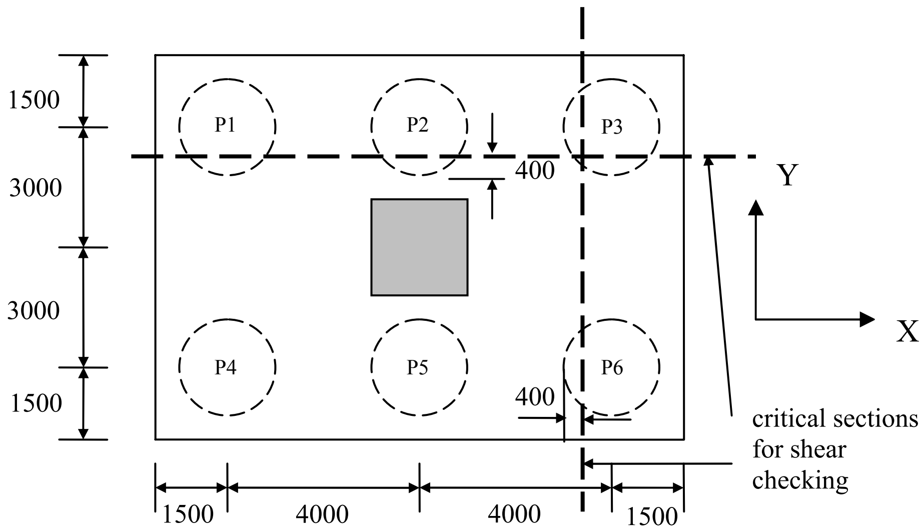


Figure 12.2 – Pile cap layout of Worked Example 12.1

(i) Factored Loads from the Column :
$P = 50000$ kN
$M_x = 2000$ kNm (along positive X-axis)
$M_y = 1000$ kNm (along positive Y-axis)
O.W. of Cap $11 \times 9 \times 2 \times 24 = 4752$ kN
Weight of overburden soil $11 \times 9 \times 1.5 \times 20 = 2970$ kN
Factored load due to O.W. of Cap and soil is

$$1.4\times(4752+2970)=10811\,\text{kN}$$

So total axial load is $50000+10811=60811\,\text{kN}$

(ii) Analysis of pile loads – assume all piles are identical

(Reference to Appendix I for general analysis formulae)

I_x of pile group = $6\times3^2=54$

I_y of pile group = $4\times4^2+2\times0=64$

Pile Loads on P1 : $\dfrac{60811}{6}-\dfrac{2000\times4}{64}+\dfrac{1000\times3}{54}=10065.72\,\text{kN}$

P2: $\dfrac{60811}{6}-\dfrac{2000\times0}{64}+\dfrac{1000\times3}{54}=10190.72\,\text{kN}$

P3: $\dfrac{60811}{6}+\dfrac{2000\times4}{64}+\dfrac{1000\times3}{54}=10315.72\,\text{kN}$

P4: $\dfrac{60811}{6}-\dfrac{2000\times4}{64}-\dfrac{1000\times3}{54}=9954.61\,\text{kN}$

P5: $\dfrac{60811}{6}-\dfrac{2000\times0}{64}-\dfrac{1000\times3}{54}=10079.61\,\text{kN}$

P6: $\dfrac{60811}{6}+\dfrac{2000\times4}{64}-\dfrac{1000\times3}{54}=10204.61\,\text{kN}$

(iii) Design for bending along the X-direction

The most critical section is at the centre line of the cap

Moment created by Piles P3 and P6 is

$(10315.72+10204.61)\times4=82081.32\,\text{kNm}$

Counter moment by O.W. of cap and soil is

$10811\div2\times2.75=14865.13\,\text{kNm}$

The net moment acting on the section is

$82081.32-14865.13=67216.19\,\text{kNm}$

$d=2000-75-60=1865$ (assume 2 layers of T40); $b=9000$

$\dfrac{M}{bd^2}=\dfrac{67216.19\times10^6}{9000\times1865^2}=2.147$; $\dfrac{z}{d}=0.926$ $p=0.58\%$

$A_{st}=97210\,\text{mm}^2$, provide T40 – 200 (2 layers, B1 and B3)

(iv) Design for shear in the X-direction

By Cl. 6.7.3.2 of the Code, the critical section for shear checking is at 20% of the diameter of the pile inside the face of the pile as shown in Figure 12.2.

Total shear at the critical section is :

Upward shear by P3 and P6 is $10315.72+10204.61=20520.33$ kN

Downward shear by cap's O.W. and soil is

$10811\times\dfrac{2.1}{11}=2063.92$ kN

Net shear on the critical section is $20520.33-2063.92=18456.41$ kN

$v=\dfrac{18456.41\times10^3}{9000\times1865}=1.10\,\text{N/mm}^2 > v_c=0.58\,\text{N/mm}^2$ by Table 6.3 of the Code.

No shear enhancement in concrete strength can be effected as per Cl. 6.7.3.3 of the Code because no shear distribution across section can be considered.

Shear reinforcements in form of links per metre width is

$\dfrac{A_{sv}}{s_v}=\dfrac{b(v-v_c)}{0.87f_{yv}}=\dfrac{1000(1.10-0.58)}{0.87\times460}=1.299$

Use T12 links – 200 in X-direction and 400 in Y-direction by which $\dfrac{A_{sv}}{s_v}$ provided is 1.41.

(v) Design for bending along the Y-direction

The most critical section is at the centre line of the cap

Moment created by Piles P1, P2 and P3

$(10065.72+10190.72+10315.72)\times3=30572.16\times3=91716.48$ kNm

Counter moment by O.W. of cap and soil is

$10811\div2\times2.25=12162.38$ kNm

The net moment acting on the section is

$91716.48-12162.38=79554.11$ kNm

$d=2000-75-60-40=1825$; (assume 2 layers of T40) $b=11000$

$\dfrac{M}{bd^2}=\dfrac{76554.11\times10^6}{11000\times1825^2}=2.09$; $\dfrac{z}{d}=0.929$ $p=0.55\%$

$A_{st}=110459\,\text{mm}^2$, provide T40 – 200 (2 layers, B2 and B4)

(vi) Checking for shear in the Y-direction

By Cl. 6.7.3.2 of the Code, the critical section for shear checking is at 20% of the diameter of the pile inside the face of the pile as shown in Figure 12.2

Total shear at the critical section is :

Upward shear by P1, P2 and P3 is 30572.16 kN

Downward shear by cap's O.W. and soil is

$$10811 \times \frac{2.1}{9} = 2522.57 \text{ kN}$$

Net shear on the critical section is $30572.16 - 2522.57 = 28049.59 \text{ kN}$

$v = \frac{28049.59 \times 10^3}{11000 \times 1825} = 1.397 \text{ N/mm}^2 > v_c = 0.579 \text{ N/mm}^2$ by Table 6.3 of the Code.

Similar to checking of shear checking in X-direction, no shear enhancement of concrete strength can be effected.

Shear reinforcements in form of links per metre width is

$$\frac{A_{sv}}{s_v} = \frac{b(v - v_c)}{0.87 f_{yv}} = \frac{1000(1.397 - 0.579)}{0.87 \times 460} = 2.044$$

As $\frac{A_{sv}}{s_v}$ in Y-direction is greater than that in X-direction, so adopt this for shear reinforcement provision.

Use T12 links – 200 BWs by which $\frac{A_{sv}}{s_v}$ provided is 2.82.

(vii) Punching shear :

Punching shear check for the column and the heaviest loaded piles at their perimeters in accordance with Cl. 6.1.5.6 of the Code :

Column : $\frac{1.25 \times 50000 \times 10^3}{4 \times 2000 \times 1825} = 4.28 \text{ MPa} < 0.8\sqrt{f_{cu}} = 4.73 \text{ MPa}.$

Pile P3 : $\frac{1.25 \times 10315 \times 10^3}{2000\pi \times 1825} = 1.12 \text{ MPa} < 0.8\sqrt{f_{cu}} = 4.73 \text{ MPa}.$

Not necessary to check punching shear at the next critical perimeters as the piles and column overlap with each other to very appreciable extents;

(viii) Checking for torsion : There are unbalanced torsions in any full width sections at X-Y directions due to differences in the pile reactions. However, as discussed in sub-section 11.2(vii) of this Manual for footing, it may not be necessary to design the torsion as for that for beams. Anyhow, the net torsion is this example is small, being $361.11 \times 3 = 1083.33 \text{ kNm}$ (361.11kN is the difference in pile loads between P3 and P4), creating torsional shear stress in the order of

$$v_t = \frac{2T}{h_{\min}{}^2\left(h_{\max} - \frac{h_{\min}}{3}\right)} = \frac{2 \times 1083.33 \times 10^6}{2000^2\left(9000 - \frac{2000}{3}\right)} = 0.065\ \text{N/mm}^2.$$ So the torsional shear effects should be negligible;

(ix) Finally reinforcement details are as shown in Figure 12.3,

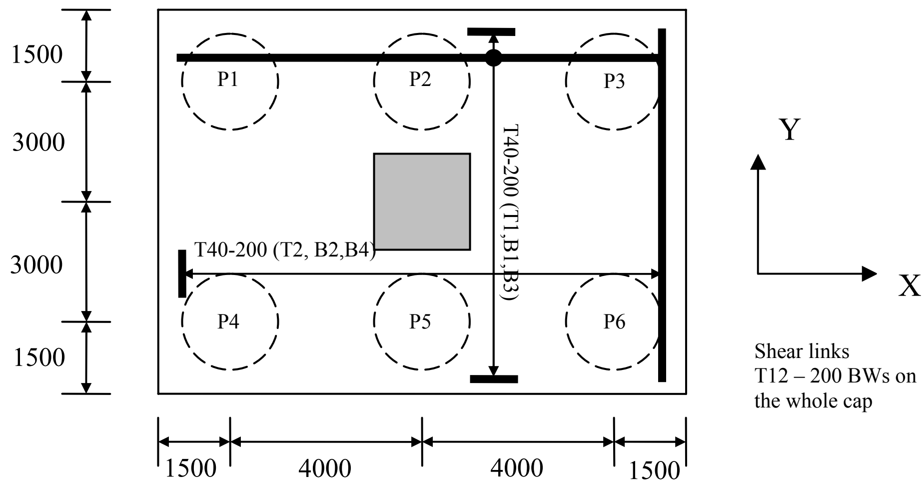


Figure 12.3 – Reinforcement Design of Worked Example 12.1

12.3 Strut-and-Tie Model

Cl. 6.7.3.1 of the Code allows pile cap be designed by the truss analogy, or more commonly known as “Strut-and-Tie Model” (S&T Model) in which a concrete structure is divided into a series of struts and ties which are beam-like members along which the stress are anticipated to follow. In a S&T model, a strut is a compression member whose strength is provided by concrete compression and a tie is a tension member whose strength is provided by added reinforcements. In the analysis of a S&T model, the following basic requirements must be met (Re ACI Code 2002):

(i) Equilibrium must be achieved;
(ii) The strength of a strut or a tie member must exceed the stress induced on it;
(iii) Strut members cannot cross each other while a tie member can cross another tie member;
(iv) The smallest angle between a tie and a strut joined at a node should exceed 25°.

The Code has specified the following requirements in Cl. 6.7.3.1 :

(i) Truss be of triangular shape;

(ii) Nodes be at centre of loads and reinforcements;

(iii) For widely spaced piles (pile spacing exceeding 3 times the pile diameter), only the reinforcements within 1.5 times the pile diameter from the centre of pile can be considered to constitute a tension member of the truss.

12.4 Worked Example 12.2 (Strut-and-Tie Model)

Consider the pile cap supporting a column factored load of 6000kN supported by two piles with a column of size 1m by 1 m. The dimension of the cap is as shown in Figure 12.4, with the width of cap equal to 1.5 m.

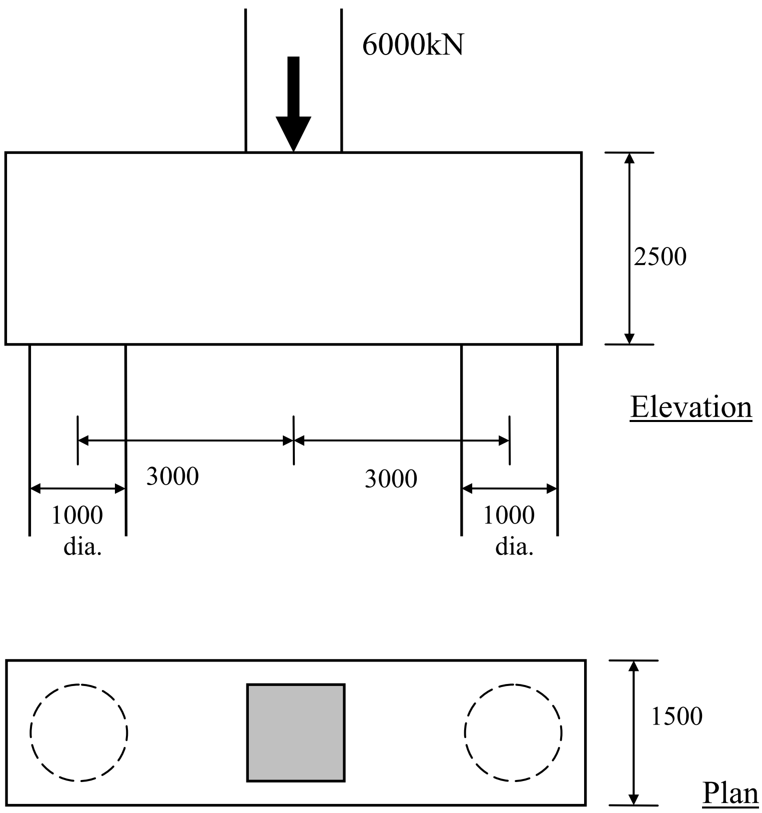


Figure 12.4 – Pile Cap Layout of Worked Example 12.2

(i) Determine the dimension of the strut-and-tie model

Assume two layers of steel at the bottom of the cap, the centroid of the two layers is at $75+40+20=135$ mm from the base of the cap. So the

effective width of the tension tie is $135 \times 2 = 270$ mm. The dimensions and arrangement of the ties and struts are drawn in Figure 12.5.

(ii) A simple force polygon is drawn and the compression in the strut can be simply worked out as (C is the compression of the strut) :
$2C\sin 38.25^0 = 6000 \Rightarrow C = 4845.8$ kN;
And the tension in the bottom tie is $T = C\cos 38.25^0 = 3805.49$ kN.

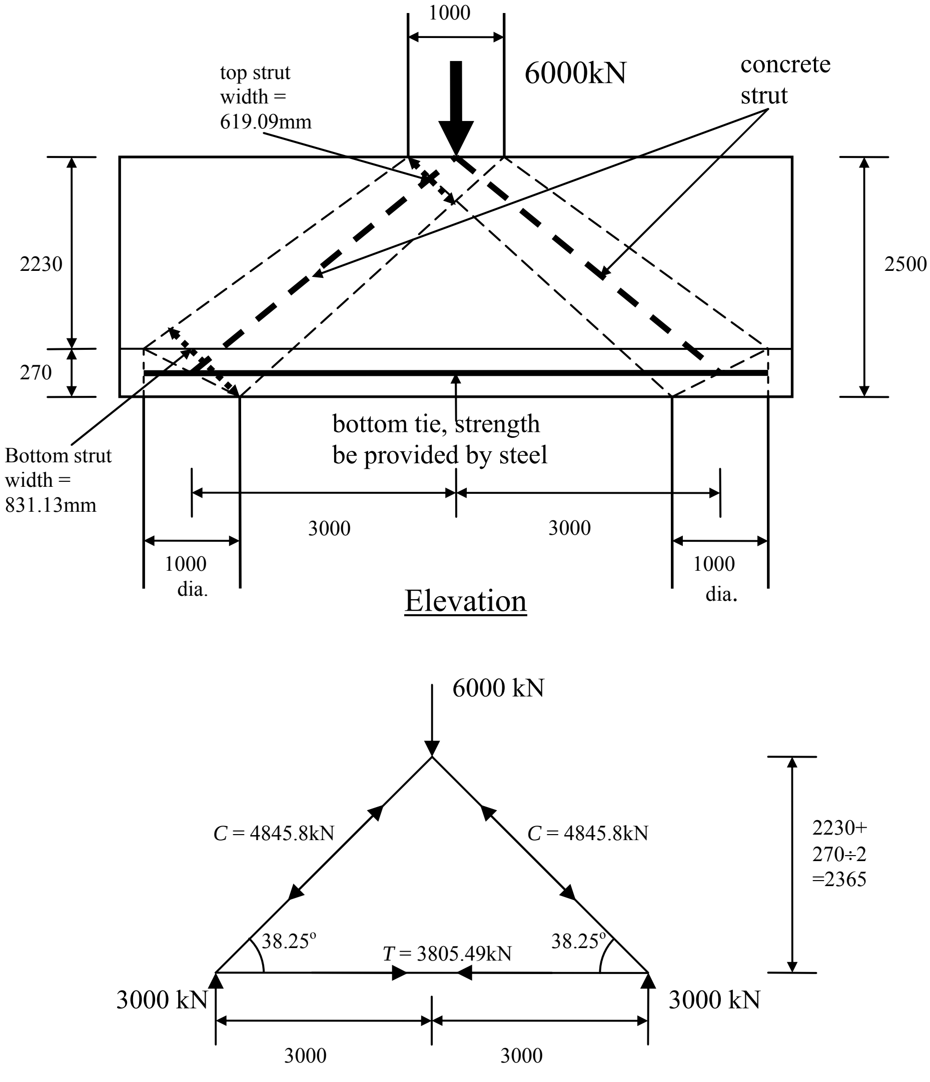


Figure 12.5 – Analysis of strut and tie forces in Worked Example 12.2

(iii) To provide the bottom tension of 3805.49 kN, the reinforcement steel required is $\dfrac{3805.49 \times 10^3}{0.87 f_y} = \dfrac{3805.49 \times 10^3}{0.87 \times 460} = 9509$ mm^2. Use 8–T40;

(iv) Check stresses in the struts :

Bottom section of the strut, the strut width at bottom is
$1000\sin 38.25^0 + 270\cos 38.25^0 = 831.13$ mm
As the bottom part is in tension, there is a reduction of compressive strength of concrete to $1.8\sqrt{f_{cu}} = 1.8\sqrt{35} = 10.08$ MPa as suggested by OAP, which is an implied value of the ultimate concrete shear strength of $0.8\sqrt{f_{cu}}$ as stated in the Code and BS8110.

As a conservative approach, assuming a circular section at the base of the strut since the pile is circular, the stress at the base of the strut is
$$\frac{4845.8\times 10^3}{831^2\pi/4} = 8.93\text{ MPa} < 10.08\text{MPa}$$
For the top section of the strut, the sectional width is
$2\times 500\sin 38.25^0 = 619.09$ mm
As the sectional length of the column is 1 m, it is conservative to assume a sectional area of 1000mm × 619.09 mm.
The compressive stress of the strut at top section is
$$\frac{4845.8\times 10^3}{1000\times 619.09} = 7.83\text{ MPa} < 0.45 f_{cu} = 15.75\text{ MPa}$$

(v) The reinforcement details are indicated in Figure 12.6. Side bars are omitted for clarity.

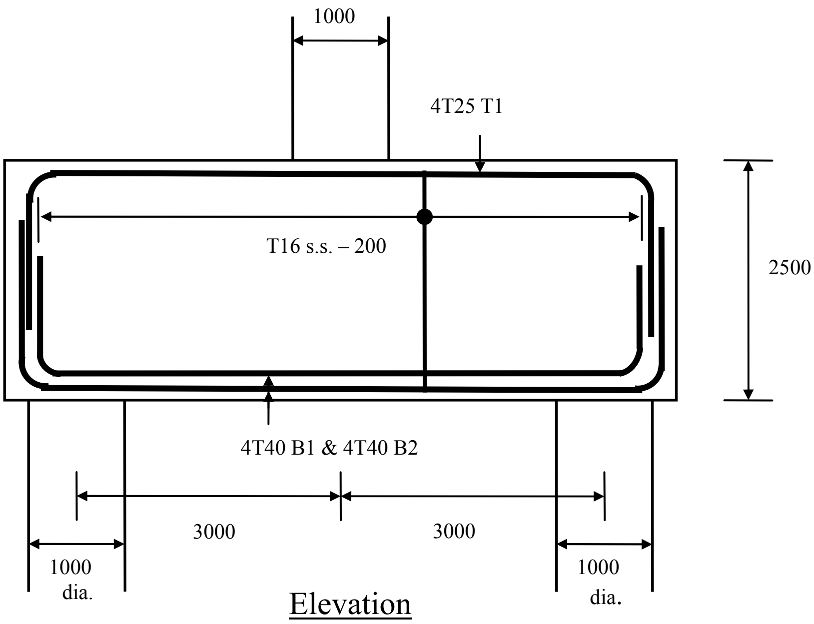


Figure 12.6 – Reinforcement Details of Worked Example 12.2

12.5 Flexible Cap Analysis

A pile cap can be analyzed by treating it as a flexible structure, i.e., as in contrast to the rigid cap assumption in which the cap is a perfectly rigid body undergoing rigid body movement only with no deformation upon the application of loads, the flexible pile cap structure will deform and the deformations will affect the distribution of internal forces of the structure and the reactions. Analysis of the flexible cap structure will require input of the stiffness of the structure which is comparatively easy. However, as similar to that of footing, the support stiffness of the pile cap which is mainly offered by the supporting pile is often difficult, especially for the friction pile which will interact significantly with each other through the embedding soil. Effects by soil restraints on the piles can be considered as less significant in end-bearing piles such large diameter bored piles.

Similar to the flexible footing, as the out-of-plane loads and deformation are most important in pile cap structures, most of the flexible cap structures are modeled as plate structures and analyzed by the finite element method.

12.6 Analysis and Design by Computer Method

Analysis and design by computer method for pile cap are similar to Section 11.3 for footing. Nevertheless, as analysis by computer methods can often account for load distribution within the pile cap structure, Cl. 6.7.3.3 of the Code has specified the followings which are particularly applicable for pile cap design :

(i) shear strength enhancement of concrete may be applied to a width of 3ϕ for circular pile, or pile width plus 2 × least dimension of pile as shown in Figure 12.7 as shear distribution across section has generally been considered in flexible cap analysis;

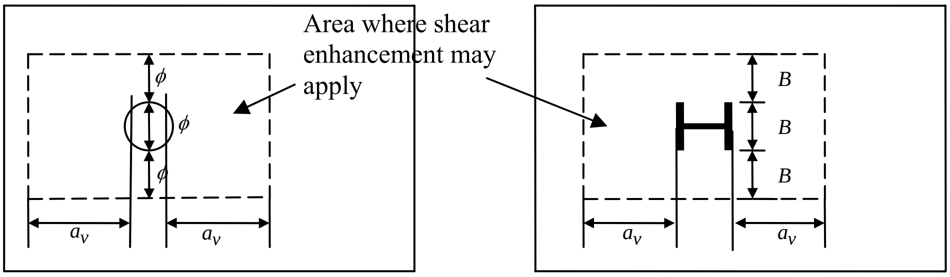


Figure 12.7 – Effective width for shear enhancement in pile cap around a pile

(ii) averaging of shear force shall not be based on a width > the effective depth on either side of the centre of a pile, or as limited by the actual dimension of the cap.

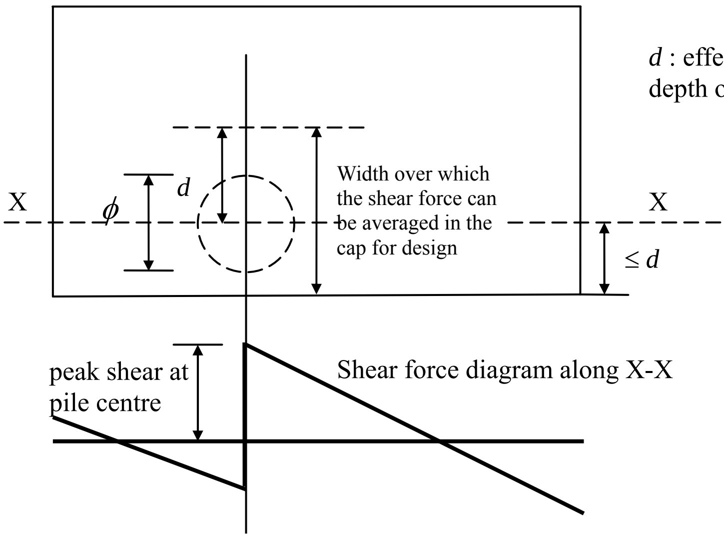


Figure 12.8 – Width in cap over which shear force at pile can be averaged for Design

Illustration in Figure 12.8 can be a guideline for determination of "effective widths" adopted in averaging "peak stresses" as will often be encountered in finite element analysis for pile cap structure modeled as an assembly of plate bending elements under point loads and point supports, as in the same manner as that for footing discussed in 11.4(ii) of this Manual.

13.0 General Detailings

13.1 In this section, the provisions of detailing requirements are general ones applicable to all types of structural members. They are mainly taken from Section 8 of the Code. Requirements marked with (D) are ductility ones for beams and columns contributing in lateral load resisting system.

13.2 Minimum spacing of reinforcements (Cl. 8.2 of the Code) – clear distance (horizontal and vertical) is the greatest of

(i) maximum bar diameter;

(ii) maximum aggregate size (h_{agg}) + 5 mm;

(iii) 20 mm.

13.3 Permissible bent radii of bars. The purpose of requiring minimum bend radii for bars are

(i) avoid damage of bar;

(ii) avoid overstress by bearing on concrete in the bend.

Table 8.2 of the Code requires the minimum bend radii to be 3ϕ for $\phi \le 20\text{mm}$ and 4ϕ for $\phi > 20\text{mm}$ (for both mild steel and high yield bar) and can be adopted without causing concrete failures if any of the conditions shown in Figure 13.1 is satisfied as per Cl. 8.3 of the Code.

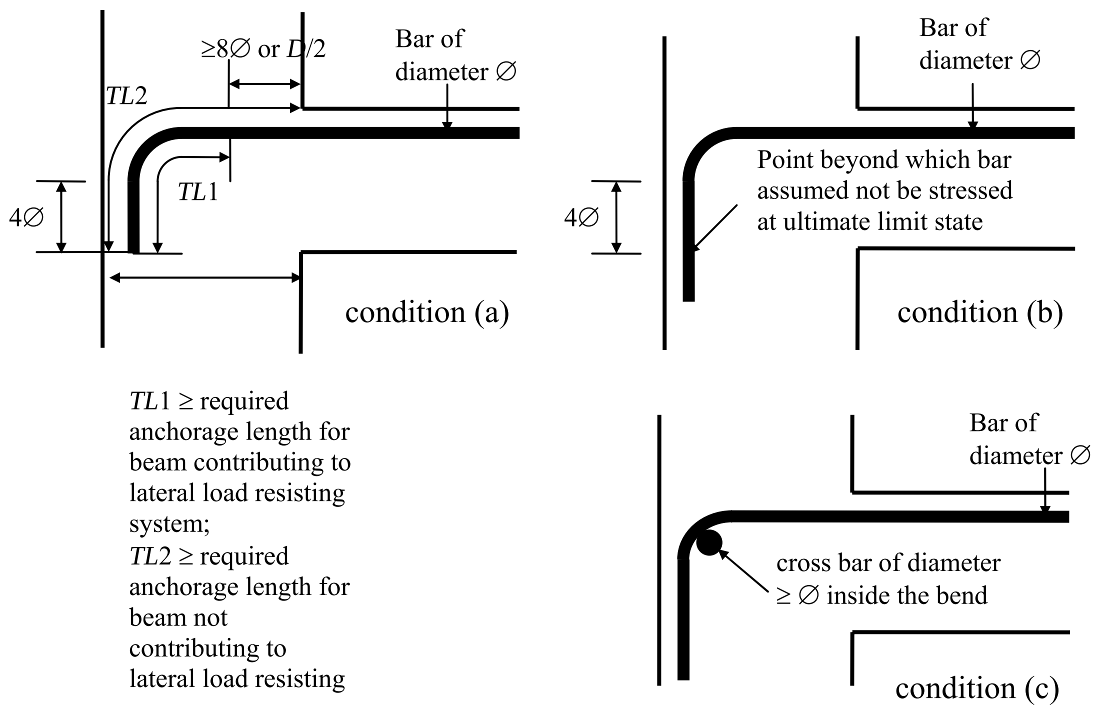


Figure 13.1 – Conditions by which concrete failure be avoided by bend of bars

If the none of the conditions in Figure 13.1 is fulfilled, (Ceqn 8.1) of the Code, reproduced as (Eqn 13.1) in this Manual should be checked to ensure that bearing pressure inside the bend is not excessive.

$$\text{bearing stress} = \frac{F_{bt}}{r\phi} \le \frac{2f_{cu}}{\left(1+2\dfrac{\phi}{a_b}\right)} \qquad \text{(Eqn 13.1)}$$

In (Eqn 13.1), F_{bt} is the tensile force in the bar at the start of the bend; r the internal bend radius of the bar; ϕ is the bar diameter, a_b is centre to centre distance between bars perpendicular to the plane of the bend and in case the bars are adjacent to the face of the member, $a_b = \phi + \text{cover}$.

Take an example of a layer of T40 bars of centre to centre separation of 100 mm and internal bend radii of 160mm in grade 35 concrete.

$F_{bt} = 0.87 \times 460 \times 1257 = 503051\,\text{N}$

$$\frac{F_{bt}}{r\phi} = \frac{503051}{160 \times 40} = 78.6 > \frac{2f_{cu}}{\left(1+2\dfrac{\phi}{a_b}\right)} = \frac{2\times 35}{\left(1+2\times\dfrac{40}{100}\right)} = 38.89$$

So (Ceqn 8.1) is not fulfilled. Practically a cross bar should be added as in Figure 13.1(c) as conditions in Figure 13.1(a) and 13.1(b) can unlikely be satisfied.

13.4 Anchorage of longitudinal reinforcements

(i) Anchorage is derived from ultimate anchorage bond stress with concrete assessed by the (Ceqn 8.3) of the Code.
$f_{bu} = \beta\sqrt{f_{cu}}$ where for high yield bars $\beta = 0.5$ for tension and $\beta = 0.65$ for compression. For example, $f_{bu} = 0.5\sqrt{35} = 2.96\,\text{MPa}$ for grade 35. For a bar of diameter ϕ, the total force up to $0.87f_y$ is $0.87f_y\left(\dfrac{\phi^2\pi}{4}\right)$. The required bond length L will then be related by $0.87f_y\left(\dfrac{\phi^2\pi}{4}\right) = \beta\sqrt{f_{cu}}\,\pi\phi L \Rightarrow L = \dfrac{0.87f_y\phi}{4\beta\sqrt{f_{cu}}} = 33.8\phi \approx 34\phi$ which agrees with Table 8.5 of the Code;

(ii) Notwithstanding provision in (i), it has been stated in 9.9.1.1(c) of the Code which contains ductility requirements for longitudinal bars of beams (contributing in lateral load resisting system) anchoring into

exterior column requiring anchorage length to be increased by 15% as discussed in Section 3.6 (v); (D)

(iii) With the minimum support width requirements as stated in Cl. 8.4.8 of the Code, bends of bars in end supports of slabs or beams will start beyond the centre line of supports offered by beams, columns and walls. By the same clause the requirement can be considered as not confining to simply supported beam as stated in Cl. 9.2.1.7 of the Code as illustrated in Figure 13.2.

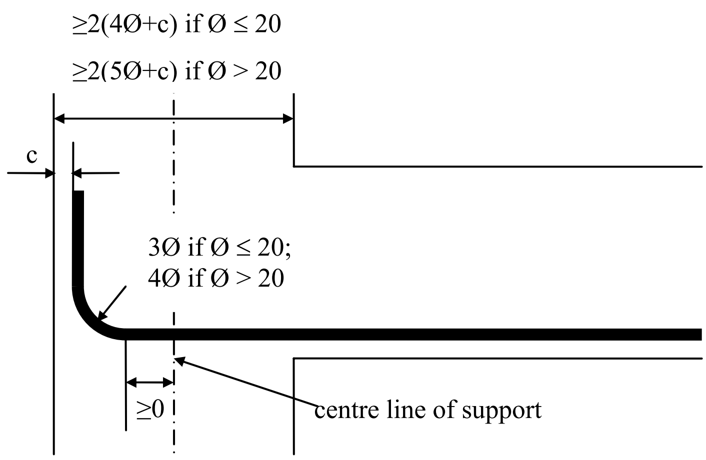


Figure 13.2 – Support width requirement

13.5 Anchorage of links – Figure 8.2 of the Code displays bend of links of bend angles from 90^o to 180^o. However, it should be noted that the Code requires anchorage links in beams and columns contributing in lateral load resisting system to have bent angles not less than 135^o as ductility requirements (D);

13.6 Laps arrangement – Cl. 8.7.2 of the Code requires laps be "normally" staggered with the followings requirement for 100% lapping in one single layer:

(i) Sum of reinforcement sizes in a particular layer must not exceed 40% of the breadth of the section at that level, otherwise the laps must be staggered;
(ii) Laps be arranged symmetrically;
(iii) Details of requirements in bar lapping are indicated in Figure 8.4 of the Code reproduced in Figure 13.3 for ease of reference;

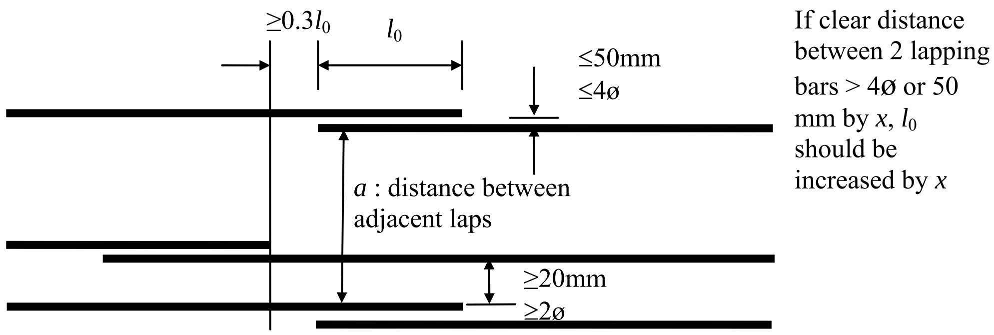


Figure 13.3 – Lapping arrangement for tension laps

(iv) When Figure 13.3 is complied with, the permissible percentage of lapped bars in tension may be 100% (but still required to be staggered, i.e. not in the same section)where the bars are all in one layer. When the bars are in several layers, the percentage should be reduced to 50%;

(v) Compression and secondary reinforcements can be lapped in one section.

The Clause effectively requires tension laps to be staggered with arrangement as shown in Figure 13.3 which is applicable in to the flexural steel bars in beams, slabs, footings, pile caps etc. Fortunately, the Code allows compression and secondary bars be lapped in one section, i.e. without the necessity of staggered laps. As such staggered laps can be eliminated in most of the locations in columns and walls.

13.7 Lap Lengths (Cl. 8.7.3 of the Code)

The followings should be noted for tension lap lengths:

(i) Absolute minimum lap length is the greater of 15ϕ and 300 mm;

(ii) Tension lap length should be at least equal to the design tension anchorage length and be based on the diameter of the smaller bar;

(iii) Lap length be increased by a factor of 1.4 or 2.0 as indicated in Figure 13.4 which is reproduced from Figure 8.5 of the Code.

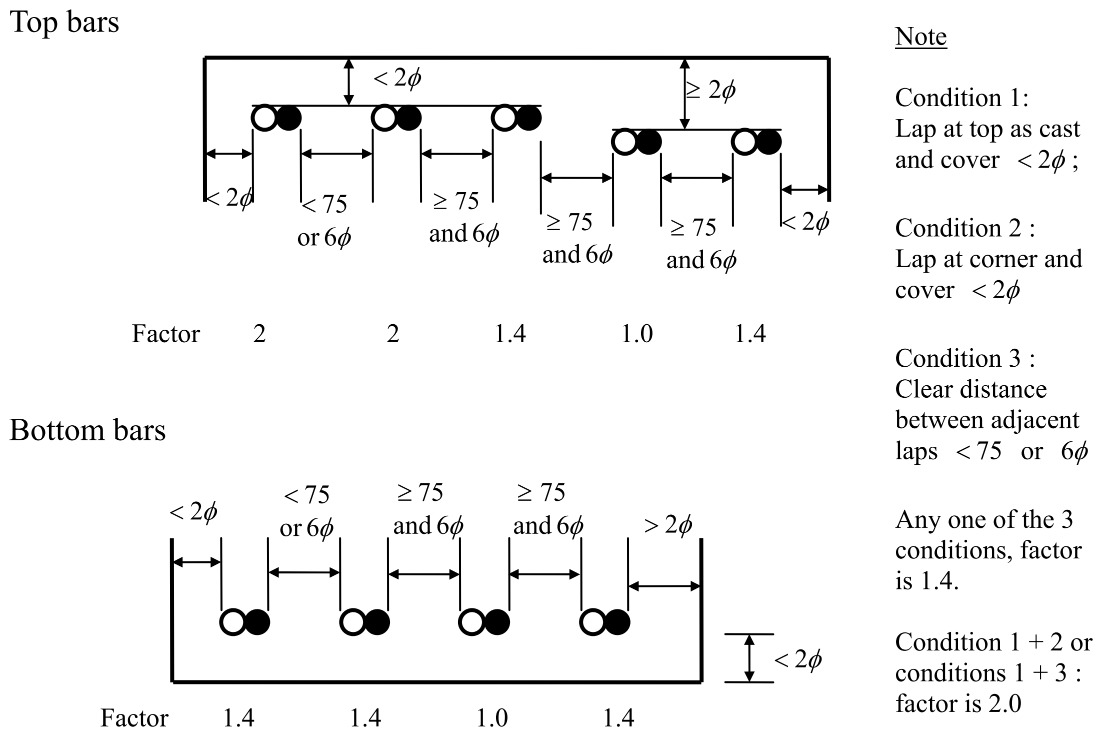


Figure 13.4 – Factors for tension lapping bars

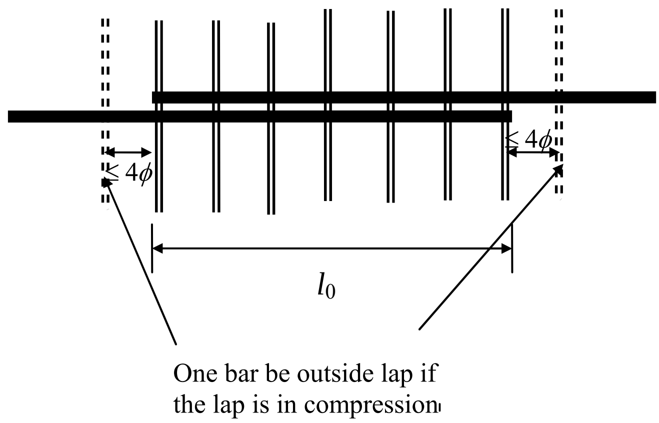
The compression lap length should be at least 25% greater than the design compression anchorage length as listed in Table 8.4 of the Code.

13.8 Transverse reinforcement in the tension lap zone (Cl. 8.7.4 of the Code)

For lapped longitudinal bars in tension, the transverse reinforcement is used to resist transverse tension forces. 3 cases be considered as :

(i) No additional transverse reinforcement is required (existing transverse reinforcement for other purpose can be regarded as sufficient to resist the transverse tension forces) when the longitudinal bar diameter $\phi < 20$ mm or percentage of lapping in any section < 25%;

(ii) When $\phi \geq 20$ mm, the transverse reinforcement should have area $\sum A_{st} \geq A_s$ where A_s is the area of one spiced bar and be placed between the longitudinal bar and the concrete surface as shown in Figure 13.5;



For T40 bars in tension lap in concrete grade 35 with spacing 200 mm (≤ 10ø = 400mm) and lap length

l_0 = 1.4×standard lap = 1920 mm.

Transverse reinforcement area required is $\sum A_{st} = 1257$ mm^2.

Use 12T12, spacing is 175 mm along the lapped length.

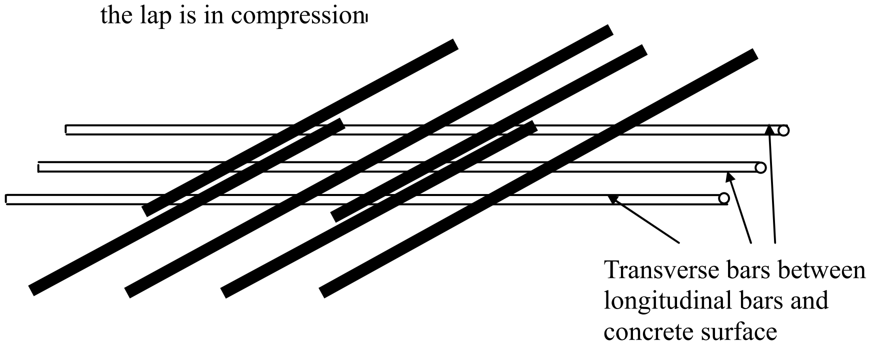


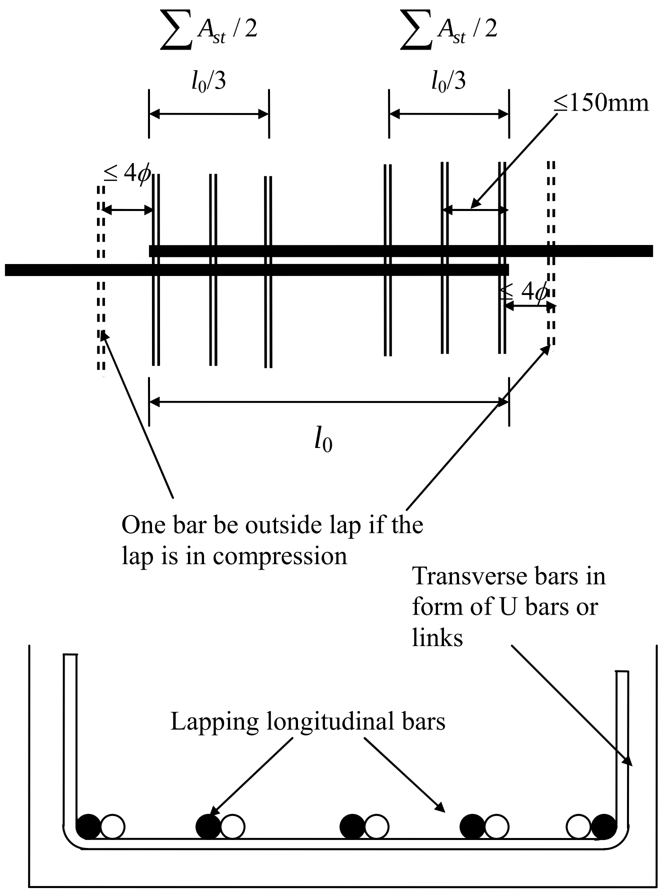
Figure 13.5 – Transverse reinforcement for lapped splices –not greater than 50% of reinforcement is lapped at one section and $\phi \geq 20$ mm

(iii) If more than 50% of the reinforcement is lapped at one point and the distance between adjacent laps $\leq 10\phi$, the transverse reinforcement should be formed by links or U bars anchored into the body of the section. The transverse reinforcement should be positioned at the outer sections of the lap as shown in Figure 13.6;

It should be noted that effectively condition (ii) requires area of transverse reinforcement identical to that of (iii), except that the bars need not be concentrated at the ends of the laps and the transverse reinforcements be in form of links or U bars.

13.9 Transverse reinforcement in the permanent compression lap zone

The requirement will be identical to that of tension lap except for an additional requirement that one bar of the transverse reinforcement should be placed outside each of the lap length and within 4ϕ of the ends of the lap length also shown in Figure 13.5 and 13.6.



For T40 bars in tension lap in concrete grade 35 with spacing 200 mm with lap length
l_0 = 1.4×standard lap = 1920 mm.
$l_0/3$=640mm

Transverse reinforcement area required is $\sum A_{st} = 1257$ mm^2.

For tension lap, on each $l_0/3$=640mm, 1257/2 = 629mm^2 is required. So use 6T12, area provided is 678mm^2 over 1950/3=640 mm zone, i.e. spacing is 128mm < 150mm.

For compression lap, also use 7T10, (628mm^2=1257/2) with 6T12 within 640mm (equal spacing = 128mm) and the 7th T10 at 160mm (=4∅) from the end of lap.

Figure 13.6 – Transverse reinforcement for lapped splices – more than 50% is lapped at one section and clear distance between adjacent laps ≤ 10∅

14.0 Design against Robustness

14.1 The Code defines the requirement for robustness in Clause 2.1.4 as "a structure should be designed and constructed so that it is inherently robust and not unreasonably susceptible to the effects of accidents or misuse, and disproportionate collapse." By disproportionate collapse, we refer to the situation in which damage to small areas of a structure or failure of single elements may lead to collapse of large parts of the structure.

14.2 Design requirements comprise :

(i) building layouts checked to avoid inherent weakness;

(ii) capable to resist notional loads simultaneously at floor levels and roof as shown in Figure 14.1. (Re Cl. 2.3.1.4(a) of the Code which also requires that applied ultimate wind loads should be greater than these notional values);

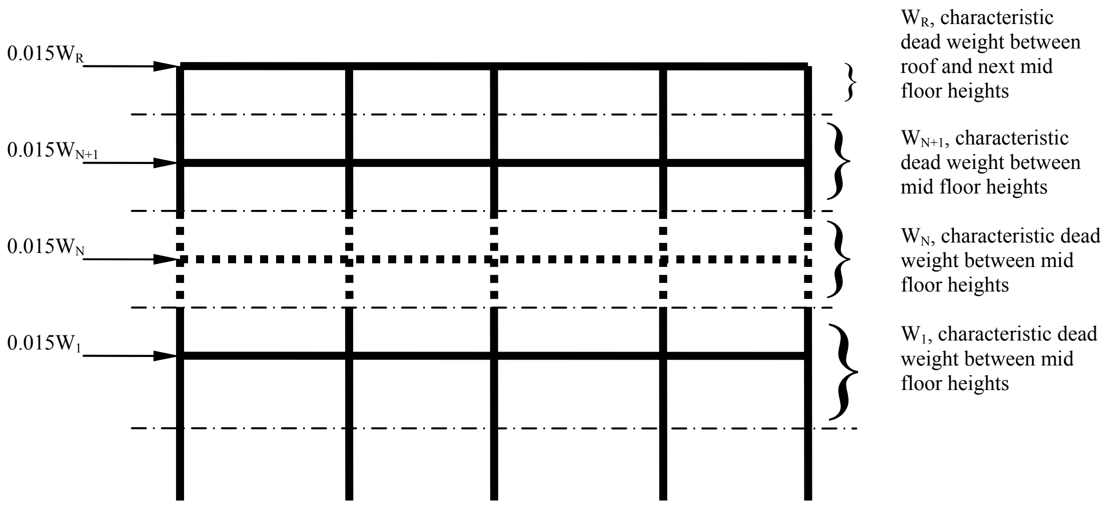


Figure 14.1 – Illustration of notional loads for robustness design

(iii) provides effective horizontal ties (in form of reinforcements embedded in concrete) (a) around the periphery; (b) internally; (c) to external columns and walls; and (d) vertical ties as per Cl. 6.4.1 of the Code, the failure of which will lead to requirement of checking key elements in accordance with Cl. 2.2.2.3 of the Code.

14.3 Principles in Design of ties (Cl. 6.4.1.2 and Cl 6.4.1.3 of the Code)

(i) The reinforcements are assumed to be acting at f_y instead of $0.87 f_y$;

(ii) To resist only the tying forces specified, not any others;

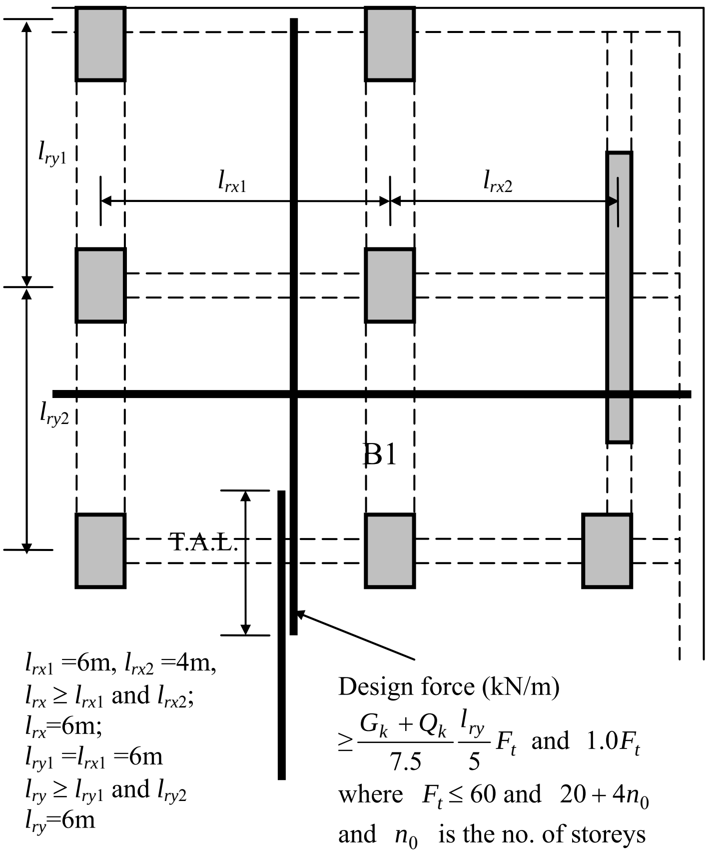
(iii) Reinforcements provided for other purpose can also act as ties;

(iv) Laps and anchorage of bars as ties similar to other reinforcements;

(v) Independent sections of a building divided by expansion joints have appropriate tying system.

14.4 Design of ties

(i) Internal Ties be provided evenly distributed in two directions in slabs – design force is illustrated in Figure 14.2 with example. The tie reinforcements can be grouped and provided in beam or wall.



<u>40 storeys building with average floor dead load as 9 kPa and average floor live load as 3 kPa</u>

<u>Internal ties in Y-direction</u>

$n_0 = 40$

$G_k = 9$ kPa; $Q_k = 3$ kPa

$l_{ry} = 6$ m

$F_t \leq 60$ and $20 + 4n_0 = 180$

$\therefore F_t = 60$

$\frac{G_k + Q_k}{7.5} \frac{l_r}{5} F_t = 115.2$

$1.0F_t = 60$

Design tensile force in ties is 115.2 kN/m

Required A_{st} for internal ties is

$\frac{F}{f_y} = \frac{115.2 \times 10^3}{460} = 250$ mm²/m

Use T10 – 300

(can likely be met by DB or rebars provided for strength purpose)

If tying bars grouped in beam, total rebars in the middle beam B1 may be $250 \times 5 = 1250$ mm². So likely can be met by the longitudinal bars.

Figure 14.2 – Derivation of internal tie reinforcement bars in slabs (evenly distributed)

(ii) Peripheral ties – Continuous tie capable resisting $1.0F_t$, located within 1.2 m of the edge of the building or within perimeter wall;

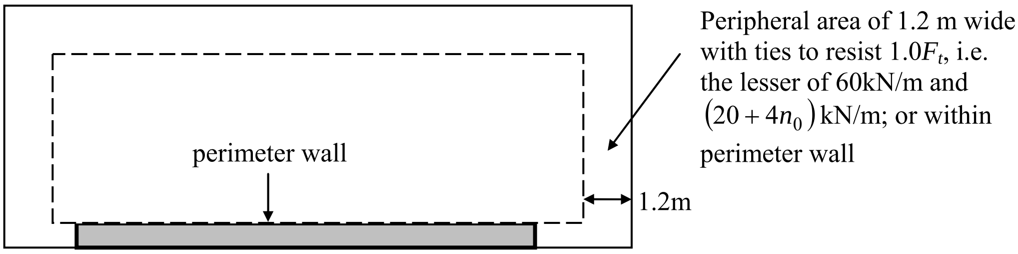


Figure 14.3 – Location and determination of Peripheral ties

(iii) External columns and wall to have ties capable of developing forces as indicated in Figure 14.4;

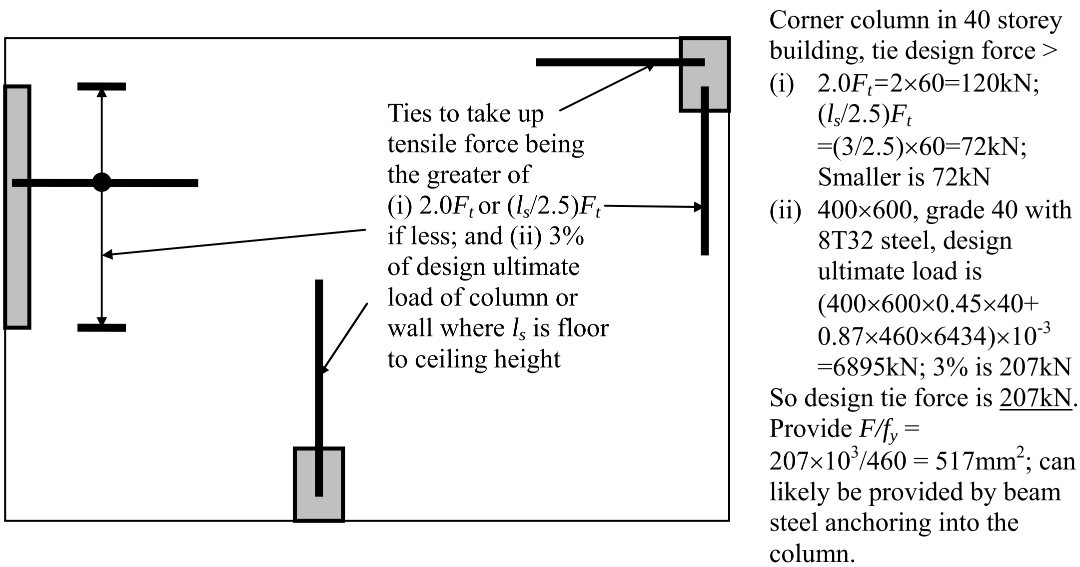
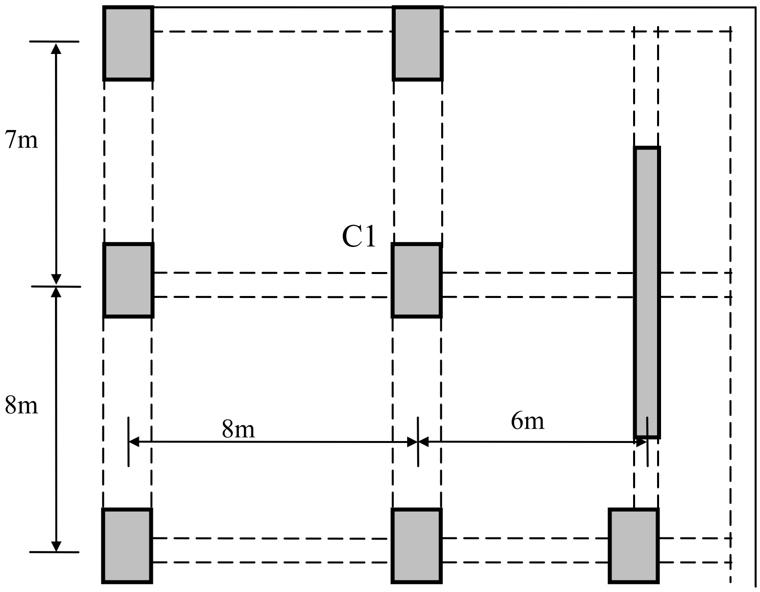


Figure 14.4 – Ties to external column and wall with example

(iv) Vertical ties provided to wall and column should be continuous and be capable of carrying exceptional load. Use γ_f ×[dead load + 1/3 imposed load + 1/3 wind load] of one floor to determine the design load for the vertical ties where γ_f = 1.05.

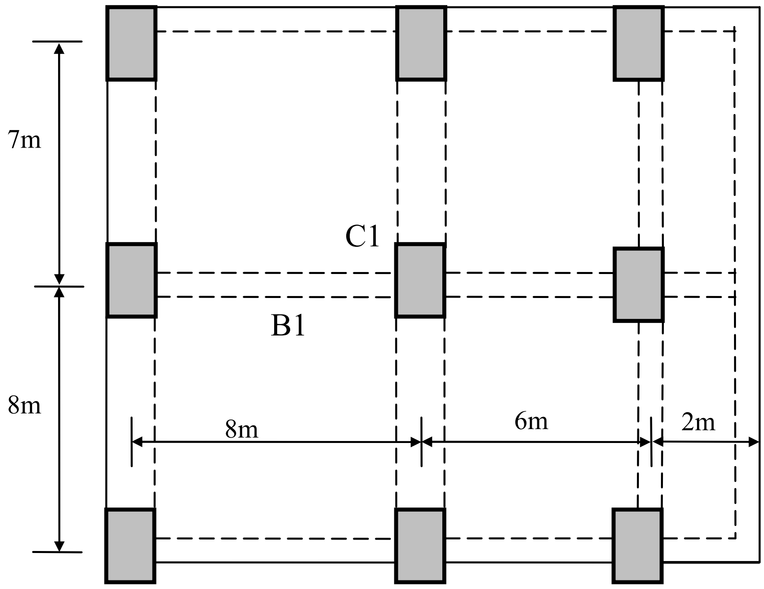


Design load for the vertical tie of the interior column C1 is
(i) Dead load from one storey 800kN;
(ii) One third of imposed load from one storey $1/3\times240 = 80$ kN;
(iii) One third of wind load $= 1/3\times90=30$kN
Designed tensile load is $1.05(800+80+30) = 956$kN.
Requiring $956000/460 = 2078\text{mm}^2$

Figure 14.5 – Design of Vertical Ties in Columns and Walls

14.5 Design of "Key Elements"

By Cl. 2.2.2.3 of the Code, when for some reasons it is not possible to introduce ties, key elements (usually columns or walls), the failure of which will cause disproportionate collapse should be identified. If layout cannot be revised to avoid them, design these elements and the supporting building components to an ultimate load of 34 kN/m^2, from any direction, to which no partial safety factor shall be applied. The Code has not defined the extent of "disproportionate collapse" for the element to be qualified as a "key element". However, reference can be made to the "Code of Practice for the Structural Use of Steel 2005" Cl. 2.3.4.3 by which an element will be considered a key element if the removal of it will cause collapse of 15% of the floor area or 70m^2, whichever is the greater. The design is illustrated in Figure 14.6.



Examples

C1 is identified as the key element as the collapse of which will lead to disproportionate collapse of area around it (more than 15% collapse of the floor).
The tributary area is $7.5\times7=52.5\text{m}^2$;
The design load is $34\times52.5=1785\text{kN}$.

Similarly, beam B1 is also required as the removal of which will cause more than 15% collapse of the floor. So B1 needs be designed for a u.d.l. of 34kPa on the linking slabs.

Figure 14.5 – Design of Key Elements

14.6 Nevertheless, it should be noted that requirements in the Code for robustness design often poses no additional requirements in monolithic reinforced concrete design in comparison with the criteria listed in 14.2 :

(i) normally no inherent weakness in the structure for a reasonable structural layout;

(ii) ultimate wind loads normally applied to the structure according to the local Wind Code can usually cover the notional loads (1.5% characteristic dead weight) specified in 14.2(ii);

(iii) requirements for various types of ties can normally be met by the reinforcements provided for other purposes. Nevertheless, continuity of the ties should be checked.

15.0 Shrinkage and Creep

15.1 Shrinkage

Shrinkage is the shortening movement of concrete as it dries after hardening. If the movement is restrained, stress and/or cracking will be created.

(Ceqn 3.5) gives estimate of drying shrinkage of plain concrete under un-restrained conditions. Together with the incorporation of the "reinforcement coefficient" K_s, the equation can be written as

$$\varepsilon_s = c_s K_L K_c K_e K_j K_s \qquad \text{(Eqn 15-1)}$$

where $c_s = 3.0$ and other coefficients can be found by Figures 3.3, 3.5, 3.6 and 3.7 and (Ceqn 3.4) of the Code, depending on atmospheric humidity, dimensions, compositions of the concrete, time and reinforcement content. It should be noted that K_j is a time dependent coefficient.

The equation and the figures giving values of the various coefficients are adopted from BS5400 which in turn are quoted from CEB-FIP *International Recommendation for the Design and Construction*, 1970 (CEB 1970) (MC-70). It should, however, be noted that the coefficient c_s is extra to CEB-FIP. The value accounts for the comparatively higher shrinkage value (3 times as high) found in Hong Kong.

Shrinkage is always in contraction.

15.2 Creep

Creep is the prolonged deformation of the structure under sustained stress. (Ceqn 3.2) and (Ceqn 3.3) give estimate of the creep strain :

$$\text{Creep strain} = \frac{stress}{E_{28}} \times \phi_c \qquad \text{(Eqn 15-2)}$$

where $\phi_c = K_L K_m K_c K_e K_j K_s$ (Eqn 15-3)

Again (Eqn 15-3) has incorporated the reinforcement coefficient K_s.

Thus creep strain depends on the *stress* in the concrete and various coefficients related to parameters similar to that of shrinkage (which can be read from

Figures 3.1 to 3.5 of the Code) with K_m and K_j dependent on time. As stress and strain are inter-dependent, it will be shown that assessment of strain will require successive time staging in some cases.

Creep creates deformation in the direction of the stress. In case shrinkage which results in tensile stress under restrained condition such as a floor structure under lateral restraints, the creep strain will serve to relax the stress due to shrinkage. Both stress and strain due to shrinkage and creep vary with time, as can be shown in the analyses that follow.

15.3 The determination of the time dependent coefficients K_m and K_j as listed in (Ceqn 3.3) and (Ceqn 3.5) will be tedious in calculation of stress and strain of a structure in a specified time step which may involve reading the figures many times. Curves in Figures 3.2 and 3.5 are therefore simulated by polynomial equations as shown in Appendix J to facilitate determination of the coefficients by spreadsheets.

15.4 <u>Worked Example 15.1</u>

A grade 35 square column of size 800 × 800 in a 4 storey building with reinforcement ratio 2% is under an axial stress from the floors as follows :

Floor	Height (m)	Time of stress creation from floor (days)	Stress (MPa)
G	4	28	3.5
1st	3	56	2.1
2nd	3	84	2.1
3rd	3	120	3.5

Strain and shortening of the G/F column due to shrinkage and creep at 360 days are determined as follows :

<u>Shrinkage</u>

The coefficients for determination of the free shrinkage strain are as follows :

$K_L = 0.000275$ for normal air from Figure 3.6;

Based on empirical formulae, for grade 35:

Water / Cement ratio = $-0.0054 f_{cu} + 0.662 = -0.0054 \times 35 + 0.662 = 0.473$

Cement content = $3.6 f_{cu} + 308 = 3.6 \times 35 + 308 = 434$ kg/m^3

From Figure 3.3 $K_c = 1.17$;

For the 800 × 800 column, the effective thickness h_e, defined as the ratio of the area of the section A, to the semi-perimeter, *u*/2 (defined in Cl. 3.1.7 of the Code) is $\frac{800 \times 800}{800 \times 4/2} = 400$ mm. So from Figure 3.7, $K_e = 0.55$;

From Figure 3.5, time at 360 days $K_j = 0.51$;

$K_s = \frac{1}{1 + \rho \alpha_e} = 0.856$; where $\rho = 0.02$ (2% steel) and $\alpha_e = \frac{200}{23.7} = 8.44$

So the shrinkage strain under perfectly free condition is :

$$\varepsilon_s = c_s K_L K_c K_e K_j K_s = 3.0 \times 0.000275 \times 1.17 \times 0.55 \times 0.51 \times 0.856 = 231.76 \times 10^{-6}$$

Creep

For estimation of creep strain, Creep strain $\varepsilon_c = \frac{stress}{E_{28}} \times \phi_c$

where $\phi_c = K_L K_m K_c K_e K_j K_s$

$E_{28} = 23.7$ GPa for grade 35 concrete.

All coefficients are same as that for shrinkage except $K_L = 2.3$ (Figure 3.1), $K_m = 1.0$ (Figure 3.2 – loaded at 28 days) and $K_e = 0.72$ (Figure 3.4)

Load from Floor	Concrete age at time of load (Day)	K_m	Time since Loading (Day)	K_j	ϕ_c	Stress by Floor (MPa)	ε_c ($\times 10^{-6}$)
1/F	28	1	332	0.489	0.811	3.5	119.73
2/F	56	0.85	304	0.467	0.658	2.1	58.30
3/F	84	0.761	276	0.443	0.560	2.1	49.61
Roof	120	0.706	240	0.412	0.482	3.5	71.15
						$\sum \varepsilon_c =$	298.80

So the creep strain at 360 days is $\varepsilon_c = 298.80 \times 10^{-6}$

Elastic strain

The elastic strain is simply $\varepsilon_e = \frac{\sigma}{E} = \frac{3.5 + 2.1 + 2.1 + 3.5}{23700} = 472.57 \times 10^{-6}$

So the total strain is

$\varepsilon = \varepsilon_s + \varepsilon_c + \varepsilon_e = 231.76\times10^{-6} + 298.80\times10^{-6} + 472.57\times10^{-6} = 1003.13\times10^{-6}$.

Total shortening of the column at G/F is at 360 days is
$\varepsilon \times H = 1003.13\times10^{-6}\times4000 = 4.01$ mm.

15.5 Estimation of shrinkage and creep effect on restrained floor structure

It is well known that shrinkage and creep effects of long concrete floor structures can be significant. The following derivations aim at providing a design approach to account for such effects based on recommendations by the Code.

Consider a floor structure spanning on vertical members of lateral support stiffness $K_{\sup 1}$ and $K_{\sup 2}$ as shown in Figure 15.1.

Let the lateral deflections at supports 1 and 2 be δ_1 and δ_2. At any time when the floor structure has an internal stress σ, a free shrinkage strain ε_s, creep strain ε_c, elastic strain ε_e, internal force, by displacement compatibility, the followings can be formulated :

$$\varepsilon_c = \frac{\sigma}{E}\phi_c,\ \ \varepsilon_e = \frac{\sigma}{E},\ \ K_{\sup 1}\delta_1 = K_{\sup 2}\delta_2 = F = \sigma A \Rightarrow \delta_1 = \frac{\sigma A}{K_{\sup 1}};\ \ \delta_2 = \frac{\sigma A}{K_{\sup 2}}$$

$$(\varepsilon_s - \varepsilon_c - \varepsilon_e)L = \delta_1 + \delta_2$$

$$\Rightarrow \left(\varepsilon_s - \frac{\sigma}{E}\phi_c - \frac{\sigma}{E}\right)L = \delta_1 + \delta_2 = \frac{\sigma A}{K_{\sup 1}} + \frac{\sigma A}{K_{\sup 2}} \Rightarrow \sigma = \frac{E\varepsilon_s}{1 + \phi_c + \dfrac{AE}{L}\left(\dfrac{1}{K_{\sup 1}} + \dfrac{1}{K_{\sup 2}}\right)}$$

(Eqn 15-4)

If the floor structure undergoes no net lateral deflection at a point P at L_e from support 2, it can be visualized as if the floor structure is divided into 2 floor structures both fixed at P and undergoes deflection δ_1 at the left portion and δ_2 at the right portion. By constant strain (implying linearly varying displacement) in the floor structure :

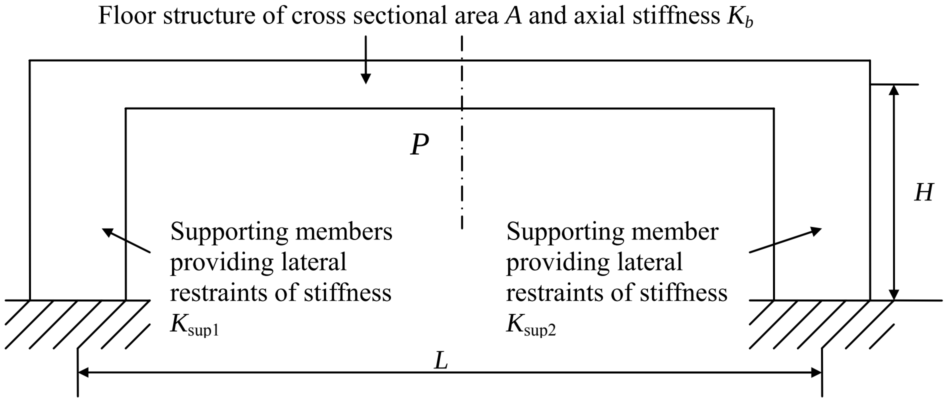
$$L_e = \frac{\delta_2}{\delta_1 + \delta_2}L = \frac{L}{K_{\sup 2}} \Bigg/ \left(\frac{1}{K_{\sup 1}} + \frac{1}{K_{\sup 2}}\right) \qquad \text{(Eqn 15-5)}$$

Substituting (Eqn 15-5) into (Eqn 15-4)

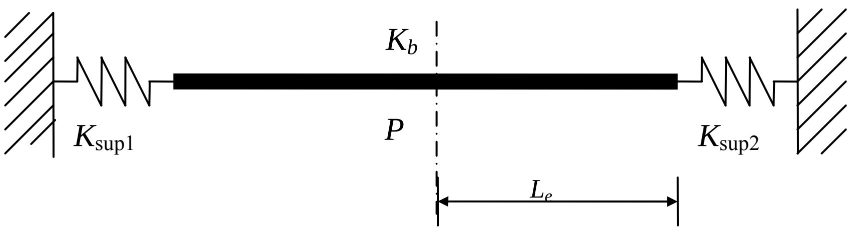
$$\sigma = \frac{E\varepsilon_s}{1+\phi_c+\dfrac{AE}{L_e K_{\sup 2}}} = \frac{E\varepsilon_s}{1+\phi_c+\dfrac{K_b}{K_{\sup 2}}} \qquad \text{(Eqn 15-6)}$$

where $K_b = \dfrac{AE}{L_e}$, the equivalent axial stiffness of the floor.

So, as an alternative to using (Eqn 15-4), we may use (Eqn 15-5) to find out L_e and (Eqn 15-6) to calculate internal stress of the floor structure.



idealized as



$$L_e = \frac{L}{K_{\sup 2}} \bigg/ \left(\frac{1}{K_{\sup 1}} + \frac{1}{K_{\sup 2}} \right)$$

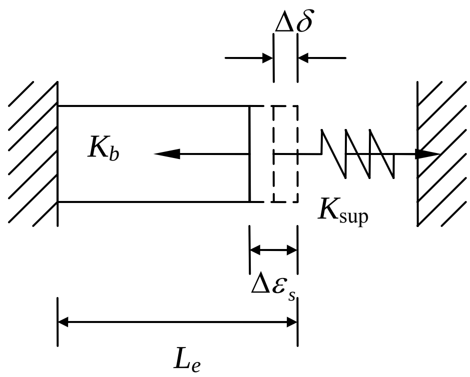
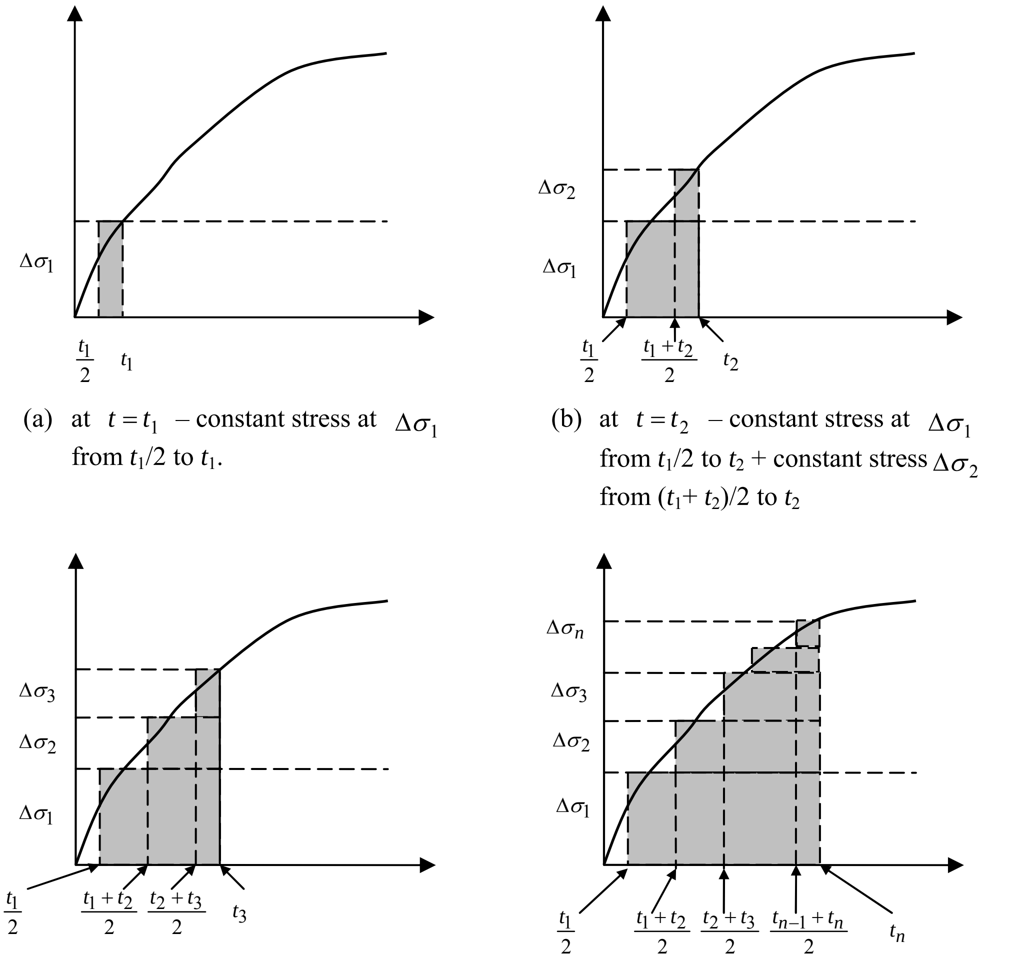


Figure 15.1 – Idealization of floor structure for shrinkage and creep estimation

In the determination of stress due to shrinkage and creep, the main difficulty lies in the determination of ϕ_c which is time dependent. Stress in concrete has therefore to be determined in successive time steps and with numerical method as demonstrated in Figure 15.2 for calculation of the creep strains. Instead of

being treated as continuously increasing, the stress is split up into various discrete values, each of which commences at pre-determined station of times. Fine divisions of time steps will create good simulation of the actual performance.



(a) at $t = t_1$ – constant stress at $\Delta\sigma_1$ from $t_1/2$ to t_1.

(b) at $t = t_2$ – constant stress at $\Delta\sigma_1$ from $t_1/2$ to t_2 + constant stress $\Delta\sigma_2$ from $(t_1 + t_2)/2$ to t_2

(c) at $t = t_3$ – constant stress at $\Delta\sigma_1$ from $t_1/2$ to t_3 + $\Delta\sigma_2$ from $(t_1 + t_2)/2$ to t_3 + $\Delta\sigma_3$ from $(t_2 + t_3)/2$ to t_3

(d) at $t = t_n$ – similarly adding up effects of all stress increments

Figure 15.2 – Estimation of Creep Strains by Numerical Method

Consider the floor structure shrinks for ε_{s1} at the time interval from time $t = 0$ to $t = t_1$, $\phi_c = K_L K_m K_c K_e K_j K_s$ should be determined at concrete age $\frac{t_1}{2}$ (for determination of K_m) and with the time since loading $\frac{t_1}{2}$ to t_1 (for determination of K_j) which is the ϕ_c value for $t_1 - \frac{t_1}{2}$ and denoted by $\phi_c\left(t_1 - \frac{t_1}{2}\right)$. The timing for determination of ϕ_c is illustrated in Figure 15-2(a). So by (Eqn 15-6)

$$\Delta\sigma_1 = \frac{E\varepsilon_{s1}}{1 + \frac{K_b}{K_{\sup 2}} + \phi_c\left(t_1 - \frac{t_1}{2}\right)} \qquad \text{(Eqn 15-7)}$$

At time t_2 after shrinkage commencement when the shrinkage strain is ε_{s2}, the creep strain ε_{c2} can be regarded as made up of two time steps with stresses $\Delta\sigma_1$ and $\Delta\sigma_2$ (increment of concrete stress between time t_1 and t_2) as $\varepsilon_{c2} = \frac{\Delta\sigma_1}{E}\phi_c\left(t_2 - \frac{t_1}{2}\right) + \frac{\Delta\sigma_2}{E}\phi_c\left(t_2 - \frac{t_1 + t_2}{2}\right)$ as illustrated in Figure 15-2(b). So, similar to the above, we can list

$$\varepsilon_{s2} - \left[\frac{\Delta\sigma_1}{E}\phi_c\left(t_2 - \frac{t_1}{2}\right) + \frac{\Delta\sigma_2}{E}\phi_c\left(t_2 - \frac{t_1 + t_2}{2}\right)\right] - \frac{\Delta\sigma_1 + \Delta\sigma_2}{E} = \frac{(\Delta\sigma_1 + \Delta\sigma_2)A}{K_{\sup}L}$$

$$\Rightarrow \Delta\sigma_1\left[\phi_c\left(t_2 - \frac{t_1}{2}\right) + 1 + \frac{EA}{K_{\sup 2}L_e}\right] + \Delta\sigma_2\left[\phi_c\left(t_2 - \frac{t_1 + t_2}{2}\right) + 1 + \frac{EA}{K_{\sup 2}L_e}\right] = E\varepsilon_{s2}$$

(Eqn 15-8)

$$\Rightarrow \Delta\sigma_2 = \frac{E\varepsilon_{s2} - \Delta\sigma_1\left[\phi_c\left(t_2 - \frac{t_1}{2}\right) + 1 + \frac{K_b}{K_{\sup 2}}\right]}{\left[\phi_c\left(t_2 - \frac{t_1 + t_2}{2}\right) + 1 + \frac{K_b}{K_{\sup 2}}\right]} \qquad \text{(Eqn 15-9)}$$

$\Delta\sigma_2$ can be determined with pre-determination of $\Delta\sigma_1$ by (Eqn 15-7)
Similarly for time t_3 with 3 time steps where

$$\varepsilon_{c3} = \frac{\Delta\sigma_1}{E}\phi_c\left(t_3 - \frac{t_1}{2}\right) + \frac{\Delta\sigma_2}{E}\phi_c\left(t_3 - \frac{t_1 + t_2}{2}\right) + \frac{\Delta\sigma_3}{E}\phi_c\left(t_3 - \frac{t_2 + t_3}{2}\right)$$

$$\therefore \Delta\sigma_1\left[\phi_c\left(t_3 - \frac{t_1}{2}\right) + 1 + \frac{K_b}{K_{\sup 2}}\right] + \Delta\sigma_2\left[\phi_c\left(t_3 - \frac{t_1 + t_2}{2}\right) + 1 + \frac{K_b}{K_{\sup 2}}\right]$$

$$+ \Delta\sigma_3\left[\phi_c\left(t_3 - \frac{t_2 + t_3}{2}\right) + 1 + \frac{K_b}{K_{\sup 2}}\right] = E\varepsilon_{s3} \qquad \text{(Eqn 15-10)}$$

$$\Delta\sigma_3 = \frac{E\varepsilon_{s3} - \Delta\sigma_1\left[\phi_c\left(t_3 - \frac{t_1}{2}\right) + 1 + \frac{K_b}{K_{\sup 2}}\right] - \Delta\sigma_2\left[\phi_c\left(t_3 - \frac{t_1 + t_2}{2}\right) + 1 + \frac{K_b}{K_{\sup 2}}\right]}{\left[\phi_c\left(t_3 - \frac{t_2 + t_3}{2}\right) + 1 + \frac{K_b}{K_{\sup 2}}\right]}$$

(Eqn 15-11)

So for any time t_n after shrinkage commencement

$$\Delta\sigma_1\left[\phi_c\left(t_n-\frac{t_1}{2}\right)+1+\frac{K_b}{K_{\sup 2}}\right]+\Delta\sigma_2\left[\phi_c\left(t_n-\frac{t_1+t_2}{2}\right)+1+\frac{K_b}{K_{\sup 2}}\right]$$

$$+\Delta\sigma_3\left[\phi_c\left(t_n-\frac{t_2+t_3}{2}\right)+1+\frac{K_b}{K_{\sup 2}}\right]+.....+\Delta\sigma_{n-1}\left[\phi_c\left(t_n-\frac{t_{n-2}+t_{n-1}}{2}\right)+1+\frac{K_b}{K_{\sup 2}}\right]$$

$$+\Delta\sigma_n\left[\phi_c\left(t_n-\frac{t_{n-1}+t_n}{2}\right)+1+\frac{K_b}{K_{\sup 2}}\right]=E\varepsilon_{sn}$$

(Eqn 15-12)

$$\Delta\sigma_n=\frac{E\varepsilon_{sn}-\Delta\sigma_1\left[\phi_c\left(t_n-\frac{t_1}{2}\right)+1+\frac{K_b}{K_{\sup 2}}\right]-.....-\Delta\sigma_{n-1}\left[\phi_c\left(t_n-\frac{t_{n-1}+t_{n-2}}{2}\right)+1+\frac{K_b}{K_{\sup 2}}\right]}{\left[\phi_c\left(t_n-\frac{t_{n-1}+t_n}{2}\right)+1+\frac{K_b}{K_{\sup 2}}\right]}$$

(Eqn 15-13)

Thus the solution for $\Delta\sigma_n$ can be obtained by successive solution of (Eqn 15-7), (Eqn 15-9), (Eqn 15-11) and (Eqn 15-13) or alternatively, in a more compact form by a system of linear simultaneous equations of (Eqn 15-7), (Eqn 15-8), (Eqn 15-10) and (Eqn 15-12). The final stress up to t_n is $\Delta\sigma_1+\Delta\sigma_2+\Delta\sigma_3+......+\Delta\sigma_n$.

15.6 Worked Example 15.2

A wide 200 mm slab of grade 35 is supported by 350×600 beams at spacing of 3000mm under restraints at both ends as shown in Figure 15.3. The span of the slab beam structure between restraints is 10 m. The longitudinal steel ratio is 0.5%. The free shrinkage strain and the stress developed due to shrinkage at 360 days after casting are to be assessed.

$$K_{\sup 1}=\frac{3EI}{H^3}=\frac{3\times 23.7\times 10^6\times 0.08}{3^3}=210667\,\text{kN/m}$$

$$K_{\sup 2}=\frac{3EI}{H^3}=\frac{3\times 23.7\times 10^6\times 0.12}{3^3}=316000\,\text{kN/m}$$

$$L_e=\frac{L}{K_{\sup 2}}\bigg/\left(\frac{1}{K_{\sup 1}}+\frac{1}{K_{\sup 2}}\right)=4\,\text{m}$$

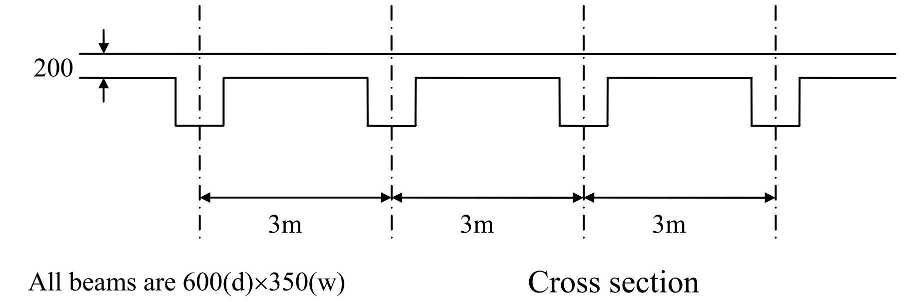
Area of a portion between centre line of two adjacent beams is

$A = 350 \times 400 + 3000 \times 200 = 740000\,\text{mm}^2$ for 3 m width;

Half Perimeter of the portion in contact with the atmosphere is

$(3000 \times 2 + 400 \times 2) \div 2 = 3400$

So the effective thickness is $h_e = \dfrac{740000}{3400} = 218\,\text{mm}$.



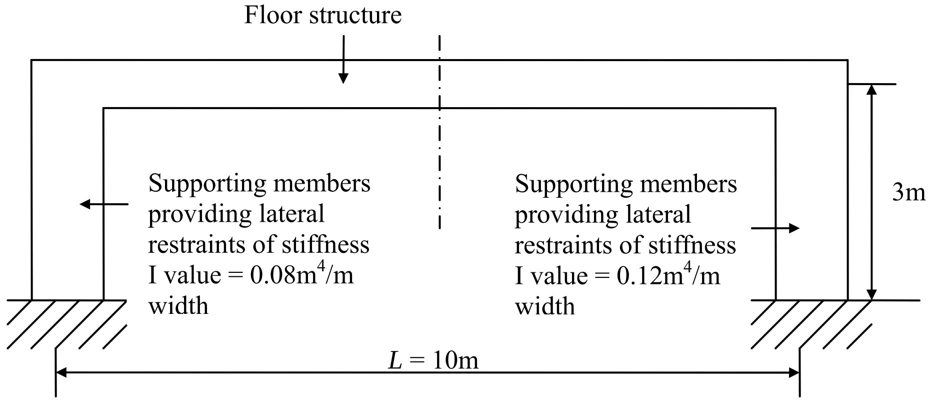


Figure 15.3 – Floor structure of Worked Example 15.2

<u>Determination of the coefficients for free shrinkage strain</u>

$\varepsilon_s = c_s K_L K_c K_e K_j K_s$,

$K_L = 0.000275$ for normal air from Figure 3.6 of the Code

Based on empirical formulae :

Water / Cement ratio = $-0.0054 f_{cu} + 0.662 = -0.0054 \times 35 + 0.662 = 0.473$

Cement content = $3.6 f_{cu} + 308 = 3.6 \times 35 + 308 = 434\,\text{kg/m}^3$

$K_c = 1.17$ from Figure 3.3 of the Code;

For $h_e = 218\,\text{mm}$ thick slab, from Figure 3.7 of the Code, $K_e = 0.768$;

K_j is time dependent and is to be read from Figure 3.5 of the Code;

$K_s = \dfrac{1}{1+\rho\alpha_e} = 0.96$ from (Ceqn 3.4).

Determination of the coefficients for creep strain

Creep strain = $\dfrac{stress}{E_{28}} \times \phi_c$ where $\phi_c = K_L K_m K_c K_e K_j K_s$,

$K_L = 2.3$ for normal air from Figure 3.1 of the Code;

K_m is time dependent and is to be read from Figure 3.2 of the Code;

$K_c = 1.17$ from Figure 3.3 of the Code, same as Shrinkage

For $h_e = 218$ mm thick slab, from Figure 3.4 of the Code, $K_e = 0.831$

K_j is time dependent and is to be read from Figure 3.5 of the Code.

$K_s = 0.96$, same as Shrinkage.

Stiffness per metre width:

$$K_b = \frac{EA}{L_e} = \frac{23.7\times10^6\times0.2467}{4} = 1461500\,\text{kN/m}$$

$$K_{\sup 2} = \frac{3EI}{H^3} = \frac{3\times23.7\times10^6\times0.12}{3^3} = 316000\,\text{kN/m};$$

$$\therefore \frac{K_b}{K_{\sup 2}} = 4.625$$

The time history to 360 days can be divided into various time points, i.e. $t_1 = 3$ days, $t_2 = 7$ days, $t_3 = 14$ days ….. up to $t_n = 360$ days in accordance with Figure 15.2. Equations in accordance with the above can be formulated numerically and stress at 360 days is calculated to be 1726.34 kN/m^2. As a demonstration, the stress increment in the first two intervals are presented :

At $t_1 = 3$, for $h_e = 218$, for shrinkage $K_j = 0.0637$ (Figure 3.5)

$$\varepsilon_{s1} = c_s K_L K_c K_e K_j K_s = 3.0\times0.000275\times1.17\times0.768\times0.0637\times0.96 = 45.22\times10^{-6}$$

$K_m = 1.743$ at $t_1 = 3$ and $K_j = 0.0546$ for time interval from 1.5 to 3 days.

$$\phi_c\left(t_1 - \frac{t_1}{2}\right) = K_L K_m\left(\frac{t_1}{2}\right) K_c K_e K_j\left(t_1 - \frac{t_1}{2}\right) K_s$$

$$= 2.3\times1.743\times1.17\times0.831\times0.0546\times0.96 = 0.2046$$

By (Eqn 15-6)

$$\Delta\sigma_1 = \frac{E\varepsilon_{s1}}{1 + \dfrac{EA}{L_e K_{\sup 2}} + \phi_c\left(t_1 - \dfrac{t_1}{2}\right)} = \frac{23.7\times10^6\times45.22\times10^{-6}}{1+4.625+0.2046} = 183.84\,\text{kN/m}^2;$$

At $t_2 = 7$, for $h_e = 218$, for shrinkage $K_j = 0.0896$ (Figure 3.5)
$\varepsilon_{s2} = c_s K_L K_c K_e K_j K_s = 3.0 \times 0.000275 \times 1.17 \times 0.768 \times 0.0896 \times 0.96 = 63.85 \times 10^{-6}$

<u>For time step 1</u>,
$K_m = 1.743$ at $t_1 = 3$ and $K_j = 0.0818$ for time interval from 1.5 to 7 days.

$$\phi_c\left(t_2 - \frac{t_1}{2}\right) = K_L K_m\left(\frac{t_1}{2}\right) K_c K_e K_j\left(t_2 - \frac{t_1}{2}\right) K_s$$
$$= 2.3 \times 1.743 \times 1.17 \times 0.831 \times 0.0818 \times 0.96 = 0.306;$$

<u>For time step 2</u>,
$K_m = 1.4667$ at $t = \frac{t_1 + t_2}{2} = 5$ and $K_j = 0.0572$ from
$t = \frac{t_1 + t_2}{2} = \frac{3+7}{2} = 5$ to $t = 7$

$$\therefore \phi_c\left(t_2 - \frac{t_1 + t_2}{2}\right) = K_L K_m\left(\frac{t_1 + t_2}{2}\right) K_c K_e K_j\left(t_2 - \frac{t_1 + t_2}{2}\right) K_s$$
$$= 2.3 \times 1.4667 \times 1.17 \times 0.831 \times 0.0572 \times 0.96 = 0.1800$$

By (Eqn 15-9)

$$\Delta\sigma_2 = \frac{E\varepsilon_{s2} - \Delta\sigma_1\left[\phi_c\left(t_2 - \frac{t_1}{2}\right) + 1 + \frac{EA}{L_e K_{\sup 2}}\right]}{\left[\phi_c\left(t_2 - \frac{t_1 + t_2}{2}\right) + 1 + \frac{EA}{L_e K_{\sup 2}}\right]}$$

$$= \frac{23.7 \times 10^6 \times 63.85 \times 10^{-6} - 183.84 \times (0.306 + 1 + 4.625)}{(0.1800 + 1 + 4.625)} = 72.85\,\text{kN/m}^2;$$

So the total stress at $t_2 = 7$ is
$\Delta\sigma_1 + \Delta\sigma_2 = 183.84 + 72.85 = 256.69$ kN/m^2

The process can be similarly repeated to calculate stress increments at later times. As the shrinkage rises rapidly in the beginning and slows down at later times, the time stations should be more frequent when t is small and be less frequent when t is high.

The stress finally arrived at 360 days is 1726.34 kN/m^2. The exercise stops at 360 days because Figure 3.2 of the Code indicates the values of the coefficient K_m up to 360 days only. The stress induced in the structure is plotted in Figure 15.4.

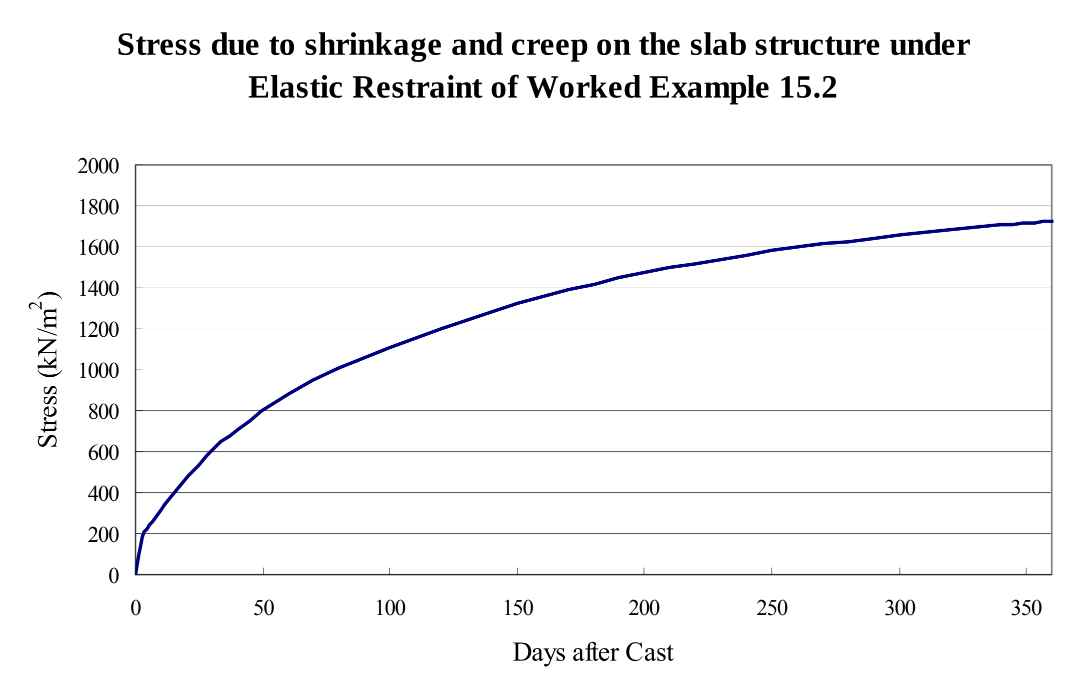


Figure 15.4 – Increase of internal stress in concrete due to shrinkage and creep of Worked Example 15.2

By assuming $K_m = 0.5$ beyond 360 days, the exercise is repeated for various span lengths up to 80 m and finally at perfect restraint where the span is set at infinity. The stress curves are plotted as indicated.

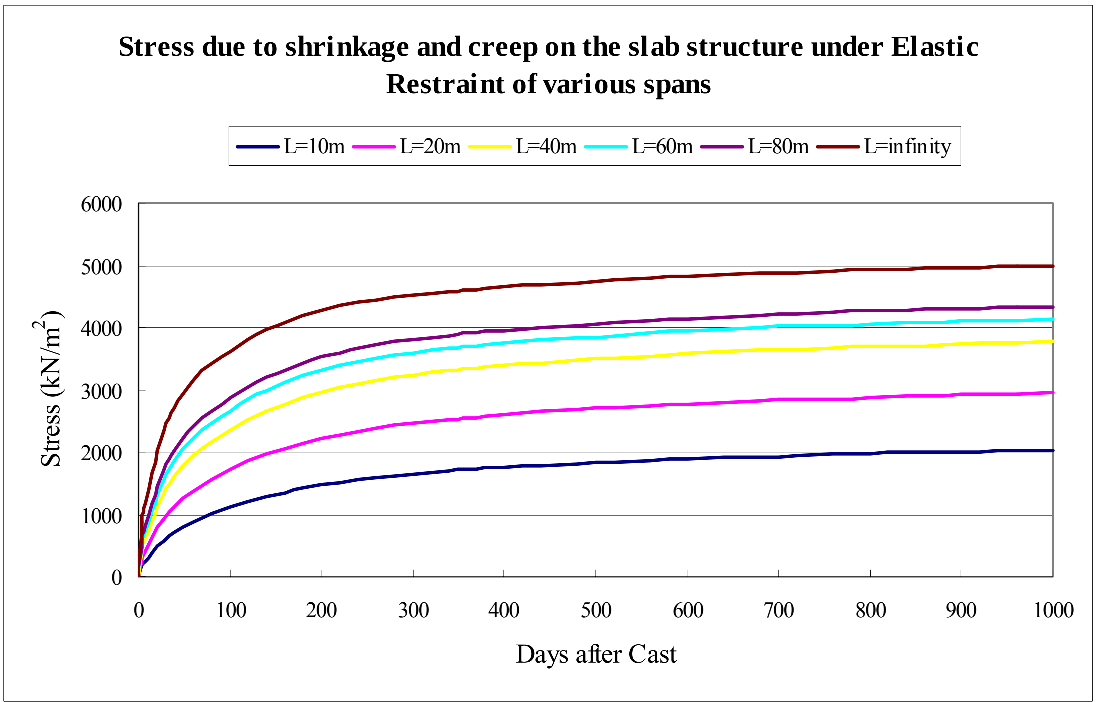


Figure 15.5 – Increase of internal stress in concrete due to shrinkage and creep of Worked Example 15.2 for various span lengths

15.7 The followings are discussed as revealed from Figure 15.5 :

(i) The stress induced increases with time and the increase becomes less significant as time goes by;

(ii) The magnitude of the stress increases with decrease of the $K_b / K_{\sup}$ ratio. As K_b decreases with increases of the floor length, longer floor length will lead to higher stress. So particular attention in relation to shrinkage and creep should be paid to long floor structures;

(iii) The particular case of perfect restraint is when $K_b / K_{\sup} = 0$, i.e. $K_{\sup}$ becomes infinity where the floor structure stress becomes maximum;

(iv) Thicker floor structures are less prone to shrinkage and creep as the coefficients K_j and K_e decrease with increase of thickness;

(v) Stronger lateral restraints will also induce higher shrinkage and creep stresses. The strong lateral restraints are often in form of core walls or shear walls whilst the columns are comparatively weak in lateral restraints. For rough analysis, the columns can be ignored. Figure 15.6 demonstrates the determination of floor span length for the assessment of shrinkage and creep effects.

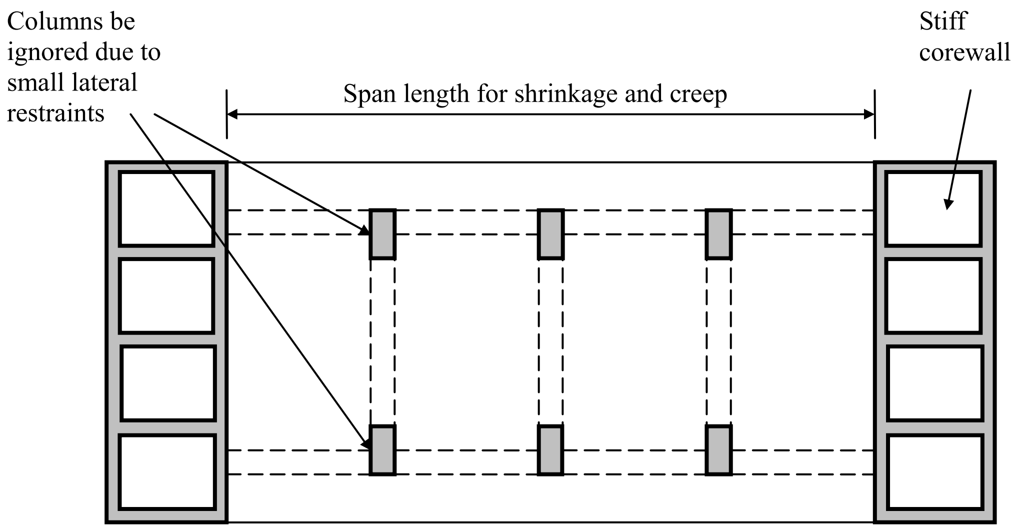
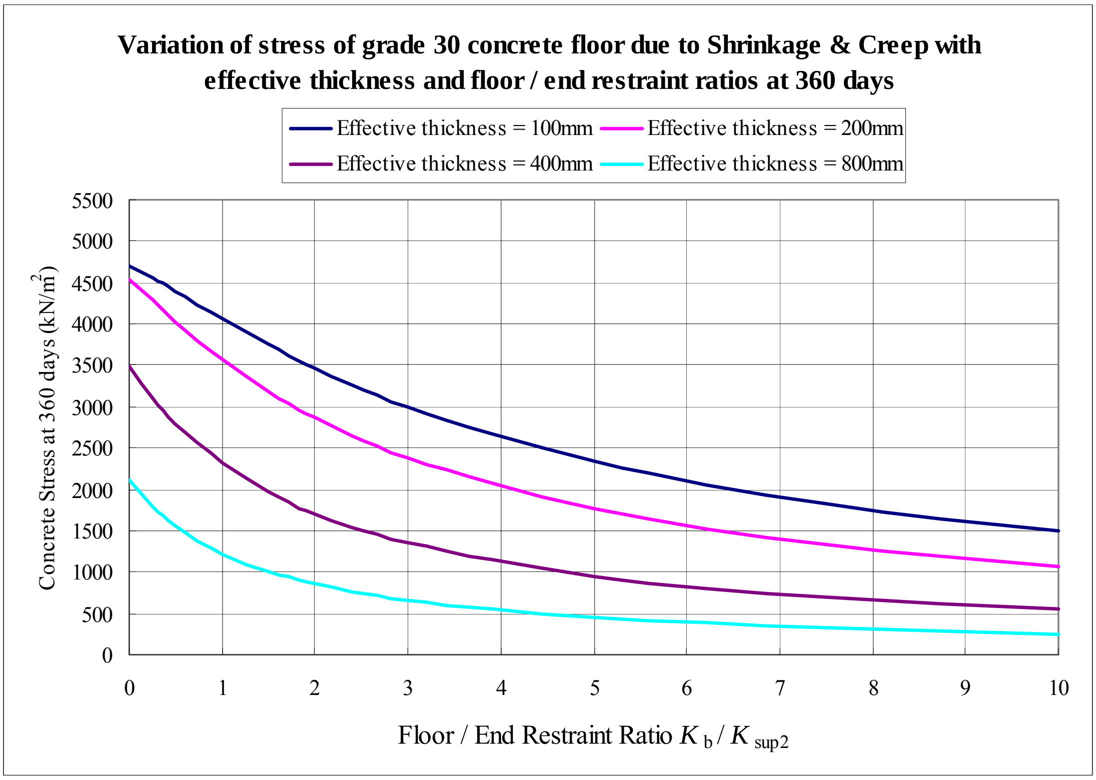
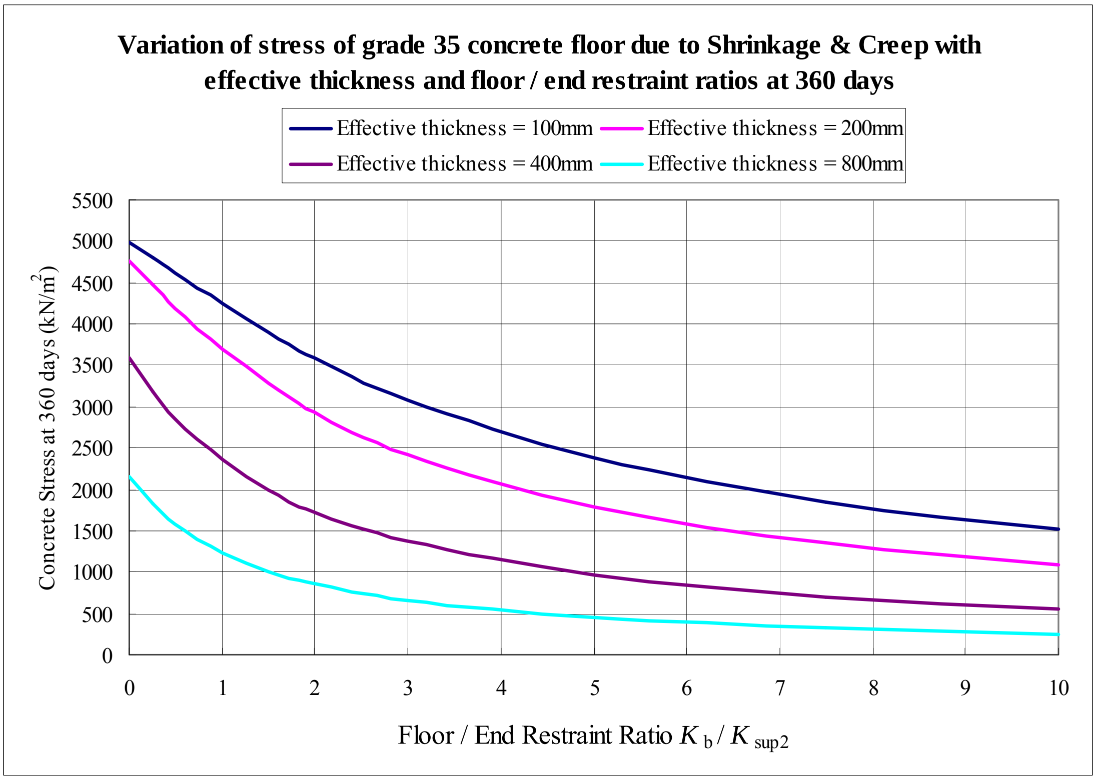


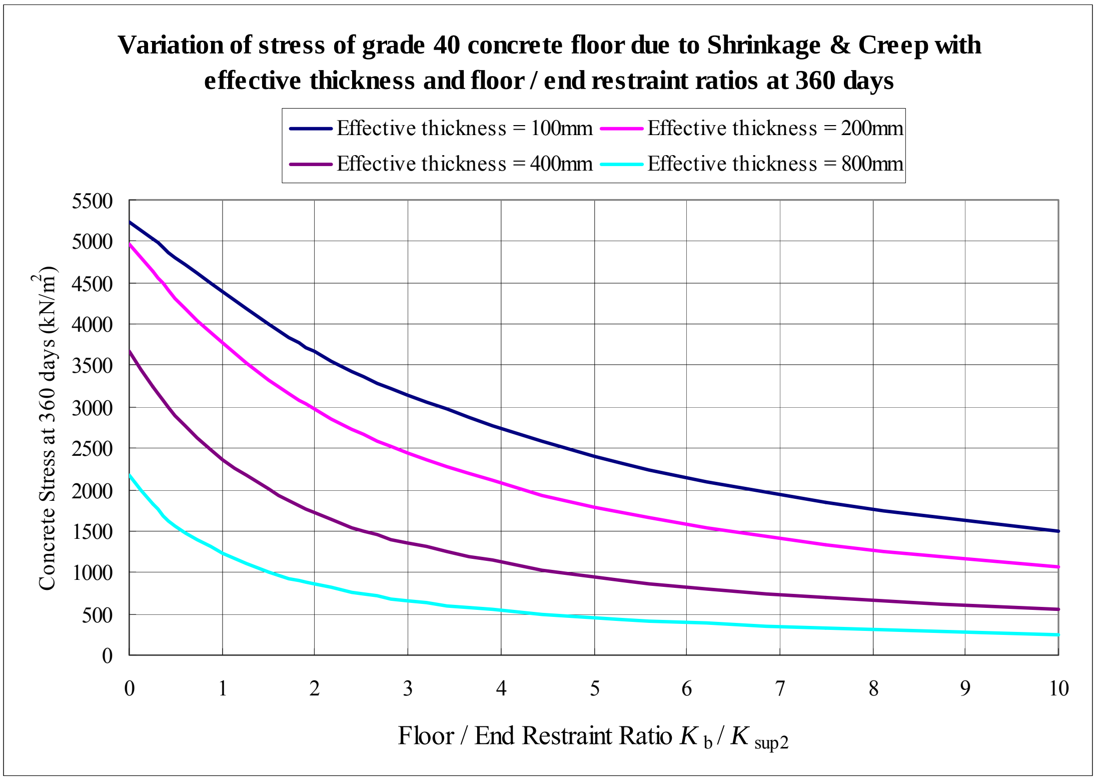
Figure 15.6 – Determination of floor span length for shrinkage and creep

15.8 For single span floor structures, if only the stress at age near to the final one such as the 360 days age is to be estimated, the design parameters for a particular concrete grade (ignoring reinforcements) can be reduced to

comprising only (i) effective thickness of the floor structure; and (ii) relative stiffness of the axial stiffness of the floor structure to the lateral stiffness of "support 2", i.e. $K_b/K_{\sup 2}$. Charts as contained in Figure 15.7 for grades 30, 35, 40 and 45 concrete are produced which can be for general use.







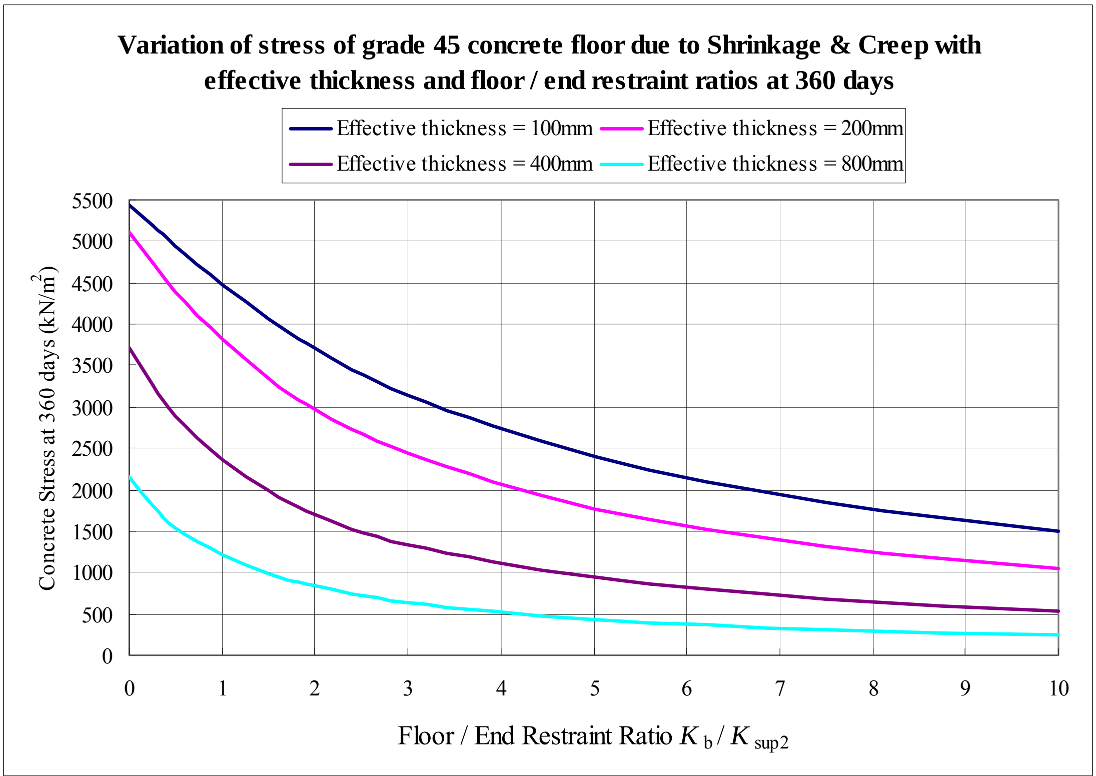


Figure 15.7 – Variation of 360 days stress due to shrinkage and creep of structural floor with effective thickness and span / support stiffness ratios

15.9 The induced stress in the concrete structure can be resisted by the tensile

strength of concrete under no cracking conditions. Or if the tensile stress is excessive, it should be resisted by reinforcements with cracks limited to various widths according to exposure conditions.

Worked Example 15.3

Consider a grade 35 floor structure of unit width under restraints at ends of the following design parameters :

Stress induced is 3MPa;

Thickness $h = 160$ mm;

Longitudinal reinforcement content : T10@100 (B.F.) $\rho = 0.982$ %;

The floor structure is now checked for pure tension created due to shrinkage and creep alone :

Crack width is checked in accordance with Cl. 3.2.2 and Appendix B of BS8007:1987 with limiting crack width of 0.2mm;

Strain for coaxial tension :

$$\varepsilon_m = \varepsilon_1 - \varepsilon_2 = \frac{\sigma}{E_s \rho} - \frac{2b_t h}{3E_s A_s} = \frac{1}{200000}\left(\frac{2.5}{0.00982} - \frac{2 \times 1000 \times 160}{3 \times 1571}\right) = 0.000934,$$

$$< \frac{0.8 f_y}{E_s} = 0.00184;$$

(ε_1 is the strain due to steel only without consideration of the tensile strength of the concrete and ε_2 represents the stiffening effect by the cracked concrete.)

Cover to reinforcement is $c_{\min} = 25$ mm;

So the greatest value a_{cr} (distance from the point under consideration to the nearest reinforcement) that will lead to greatest crack width is $\sqrt{50^2 + 25^2} = 55.9$ mm;

By equation 4 of Appendix B of BS8007, the crack width is

$\omega = 3a_{cr}\varepsilon_m = 3 \times 55.9 \times 0.000934 = 0.157$ mm < 0.2mm;

The crack width is acceptable for all exposure conditions as required by Table 7.1 of the Code.

16.0 Summary of Aspects having significant impacts on current Practice

16.1 General

Though some of the new practices in the Code as different from BS8110 have significant impacts on our current design, detailing and construction practices, these practices are however generally good ones resulting in better design and workmanship. The improvement in design lies mainly in enhancing ductility of the structure which should be regarded as another “limit state” equally as important as the “ultimate” and “serviceability” limit states. This section tends to summarize all these new practices and discuss the various impacts so as to alert the practitioners in switching from BS8110 to the Code.

The aspects with the most significant impacts by the Code on our current design are obviously the incorporation of the ductility requirements in Cl. 9.9 of the Code for beams and columns contributing in lateral load resisting system, and the design of beam column beam joints in Cl. 6.8. Others include checking building accelerations in Cl. 7.3.2. Nevertheless, minor ones such more stringent requirements in locations and provisions of transverse reinforcements in lapping of longitudinal bars should also be noted. In addition, there are relaxations in design requirements such as raising the absolute ultimate design shear stress (v_{tu}) to 7N/mm^2 and giving clear guidelines in choosing design moments at or near column faces in Cl. 5.2.1.2. These aspects are highlighted and briefly discussed in this Section. The effects of different concrete stress strain curve as indicated in Figure 3.8 of the Code from that of BS8110 are, however, found to be insignificant on the calculation of longitudinal bars required in beams and columns.

16.2 Ductility Requirements

The followings are highlighted :

(i) Bending and lapping of reinforcement bars

(a) Though the Code includes BS8666 : 2000 in its list of acceptable standards for the specifications of bending and dimensioning of reinforcing bars, Table 8.2 of the Code has, however, indicated simple rules for the minimum internal bend radii of bar diameter as 3Ø for Ø ≤

20 mm and 4Ø for Ø > 20 mm where Ø is the diameter of the reinforcing bar. The minimum internal bend radii are all greater than that required by BS8666 : 2000 which ranges from 2Ø to 3.5Ø. Furthermore, as unlike the British Standards (including the former BS4466), no distinction is made for mild steel and high tensile steel bar in the Code. The more stringent requirement in minimum bend radii creates greater difficulties in r.c. detailings;

(b) Cl. 9.9.1.2 and Cl. 9.9.2.2(c) of the Code under the heading of ductility requirements for "Beams" and "Columns" (contributing in lateral load resisting system) state that "Links should be adequately anchored by means of 135^{o} or 180^{o} hooks in accordance with Cl. 8.5. (Presumably 180^{o} bent hooks can be accepted as better anchorage is achieved.) So the all links in such beams and columns contributing in lateral load resisting system should be anchored by links with bent angle ≥ 135^{o} as indicated in Figure 16.1.

Link with 180^{o} bent hooks   Link with 135^{o} bent hooks

Figure 16.1 – Links with hooks for beams and columns contributing to lateral load resisting system and for containment of beam compression reinforcements

For structural elements other than beams and columns contributing in lateral load resisting system, the 90^{o} anchorage hooks can still be used except for containment of compression reinforcements in beams which should follow Figure 16.1 (Re Cl. 9.2.1.10 and Cl. 9.5.2.2 of the Code.)

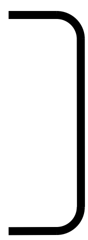


Figure 16.2 – 90^{o} bent links : used in structural elements other than beams / columns contributing in lateral load resisting system and except compression bar containment

(c) Cl. 9.9.1.1(c) of the Code under the heading of ductility requirement for anchorage of longitudinal bars in beams (contributing in lateral load resisting system) into exterior column states that "For the calculation of anchorage length the bars must be assumed to be fully stressed". The calculation of anchorage length of bars should therefore be based on f_y instead of $0.87f_y$ as discussed in Sections 8.4.4 and 8.4.5 of this Manual, resulting in some 15% longer in anchorage and lap lengths as compared with Table 8.4 of the Code. Thus longer anchorage length is required for longitudinal bars in beams contributing in lateral load resisting system anchoring into exterior column;

(d) Cl. 8.7.2 and Figure 8.4 of the Code have effectively required all tension laps to be staggered which are generally applicable in the flexural steel bars in beams, slabs, pile caps, footings etc. as per discussion in Section 13.6 of this Manual. The practice is not as convenient as the practice currently adopted by generally lapping in one single section. Nevertheless, if staggered lapping is not adopted, lapping will likely be greater than 50% and clear distance between adjacent laps will likely be $\leq 10\phi$, transverse reinforcement by links or U bars will be required by Cl. 8.7.4.1 of the Code which may even be more difficult to satisfy. Fortunately, the requirements for staggered lapping (in Cl. 8.7.2) do not cover distribution bars and compression bars. So most of the longitudinal bars in columns and walls can be exempted;

(e) Cl. 8.7.4 of the Code requires additional transverse reinforcements generally in lap zones of longitudinal bars which is not required by BS8110. Arrangement and form of transverse reinforcements (straight bars or U-bars or links) required are in accordance with the longitudinal bar diameter ϕ, spacing of adjacent laps and percentage of lapping at one point. Take an example : when T40 bars of transverse spacing ≤ 400 mm (10ϕ) are lapped at one section, total area of transverse reinforcements equal to 1 longitudinal bar which is 1257 mm^2 should be spaced along the lapped length of some 2000 mm. The transverse reinforcement is therefore T12 – 125 mm spacing (providing 2261mm^2) in form of U-bars or links at ends of the laps as demonstrated in Figure 13.6. Apparently these transverse reinforcements should be in addition to the transverse reinforcements already provided for other purposes unless

$\phi < 20$ mm or percentage of lapping at a section < 25%. As it is difficult to perform lapping with percentage < 25% in any one section, such extra transverse reinforcement will normally be required for $\phi \geq 20$ mm. Nevertheless, with lapping ≤ 50% at one section, at least U-bars or links can be eliminated.

(ii) Beam

(a) Limitation of neutral axis depths

Neutral axis depths have been reduced from 0.5 to 0.4 for concrete grades 45N/mm^2 < f_{cu} ≤ 70 N/mm^2 and further reduced to 0.33 if f_{cu} > 70 N/mm^2 as per Cl. 6.1.2.4(b) of the Code under Amendment No. 1. The effects should be insignificant as it is uncommon to design flexural members with grade higher than 45.

(b) Reduction of moment arm factors for high grades concrete in sectional design of beam by the Simplified Stress Block from 0.9 to 0.8 and 0.72.

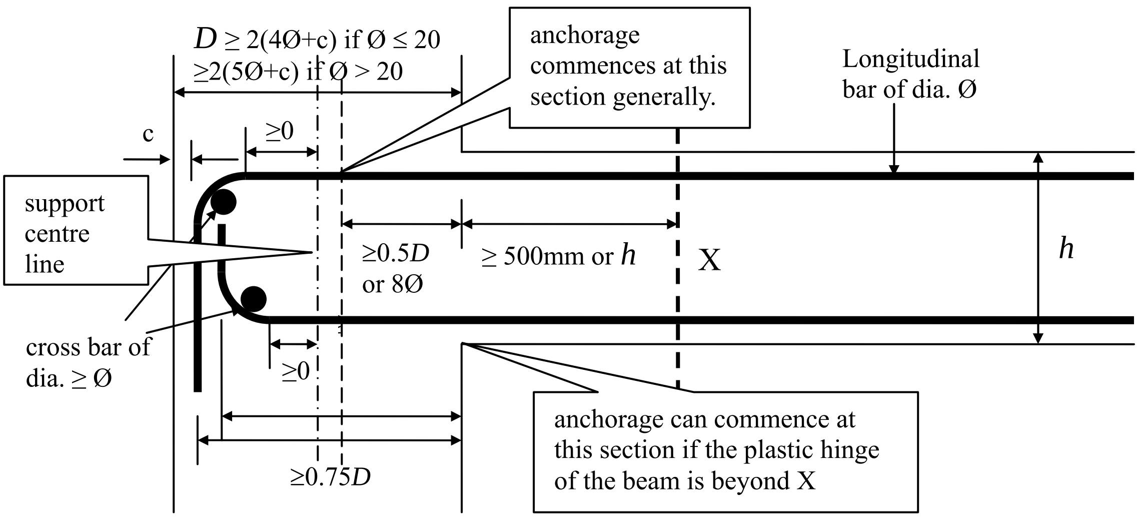
(c) Steel Percentages

The maximum and minimum tension steel percentages are respectively 2.5% and 0.3% in Cl. 9.9.1.1(a) of the Code for beams contributing to lateral load resisting system. The lower maximum tension steel percentages may force the designer to use larger structural sections for the beams contributing in lateral load resisting system.

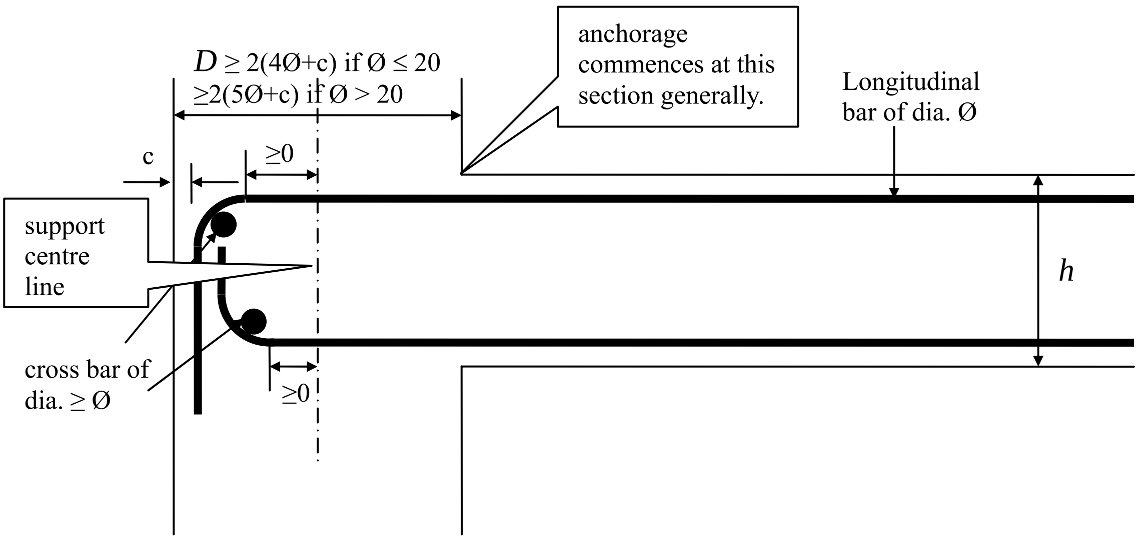
In addition, Cl. 9.9.1.1(a) also imposes that "At any section of a beam within a critical zone (the Handbook gives an example of that "plastic hinge zone" is a critical zone), the compression reinforcement should not be less than one-half of the tension reinforcement at the same section." The "critical zone" should likely include mid-spans and/or internal supports in continuous beam. As plastic hinges will likely be extensively in existence in normal floor beams as per the discussion in Section 2.4, the requirement is expected to be applicable in many locations in beams contributing in lateral load resisting system. The adoption of this clause will obviously increase amounts of longitudinal bars significantly for these beams.

(d) End Support Anchorage

The Code has clarified support anchorage requirements of reinforcement bars of beams as summarized in the following Figure 16.3 which amalgamates contents in Figures 3.19, 3.20 and 13.1 of this Manual :



Beam contributing to lateral load resisting system



Beam not contributing to lateral load resisting system

Figure 16.3 – Summary of longitudinal bar anchorage details at end support

(1) Cl 8.4.8 clarifies the support widths to beams in form of beams,

columns and walls as in excess of 2(4Ø+c) if Ø ≤ 20 and 2(5Ø+c) if Ø > 20 where Ø is the diameter of the longitudinal bar and c is the concrete cover to the bar. The clause has effectively required bend of bars to commence beyond the centre-line of support which is an existing requirement in BS8110 stated for simply supported end (Cl. 3.12.9.4 of the BS). The clause has extended the requirement to cover beam at supports restrained against rotation. Support widths may then require to be increased or bar size be reduced to satisfy the requirement, giving constraints in design;

(2) Cl. 9.9.1.1(c) of the Code requires anchorage of longitudinal bar of beam contributing to lateral load resisting system to commence at the centre line of support or 8 times the longitudinal bar diameter whichever is the smaller unless the plastic hinge at the beam is at the lesser of 500mm or a beam structural depth from the support face of the beam. Effectively the requirement covers all such beams designed to be having rotational restraints at the exterior columns or walls unless it can be shown that critical section of plastic hinge is beyond X as shown in Figure 16.3. By the requirement, anchorage length needs be increased by the lesser of the half of the support width and or 8 times the longitudinal bar diameter for most of the end span beams contributing to lateral load resisting system and anchored into exterior column which is often relatively stiffer than the beam as per discussion in Section 2.4;

(3) A method of adding a cross bar so as to avoid checking of internal stress on concrete created by the bend of the longitudinal bar (by (Ceqn 8.1) of the Code) has been added in Cl. 8.3 of the Code, even if checks on the bar indicates that anchorage of the longitudinal bar is still required at 4Ø beyond the bend. The method is not found in BS8110. The method is quite helpful as the designer can avoid using large bends of bars to reduce bearing stress in concrete which may otherwise result in non-compliance with Cl. 8.4.8 of the Code;

(4) Cl. 9.9.1.1 (c) states clearly that top beam bars be bent downwards and bottom beam bars bent upwards, again applicable to beams contributing in lateral load resisting system. The requirement may create difficulties to the conventional construction work as, apart from aggravating steel bar congestion problems in the column

beam joint, the top bars may be required to be fixed prior to column concreting if they have to be bent into the column shaft to achieve adequate anchorage. The practice is obviously not convenient in the current construction sequence for buildings.

A Solution to anchorage problem may be adding an “elongation” of the structural beam, if possible, beyond the end column as shown in Figure 16.4.

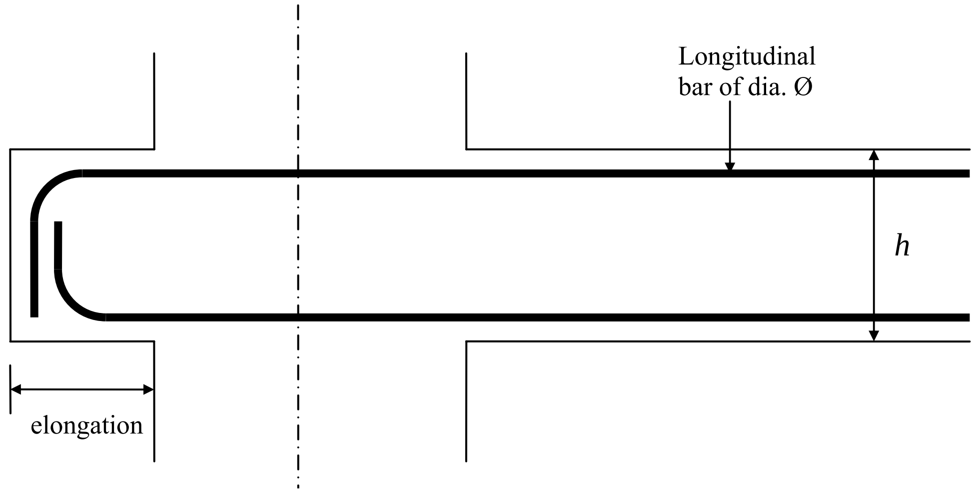


Figure 16.4 – “Elongation” for anchorage of longitudinal bars beyond end supports

(iii) Column

(a) Steel Percentages

Cl. 9.9.2.1(a) of the Code has required the maximum longitudinal reinforcements to be 4% of the gross sectional area for columns contributing to lateral load resisting system which are more stringent than columns not contributing to lateral load resisting system (6% to 10% in accordance with 9.5.1 of the Code). In addition, the clause also clarifies that the maximum longitudinal bar percentage at laps is 5.2% which effectively reduces the maximum steel percentage to 2.6% if the conventional lapping at single level (not staggered lap) is adopted in construction for columns contributing to lateral load resisting system.

(b) Anchorage and lapping of longitudinal bars in supporting beams or foundations

Figures 5.9, 5.10 and 5.11 of this Manual illustrate anchorage of longitudinal bars of columns in supporting beam or foundations as required by Cl. 9.9.2.1(c) of the Code where the columns contribute in lateral load resisting system and plastic hinges will occur in the column. Generally anchorage lengths will be increased as anchorage commences inside the beam and foundation element instead of column foundation interfaces for such columns. Furthermore, bars in columns anchored into intersecting beams must be terminated with 90^o standard hooks (or equivalent anchorage device) and have to be bent inwards unless the column is designed only for axial loads. All these lead to longer anchorage lengths and stability problem in reinforcing bars erection.

(c) Splicing of longitudinal bars

To reduce weakening of the column in reinforcement splicing (lapping and mechanical coupling) in "critical zones" (potential plastic hinge formation zones), Cl. 9.9.2.1(d) of the Code requires the longitudinal splicing locations of columns contributing in lateral load resisting system should, as far as possible, be away from these "critical zones" which are near the mid-storey heights as illustrated in Figure 5.9. For such columns, the current practice of lapping at floor levels in building construction requires review.

(d) Transverse Reinforcements

Cl. 9.9.2.2 of the Code which is applicable to column contributing to lateral load resisting system defines "critical regions" along a column shaft which are near the ends of the column resisting high bending moments and specifies more stringent transverse reinforcement requirements in the same clause than the normal region near mid-heights of the column. In this clause, the definition of "critical regions" relies on axial stress in the column and has made no reference to any potential plastic hinge formation zone. As it is not our usual practice of specifying different transverse reinforcements along the column shaft and the lengths of the "critical regions" are often more than half of the column shaft (dictated also by the requirement of one to two times the greater lateral dimension of the column), it seems

sensible to adopt the more stringent requirements along the whole column shaft. The more stringent requirements of transverse reinforcements comprise closer spacing and that every longitudinal bar (instead of alternate bar) must be anchored by a link. Whilst the maximum spacing in the normal region is 12Ø where Ø is the longitudinal bar diameter and that in the critical region is the smaller of 6Ø and 1/4 of the least lateral dimension in case of rectangular or polygonal column and 1/4 of the diameter in case of circular column, the quantities of transverse reinforcements can be doubled.

(iv) Column Beam Joints

The requirement of providing checking and design in column beam joints as discussed in Section 6 constitutes a significant impact on the current design and construction as, apart from increase of construction cost due to increase of steel contents, the requirement aggravate the problem of steel congestions in these joints. Enlargement of the column head as indicated in Figure 16.6 may be required in case the shear stress computed by Ceqn 6.71 is excessive or the required reinforcements are too congested. In addition, it should also be noted that even no shear reinforcement is required as per checking of shear in the joints in accordance with Cl. 6.8 of the Code, transverse reinforcements in accordance with Cl. 9.5.2 which are installed in the column shaft outside "critical regions" shall also be installed within the column beam joints as shown in Figure 6.3 of this Manual.

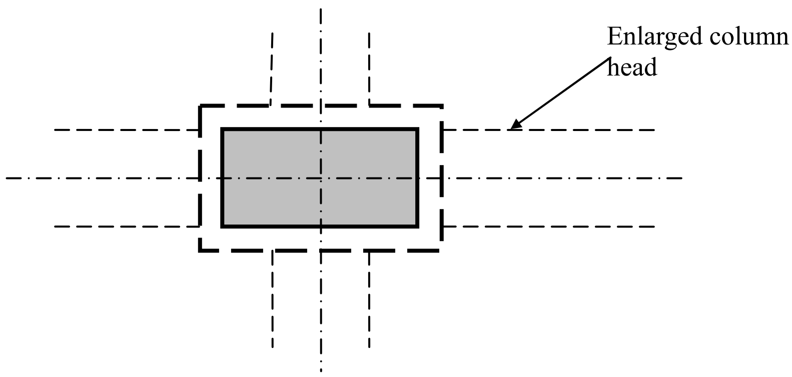


Figure 16.6 – Column head enlargement for column beam joint

16.3 Building Accelerations

Cl. 7.3.2 of the Code specifies that “where a dynamic analysis is undertaken, the maximum peak acceleration should be assessed for wind speeds based on a 1-in-10 year return period of 10 minutes duration with the limits of 0.15m/sec^2 for residential buildings and 0.25m/sec^2 for office or hotel. The term “dynamic analysis” is not defined in the Code. However, if it is agreed that computation of wind loads in accordance with Wind Code 2004 Appendix F (titled “Dynamic Analysis”) is a dynamic analysis, the requirement will be applicable to all buildings defined as ones with “significant resonant dynamic response” in Clause 3.3 of the Wind Code, i.e. (i) taller than 100 m; and (ii) aspect ratio > 5 unless it can be demonstrated that the fundamental natural frequency > 1 Hz. Thus most of the high-rise buildings are included.

Empirical approaches for assessment of building accelerations are described in Appendix B. The second approach which is taken from the Australian Code should be compatible to the Hong Kong Wind Code as it is based on the Australian Code that the Hong Kong Wind Code determines approaches of dynamic analysis in its Appendix F. Furthermore, it can be shown in the chart attached in the Appendix that building acceleration generally increases with building heights and thus pose another compliance criterion. Fortunately, the accelerations approximated are not approaching the limiting criterion as per the exercise on a square plan shaped building. However, the effects should be more significant for buildings with large plan length to breadth ratios.

References

This Manual has made reference to the following documents :

1. Code of Practice for the Structural Use of Concrete 2004
2. Concrete Code Handbook by HKIE
3. Hong Kong Building (Construction) Regulations
4. The Structural Use of Concrete 1987
5. Code of Practice on Wind Effects in Hong Kong 2004
6. Code of Practice for the Structural Use of Steel 2005
7. The Code of Practice for Dead and Imposed Loads for Buildings (Draft)
8. BS8110 Parts 1, 2 and 3
9. BS5400 Part 4
10. Eurocode 2
11. Code of Practice for Precast Concrete Construction 2003
12. New Zealand Standard NZS 3101:Part 2:1995
13. ACI Code ACI 318-05
14. Code of Practice for Fire Resisting Construction 1996
15. BS8666 : 2000
16. BS4466 : 1989
17. PNAP 173
18. Practical design of reinforced and prestressed concrete structures – based on CEP-FIP model code MC78
19. CEB-FIP Model Code 1990
20. Standard Method of Detailing Structural Concrete – A Manual for best practice – The Institution of Structural Engineers
21. The "Structural Design Manual" – Highways Department of HKSAR
22. Reinforced and Prestressed Concrete 3rd edition – Kong & Evans
23. Reinforced Concrete Design to BS110 – Simply Explained A.H. Allen
24. Design of Structural Concrete to BS8110 – J.H. Bungey
25. Handbook to British Standard BS8110:1985 Structural Use of Concrete – R.E. Rowe and others – Palladian Publication Ltd.
26. Ove Arup and Partners. *Notes on Structures: 17* April 1989
27. Australian/New Zealand Standard – Structural Design Actions Part 2 : Wind Actions AS/NZS 1170.2.2002
28. Examples for the Design of Structural Concrete with Strut-and-Tie Models – American Concrete Institute
29. Tables for the Analysis of Plates, Slabs and Diaphragms based on the

Elastic Theory – Macdonald and Evans Ltd.
30. Some Problems in Analysis and Design of Thin/Thick Plate, HKIE Transactions – Cheng & Law, Vol. 12 No. 1 2004

Appendix A

Clause by Clause Comparison between “Code of Practice for Structural Use of Concrete 2004” and BS8110

Comparison between Code of Practice for Structural Use of Concrete 2004 and BS8110:1997 (and 1985) Appendix A

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
1.1 – Scope	The clause has explicitly stated that the Code applies only to normal weight concrete, with the exclusion of (i) no fines, aerated, lightweight aggregate concrete etc; (ii) bridge and associated structures, precast concrete (under the separate code for precast concrete); and (iii) particular aspects of special types of structures such as membranes, shells.	Pt. 1 1.1 – Scope	The clause only explicitly excludes bridge structures and structural concrete of high alumina cement.	The exclusion of CoPConc2004 should also be applied to BS8110. In addition, BS8110 does not apply to high strength concrete.
2.1.5 – Design working life	The clause states that the Code assumes a design working life of 50 years. Where design working life differs from 50 years, the recommendations should be modified.	–	Nil	No similar statement in BS8110.
2.2.3.3 – Response to wind loads	The clause refers to clause 7.3.2 for the usual limits of H/500 to lateral deflection at top of the building and accelerations of 1-in-10 year return period of 10 minutes duration of 0.15m/sec^2 for residential and 0.25m/sec^2 for office. However, there is no requirement on the inter-storey drift, though the draft steel code has a requirement of storey height/400.	Pt. 1 2.2.3.3 – Response to wind loads	Reference to specialist literature is required. In addition Pt. 2 3.2.2.2 stipulates a limit on inter-storey drift of Storey height/500 for avoidance of damage to non-structural elements.	CoPConc2004 is more specific. However, method for determination of the acceleration is not given in the Code and in the HKWC-2004.
2.3.2.1 – Loads for ultimate limit state	Table 2.1 is generally reproduced from Table 2.1 of BS8110 except that the partial factor for load due to earth and water is 1.0 for the beneficial case.	Pt. 1 2.4.3.1.2 – Partial factors for earth pressures	It is stated in the clause that when applying the load factor, no distinction should be made between adverse and beneficial loads.	1.0 in CoPConc2004 may not be adequately conservative as there may even be over-estimation in the determination of the unfactored soil load. It is even a practice to set the load to zero in beneficial case. ICU has raised this comment during the comment stage when the draft Code has exactly the content of BS8110.
2.3.2.3 &	The clauses explicitly state that these effects need	–	No similar clauses in BS8110	The clause in CoPConcrete2004

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
2.3.2.4 – Differential settlement of foundations, creep, shrinkage, temperature effects	only be considered when they are significant for ULS. In most other cases they need not be considered provided ductility and rotational capacity of the structure sufficient.			affirms engineers to ignore consideration of these effects in normal cases which are the usual practices.
2.4.3.2 – Values of γ_m for ULS	Table 2.2 gives γ_m for ULS for concrete and re-bars. γ_m for re-bars is 1.15, implying strength of re-bars for design remain as $0.87f_y$.	Pt. 1 2.4.4.1 – Values of γ_m for ULS	Table 2.2 gives γ_m for ULS for concrete and re-bars. γ_m for re-bars is 1.05, implying strength of re-bars for design remain as $0.95f_y$.	BD has been insisting on the use of $0.87f_y$ even if BS8110 was used before the promulgation of the new concrete code
3.1.3 – Strength grades	Table 3.1 states concrete strength grades from 20 MPa up to 100 MPa which is the range covered by the Code.	–	BS8110 has not explicitly stated the concrete grades covered by the BS, However, concrete grades covered by the design charts in Part 3 of the Code range from grade 25 to 50 whilst other provisions such as v_c (Pt. 1 Table 3.8), lap lengths (Pt. 1 Table 3.27) are up to grade 40.	The coverage of CoPConc2004 is wider.
3.1.4 – Deformat-ion of Concrete	It is stated in the 1st paragraph of the clause that for ULS, creep and shrinkage are minor, and no specific calculation are required.	Pt. 1 2.4.3.3 – creep, shrinkage and temperature effects	The clause states that “For the ULS, these effects will usually be minor and no specific calculations will be necessary.	BS 8110 has included temperature effects be a minor one that can be ignored in calculation.
3.1.5 – Elastic deformat-ion	Table 3.2 stipulates short term static Young’s Modulus of concrete of various grades based on the formula $3.46\sqrt{f_{cu}}+3.21$ in MPa derived from local research.	Pt. 1 Figure 2.1	The determination of short term static Young’s Modulus of concrete is given by the slope gradient in the figure which is $5.5\sqrt{(f_{cu}/\gamma_m)}$.	Values in the CoPConc2004 should be used as it is based on local research and concrete E values are affected by constituents which are of local materials. Nevertheless, it

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
				should be noted that E values in the new Code are slightly higher than the previous ones in "The Structural Use of Concrete – 1987" (Table 2.1).
3.1.7 & 3.1.8 – Creep and shrinkage	Though it is stated in 3.1.4 that for ULS creep and shrinkage are minor and require no specific calculations, these clauses contain detailed formulae and charts for prediction of creep and shrinkage strain. The approach is identical to the "Structural Design Manual" issued by Highways Department and the charts are extracted from BS5400:Pt 4:1990. As identical to the previous version of "Structural Design Manual" (SDM), the c_s value is 4.0 to suit local crushed granite. However, it is noted that recently the SDM has reduced the factor to 3.0 and the Code has adopted the change in Amendment No. 1.	–	No account has been given for creep and shrinkage, as stated in 2.4.3.3.	As stated in the CoPConc2004 Cl. 2.3.2.4, effects need only be considered if they are significant.
3.1.9 – Thermal Expansion	The linear coefficient of thermal expansion given in the Code for normal weight concrete is 10×10^{-6}/^{o}C whilst that stated in the "Structural Design Manual" issued by Highways Department is 9×10^{-6}/^{o}C in Clause 2.4.4.	–	No account has been given for temperature, as stated in 2.4.3.3.	The linear coefficient of thermal expansion given by both CoPConc2004 is slightly higher than SDM and both are independent of concrete grades.
3.1.10 – Stress -strain relationship for design	The short term design stress-strain curve of concrete (Fig. 3.8) follows closely the traditional one in BS8110, though there are differences in the values of the Young's Moduli. Furthermore, the ultimate strain is limited to below 0.0035 for concrete grade exceeding C60. Nevertheless, the "plastic strain", strain beyond which stress is constant remains identical as BS8110;	Pt. 1 2.5.3 – Analysis of section for ULS	Pt. 1 Fig. 2.1 shows stress-strain relation of normal weight concrete.	(i) The Young's Moduli of concrete stipulated in CoPConc2004 should be adopted as they are based on local data and cover up to grade 100 concrete, together with the decrease of ultimate strain for grade

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
				above C60, to account for the brittleness of high strength concrete; (ii) "Smooth" connection between the parabolic curve and the straight line portion of the stress-strain curve cannot be effected if $\varepsilon_0 = 2.4\times10^{-4}\sqrt{f_{cu}/\gamma_m}$ is kept and the Young's moduli in Table 3.2 of CoPConc2004 are used. For smooth connection, ε_0 should be revised $2\sigma_{ult}/E_c$ or $1.34f_{cu}/\gamma_mE_c$ where $\sigma_{ult}=0.67f_{cu}/\gamma_m$. Nevertheless, ε_0 has been rectified to $1.34f_{cu}/\gamma_mE_c$ in Amendment No. 1.
3.2.7 – Weldability (of re-bars)	The clause states that re-bars can be welded provided the types of steel have the required welding properties given in acceptable standards and under Approval and inspection by competent person. Further provisions for welding are given in 10.4.6.	Pt. 1 3.12.8.16 7.6	There are provisions for lapping re-bars by welding in Pt. 1 3.12.8.16, and general welding requirements in Pt. 1 7.6. Nevertheless, another BS7123 titled "Metal arc welding of steel for concrete reinforcement" requires the re-bars be in compliance with BS4449 or BS4482.	Steel complying CS2 is likely weldable as CS2 does not differ significantly from BS4449.
Section 4 – Durability and fire resistance	The requirements are general. The followings are highlighted :- (i) In 4.1.1, it is stated that requirements are based on design working life of 50 years; (ii) In Table 4.1 under 4.2.3.2, exposure conditions 1 to 5 are classified with headings similar to that	Pt. 1 2.2.4 – Durability; Pt. 1 2.2.6 – Fire resistance; Pt. 1 3.1.5.2 –	The requirements are general. However, the followings are highlighted : (i) 3.1.5.2 states that when cement content > 400 kg/m^3 and section thicker than 600 mm, measures for temperature control should be implemented;	The approaches of the two codes are quite different. However, CoPConc2004 is more related to local practice.

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	in Pt. 1 3.3.4 of BS8110. However, detailed descriptions are different except the last one "abrasive"; (iii) In 4.2.7.3, control of AAR has been incorporated from the previous PNAP 180; (iv) Concrete covers are given in Table 4.2 for various concrete grades and exposure conditions; (v) By 4.3, the user has to refer to the current fire code for additional requirements against fire resistance.	Design for Durability Pt. 1 3.3 – Concrete cover to rebars Pt. 2 Section 4	(ii) A full description for fire resistance control is given in Pt. 2 Section 4, outlining methods of determining fire resistance of structural elements and with reference to BS476 Pt. 8;	
5.1.3.2 – Load cases and combinations for beams and slabs	3 simplified load cases for Dead + Live loads are recommended (with DL always present): (i) all spans loaded with LL; (ii) alternate spans loaded with LL; (iii) adjacent spans loaded with LL. The 2nd case is to seek for max. sagging moment and the 3rd case is likely to seek for max. hogging moment. But true max hogging moment should also include alternate spans from the support being loaded.	Pt. 1 3.2.1.2.2	2 simplified load cases are considered sufficient for design of spans and beams (with DL always present): (i) all spans loaded with LL; (ii) alternate spans loaded with LL;	CoPConc2004 more reasonable, though not truly adequate. Nevertheless, the current softwares mostly can account for the load case to search for max. hogging moment at support.
5.2 – Analysis of Structure	The clause contains definition of beam, slab (included ribbed, waffle, one-way or two-ways), column in accordance with their geometries. The following differences with BS8110 are highlighted : (i) Conditions in relations to rib spacings and flange depths for analysis of a ribbed or waffle slab as an integral structural unit are given in 5.2.1.1(d); (ii) By definition, the effective flange widths in T and L-beams included in 5.2.1.2 (a) are slightly greater than that in BS8110 by 0.1×clear rib spacing unless for narrow flange width controlled by rib spacing. It is also stated that	3.2 – Analysis of Structures 3.4 – Beams	The followings are highlighted : (i) Generally provisions are applicable to normal strength concrete; (ii) Consideration for high-rise buildings (second order effects) and structures such as shear walls, transfer structures are not given; (iii) The effective span of a simply supported beam is the clear span plus the lesser of half effective depth and half of support width whilst that of continuous beam and cantilever are span between supports (presumably mid-support width) except at end span in case of continuous beam and a cantilever forming the end of a	The following remarks are made : (i) CoPConc2004 is more suitable for use in Hong Kong as it cover high strengths concrete, high rise buildings, shears, transfer structures. (ii) Extended use of effective flange in beam in structural analysis and moment reduced to support shear are explicitly

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	the effective flange can be adopted in structural analysis; (iii) Clearer definition of effective spans of beams is also included in 5.2.1.2(a) with illustration by diagrams, together with reduction of span moments due to support width in 5.2.1.2(b). In principle, the effective span of a simply supported beam, continuous beam or cantilever is the clear span plus the minimum of half effective depth and half support width except that on bearing where the span should be up to the centre of the bearing. (iv) Furthermore reduction in support moments due to support width is allowed which is not mentioned in BS8110; (v) The definition of effective flange for T- and L-beams are slightly different from BS8110 though the upper bound values are identical to that of BS8110. So more stringent. (vi) Moment redistribution is restricted to grade below C70 as per 5.2.9.1; (vii) Condition 2 in 5.2.9.1 relation to checking of neutral axis depths of beam in adopting moment re-distribution is different from Cl. 3.2.2.1 b) of BS8110. More stringent limit on neutral axis is added for concrete grade higher than C40 as per 6.1.2.4(b); (viii) Provision for second order effects with axial loads in 5.3 is added. The provisions are quite general except the statement "second order effects can be ignored if they are less than 10%"; (ix) Provisions for shears walls and transfer structures are added in 5.4 and 5.5 though the		continuous beam, the effective span is the clear span plus mid-support width should be used.	stated in CoPConc2004. (iii) Method of analysis in both codes are old-fashioned ones that can be performed by hand calculations. Use of computer methods (extensively adopted currently in Hong Kong) are not mentioned.

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	provisions are too general.			
6.1 – Members in Flexure	The followings differences with BS8110 are identified : (i) Ultimate strains are reduced below 0.0035 for concrete grade exceeding C60 as per Fig. 6.1; (ii) In Figure 6.1, the factor 0.9, concrete stress block depth ratio is revised to 0.8 for $45<f_{cu}\le70$ and 0.72 for $70<f_{cu}$ in Amendment No. 1; (iii) Neutral axis depths limited to less than 0.5d for concrete grade higher than C45 as per 6.1.2.4(b) (Originally the Code limits demarcate neutral axis depths from 0.5d to 0.4d at C40. But the demarcation is revised to C45 at Amendment No. 1); (iv) The limitations on neutral axis depth ratio under redistribution of moments are further restricted for grade > 45 (revised as per Amendment No. 1), as different from BS8110 which makes no differences among concrete grades in Cl 3.4.4.4 of BS8110 which if redistribution of moment exceed 10%; (v) Different design formulae are used for the higher grades ($45\le f_{cu}\le70$; $70\le f_{cu}<100$, as revised by Amendment No. 1) concrete based on simplified stress blocks as per 6.1.2.4(c). Design charts similar to that BS8110 based on stress-strain relationship in figure 3.8 are not available; (vi) Table 6.2 in relation to v_c of concrete as related to tensile reinforcements is identical to Pt. 1 Table 3.8 of BS8110 except that applicability is extended to C80 whereas BS8110 limits to grade 40. The minimum amount of shear	Pt. 1 3.4 – Beams	In the design aspects, the Code is limited to grade 40.	The followings are highlighted : (i) Provisions are made in CoPConc2004 for concrete grades higher than 40 including limitation of neutral axis depths etc. However, it is also noted that the ultimate stress in Fig. 6.1 remains unchanged for high concrete grade though the ultimate strain is reduced; (ii) Ultimate concrete shear strengths increased in CoPConc2004; (iii) Due to the more stringent limitation on neutral axis depth for high grade concrete in CoPConc2004, different formulae have been devised for $x > 0.5d$ for rectangular beams. However, similar formulae are not given for flanged beams; (iv) 6.1.4.2 of CoPConc2004 is similar to BS8110 Pt. 1 3.6.2. However, it is ICU's comment to qualify that if ribbed slabs are to be designed as two-ways spanning as similar to a

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	reinforcements required is reduced to that can provide shear resistance of $0.4(f_{cu}/40)^{2/3}$ MPa for grade over 40 whilst that for concrete at and below grade 40 remains to that can provide 0.4 MPa; (vii) Partial strength factor for steel remains as $0.87f_y$ in accordance with BS8110:1985 instead of $0.95f_y$ as in accordance with BS8110:1997, for both flexure and shear; (viii) Ultimate shear strength of concrete increased to 7.0 MPa as compared 5.0 MPa in BS8110. The other limitation of $0.8\sqrt{f_{cu}}$ is identical to BS8110; (ix) Table 6.3 is identical to Table 3.8 of BS8110 except, the last note under Notes 2 for the effect of effective depth (d) on v_c where shear reinforcement is required. Whilst BS8110 effectively gives 1 for the factor $(400/d)^{1/4}$ for $d \geq 400$mm, Table 6.3 requires the factor to decrease beyond d=400mm. However as subsequent to discussion with experts, it is advisable to keep the factor to 1 for $d \geq 400$mm as there are no tests to justify the decrease; (x) By 6.1.3.5, the minimum shear reinforcement to cater for shear strength of 0.4 MPa is for concrete grade below C40 (requirement by BS8110). Above grade C40, the required shear strength is increased by factor $(f_{cu}/40)^{2/3}$ as per Table 6.2; (xi) By 6.1.4.2, ribbed slabs (6.1.4.2) can be designed as two-ways spanning as similar to flat slab <u>if they have equal structural properties in two mutually perpendicular directions</u>. BS8110 does not explicitly require equal structural			flat slab, it should be qualified that the slab has equal structural properties in two mutually perpendicular directions.

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	properties in mutually perpendicular directions in adopting design method as flat slab (BS8110, 3.6.2).			
6.2 – Members axially loaded with or without flexure	The followings are highlighted: (i) The CoP contains no design charts for column design with moments. Strictly speaking, the design charts in BS8110 are not applicable (a) the Young's Moduli of concrete are different; and (b) the ultimate strain for concrete grade > C60 are reduced; (ii) Due to the use of lower partial strength factor of steel (γ_m = 1.15), the factors on steel in equations for strengths of column sections (6.55, 6.56) are lower than BS8110 (Eqn 28, 29); (iii) The provisions for more accurate assessment of effective column height in accordance with BS8110 Pt. 2 2.5 are not incorporated in the Code.	Pt. 1 3.8 – Columns Pt 2 2.5 – Effective column height	The followings are highlighted : (i) In the design aspects, the provisions are limited to grade 40; (ii) A more tedious approach for determination of effective height of column by consideration stiffness of the connecting beams and columns is outlined in Pt. 2 2.5.	The followings are highlighted : (i) The limitation of neutral axis depth as for beam is clearly not applicable to columns. However, it should be noted that members with axial loads creating axial stress < $0.1f_{cu}$ may be regarded as beam in design. Re 6.1.2.4 of CoPConc2004.
6.3 – Torsion and Combined Effects	The followings are highlighted : Table 6.17 in relation to limitation of torsional shear stresses contains specific values for grades 25 to 80. For values below grade 40, they are identical to BS8110. Above grade 40, the Code provides values for v_{tmin} and v_t for different grades up to 80, beyond which the values remain constant whilst BS8110 set the values to that of grade 40 for grades above 40;	Pt. 2 2.4	The followings are highlighted (i) Table 2.3 contains specific values for v_{tmin} and v_t up to grade 40. The values remain constant from grade 40 thereon; (ii)	CoPConc2004 contains more specific values for ultimate concrete shear stress.
6.4 – Design for robustness	The followings are highlighted : (i) In 6.4.2 in relation to design of "bridging elements" which is identical to BS8110 Pt.2 2.6.3, the words "where required in buildings of	Pt 2 2.6.3	The followings are highlighted : (i) Pt.2 2.6.3 in relation to design of "bridging elements" applies to buildings of five or more storeys;	CoPConcrete 2004 is more stringent.

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	five or more storeys" have been deleted. So the Code is more stringent as consideration to loss of elements is required for all buildings;			
6.6 – Staircase	The followings are highlighted : (i) 6.6.1 is in relation to design of staircase. There is no stipulation that the staircase may include landing;	Pt 1 3.10.1	The followings are highlighted : (i) A note in 3.10.1 in relation to design of staircase has stipulated that a staircase also include a section of landing spanning in the same direction and continuous with flight;	It is more reasonable to assume staircase should include landing, as in CoPConc2004.
6.7 – Foundations	The following differences with BS8110 are highlighted : (i) In 6.7.1.1, the assumptions of uniform reaction or linearly varying reaction of footing and pile cap are based on use of rigid footings or pile caps. So a pre-requisite for the use of these assumptions is stated at the end of the clause which reads "if a base or pile cap is considered be of sufficient rigidity."; (ii) In 6.7.3.1, a statement has been inserted that a pile cap may be designed as rigid or flexible, depending on the structural configuration. No similar provision is found in BS8110; (iii) In 6.7.3.3, 2nd dot, it is stated that "where the shear distribution across section has not been considered, shear enhancement shall not be applied." No similar provision is found in BS8110; (iv) In 6.7.3.3 3rd dot, shear enhancement in pile cap can only be applied where due consideration has been given to shear distribution across section. No similar provision is found in BS8110; (v) In 6.7.3.3 4th dot, determination of effective	Pt 1 3.11	The followings are highlighted : (i) In Pt 1 3.11.2.1, the assumption of uniform or linearly varying assumption is without the pre-requisite that the footing or pile cap is sufficiently rigid. This is not good enough as significant errors may arise if the footing or pile cap is flexible; (ii) In Pt 1 3.11.4.1, there is no mention of pile cap rigidity which affects design; (iii) In Pt 1 3.11.4.3, there is no mention that shear enhancement shall not be applied to where shear distribution across section has not been considered; (iv) In Pt 1 3.11.4.4 b), there is no mention that shear enhancement in pile cap shall be applied to under the condition that shear distribution across section has not been duly considered; (v) There is no explicit stipulation on the limit of effective width for checking shear; (vi) No explicit statement that shear reinforcement is required if $v<v_c$ though it is a normal practice of not providing shear stirrups in pile caps; (vii) No explicit statement that torsion is required to be checked, if any.	The followings are highlighted : (i) CoPConc2004 is generally more reasonable as it makes provision for the modern analysis by treating the pile cap as a flexible structure by computer methods; (ii) Apparently CoPConc2004 forces the designer to check torsion as if the cap or footing is a beam under the rigid cap (footing). This is not too sound as the formulae for beam are under the assumption that torsional cracks can be fully developed for a beam length of b+d. If such length is not available which is very common for cap or footing structures which are usually wide structures. There should be

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	width of section for resisting shear is included. No similar provision is found in BS8110; (vi) In 6.7.3.3 5th dot, it is explicitly stated that no shear reinforcement is required if $v<v_c$. No similar stipulation is found in BS8110; (vii) In 6.7.3.4, the ultimate shear stress is increased to 7 MPa, as compared with BS8110; (viii) In 6.7.3.5, it is stated that torsion for a rigid pile cap should be checked based on rigid body theory and where required, torsional reinforcements be provided.			shear enhancement if the full length cannot mobilized, as similar to direct stress. However, no study data is available for torsional shear enhancement.
6.8 – Beam Column Joints	The Code contains detailed provisions for design of beam column joints. No similar provision found in BS8110.	–	No similar provision.	Design checking on beam-column shall be done if CoPConc2004 is used.
Section 7 – Serviceability Limit States	The followings are highlighted : (i) This section contains provisions in BS8110 Pt. 2 Section 3 and the deem-to-satisfy requirements (for deflections) in BS8110 Pt. 1 Section 3; (ii) Limits of calculated crack widths are given in 7.2.1 for member types and exposure conditions. The limited values are mostly 0.3mm as compared with BS8110 except water retaining structures and pre-stressed concrete (0.2 mm); (iii) 7.2.4.1 is identical to BS8110 Pt. 2 3.8.4.1 except that the last paragraph and Table 3.2 of BS8110 Pt. 2 3.8.4.1 have been omitted. The omitted portion is in relation to estimated limiting temperatures changes to avoid cracking; (iv) Equation 7.3 in 7.2.4.2 is different from	Pt. 2 Section 3	The followings are highlighted : (i) General provisions for determination of deflections, cracks (including thermal cracking) are given; (ii) Some of the provisions may be applicable to UK only; (iii) Limit of calculated crack widths is given as 0.3mm in 3.2.4.1 as a guide.	The following remarks are made : (i) The omission of the last paragraph and Table 3.2 of BS8110 Pt. 2 3.8.4.1 is likely because of the different climate and material properties in Hong Kong; (ii) Equation 7.3 in 7.2.4.2 appears to be a refined version of Equation 14 of BS8110 Pt. 2 3.8.4.2. The factor 0.8 accounts for discount of strain due to the long term strain.

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	Equation 14 of BS8110 Pt. 2 3.8.4.2 where the difference in temperatures is divided into 2 parts and a factor of 0.8 is employed in the estimation of thermal strain. Also it is allowed in the clause to take reinforcements into consideration; (v) In 7.3, pre-camber is limited to span/250 to compensate excessive deflection. The limit is not given in BS8110 Pt. 2 3.2.1. Also, deflection limit after construction for avoidance of damage to structure is limited to span/500, whilst BS8110 Pt. 2 3.2.1.2 specifies span/500 or span/350 in accordance with brittleness of finishes for avoidance of damage to non-structural elements. In addition, 20 mm as an absolute value is also imposed in BS8110; (vi) In 7.3.2, limits (0.15 and 0.25 m/s^2) on accelerations (10 years return period on 10 min. duration) are given as avoidance of “excessive response” to wind loads whilst no numerical values are given in BS8110 Pt. 2 3.2.2.1. Furthermore, deflection limit due to wind load is given as H/500 whilst BS8110 Pt. 2 3.2.2.2 indicates limit of h/500 as inter-storey drift for avoidance of damage to non-structural members; (vii) In 7.3.3, excessive vibration should be avoided as similar in BS8110 Pt. 2 3.2.3. However, there is extra requirement in 7.3.3 that dynamic analysis be carried out in case structural frequency less than 6 Hz for structural floor and footbridge less than 5 Hz; (viii) Table 7.3 under 7.3.4.2 in relation to deem-to-satisfy requirement for basic			

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	span/depth ratios of beam and slab contains, requirements for "two-way slabs" and "end spans" are included, as in comparison with Table 3.9 of BS8110 Pt 1; (ix) Table 7.4 in relation to modification factor to effective span depth ratio by tensile reinforcement is identical to Table 3.10 of BS8110 except that the row with service stress 307 (f_y = 460) has replaced that of 333 (f_y = 500); (x) 7.3.4.6 is identical to BS8110 Pt. 1 3.4.6.7 except that the last sentence in BS8110 is deleted; (xi) The provision of deflection calculation in 7.3.5 is identical to BS8110 Pt 2 Cl. 3.7; (xii) Equation 7.7 in 7.3.6 is not identical to equation 9 in BS8110 Pt. 2 in the derivation of shrinkage curvature;			
Section 8 – Reinf't requirements	The followings are highlighted : (i) In 8.1.1, it is declared that the rules given in the Section for re-bars detailings do not apply to seismic, machine vibration, fatigue etc and epoxy, zinc coated bars etc. No similar exclusion is found in BS8110; (ii) In 8.1.2, it is stated that bar scheduling should be in accordance with acceptable standards whilst BS8110 Pt. 1 3.12.4.2 requires standard be in accordance with BS4466; (iii) In 8.2, the minimum spacing of bars should be the lesser of bar diameter, h_{agg}+5 mm and 20 mm. BS8110 Pt. 1 3.12.11.1 does not include 20 mm and bar diameter is only required when bar size > h_{agg}+5 mm;	Pt. 1 Section 3 Pt. 1 4.10 in relation to anchorage of tendons in prestressed concrete	The followings are highlighted : (i) The provisions are general; (ii) Consideration for ductility is not adequate.	The followings are highlighted : (i) CoPConc2004 requires bar scheduling to acceptable standards. However, provisions in 8.3, 9.2.3 etc have requirements for bend of bars; (ii) ICU has commented that requirement in BS8110 Pt. 1 3.12.9.4(a) should extend to all support conditions. 8.4.8 of the Code seems to incorporate the comment; (iii) ICU has also suggested the use of torsional links

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	(iv) In 8.2, it is stated that the distance between horizontal layer of bars should be sufficient without quantification whilst BS8110 Pt. 1 3.12.11.1 requires minimum be $2h_{agg}/3$; (v) 8.3 is essentially identical to BS8110 Pt. 1 3.12.8.22, 24, 25, except that (a) an additional provision of installing a cross bar inside a bend can eliminate checking of bearing stresses of the bend in concrete; (b) a single Table 8.2 (in relation to minimum bend radii of re-bars) to replace Table 3 of BS4466 to which BS8110 is making reference. As such, no distinction is made between mild steel bars and HY bars – both adopting minimum radii of HY bars. Provisions to the newer BS – BS8666 where the minimum bend radii are generally smaller is not adopted; (vi) 8.4 is essentially identical to BS8110 Pt. 1 3.12.8 except (a) It is mentioned in 8.4.1 that when mechanical device is used, their effectiveness has to be proven. No similar provision is found in BS8110; (b) Type 1 bars are not included in the Code; (c) 8.4.6 is added with Figure 8.1 for illustration of bend anchorage; (d) 8.4.8 in relation to minimum support widths requires any bend inside support be beyond the centre line of the support. This effectively extend the requirement of BS8110 Pt. 1 3.12.9.4(a) to support conditions other than simply support; (vii) 8.5 in relation to anchorage of links and shear			similar to that in ACI code (135^{o} hood) which is of shape other than shape code 77 of BS4466.

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	reinforcements contains more stringent requirement for the length of the link beyond bends of bars than BS8110 Pt. 1 3.12.8.6. – (1) the greater of 4ϕ or 50 mm for 180^o bend in the Code but only 4ϕ in BS8110; (2) the greater of 10ϕ or 70 mm for 90^o bend in the Code but only 8ϕ in BS8110. Provisions for 135^o bend and welded bars are also added; (viii) 8.6 contains requirements for anchorage by welded bars which is not found in BS8110; (ix) Except for the requirement that the sum of re-bar sizes at lapping < 40% of the breadth of the section, 8.7 contains more requirements in terms of “staggering laps”, minimum longitudinal and transverse distances between adjacent lapping of bars which are not found in BS8110. The requirements are also schematically indicated in Fig. 8.4. Effectively the clause requires tension laps be always staggered. Fortunately compression laps and secondary rebar lapping can be in one section; (x) 8.7.4.1 contains different and more detailed requirements for transverse reinforcements in lapped zone than BS8110 Pt. 2 3.12.8.12; (xi) 8.8 and 8.9 in relation to large diameter bars (>40ϕ) and bundle bars which are not found in BS8110; (xii) The provision in BS8110 Pt. 1 3.12.8.16 for butt joints of re-bars is not found in the Code.			
Section 9 – Detailing of Members and	The followings are highlighted : (i) Table 9.1 under Cl. 9.2 tabulates minimum steel percentage equal to Table 3.25 of BS8110 for	Pt. 1 Section 3 and 5.2.7	The followings are highlighted : (i) The analysis procedures are largely old-fashioned relying on old theories of	The followings are highlighted : (i) The stress reduction in design of cantilevered

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
particular rules	beams; (ii) In 9.2.1.4 in relation to maximum distance of bars in tension as similar to BS8110 Pt. 1 3.12.11.2, the stipulation in BS8110 that demonstration of crack width < 0.3 mm can be accepted is omitted; (iii) In 9.2.1.5, a requirement of 15% span moment be used to design beam support even under simply supported assumption is not found in BS8110. Furthermore, it is also stated in the clause that total tension re-bars of a flanged beam over intermediate supports can be spread over the effective width of the flange provided that half of the steel within the web width. There is also no such provision in BS8110; (iv) 9.2.1.8 requiring 30% of the calculated mid-span re-bars be continuous over support appears to be adopted from Fig. 3.24 a) of BS8110. However, the circumstances by which the Figure is applicable as listed in 3.12.10.2 of the BS is not quoted; (v) 9.2.1.9 requires top steel of cantilever to extend beyond the point of contraflexure of the supporting span whilst Fig. 3.24 c) requires at least half of the top steel to extend beyond half span of the cantilever or 45ϕ; (vi) In 9.2.2, maximum spacing of bent-up bars is stipulated whilst no such requirement is found in BS8110; (vii) Torsional links has to be closed links (shape code 77 of BS4466) as required by BS8110 Pt. 2 2.4.8. However, 9.2.3 of the Code provides an alternative of using closed links of 135^{o} bend; (viii) In 9.3.1.1 (b) in relation to maximum spacing		Johansen, Hillerborg. Detailings to cater for behaviours not well understood or quantified are thus provided, though the determination of which are largely empirical or from past experiences; (ii) Though ductility is not a design aid explicitly stated in the BS, the BS does requires 135^{o} bend of links in anchoring compression bars in columns and beams (Pt. 1 3.12.7.2).	projecting structures in PNAP173 is not incorporated is likely because the PNAP is based on working stress design method. So there should be some other approaches and this is not mentioned in the CoPConc2004; (ii) Ductility is more emphasized in CoPConc2004 9.9 which largely stem from seismic design.

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	of re-bars in slab is more detailed than BS8110 Pt. 1 3.12.11.2.7 and appears more reasonable. The provisions are, in fact, more stringent : (a) for principal re-bars, 3h ≤ 400 mm whilst BS8110 is 3h ≤ 750 mm; (b) for secondary re-bars 3.5h ≤ 450 mm whilst no provision in BS8110; (c) more stringent requirements are added for slabs with concentrated loads or areas of maximum moments whilst no similar requirements are found in BS8110; (ix) The first para. in 9.3.1.3 requires half of the area of the calculated span re-bars be provided at the simply supported slabs and end support of continuous slabs. The requirement is identical to BS8110 Pt. 1 3.12.10.3.2. However, the provision in BS8110 is under the condition listed in 3.12.10.3.1 that the slabs are designed predominantly to carry u.d.l. and in case of continuous slabs, approximately equal span. These conditions are not mentioned in the Code; (x) In 9.3.1.3, there is also a provision that if the ultimate shear stress < $0.5v_c$ at support, straight length of bar beyond effective anchorage for 1/3 of support width or 30 mm (whichever is the greater) is considered effective anchorage. No similar provision is found in BS8110; (xi) 9.3.1.6 requiring closed loops of longitudinal rebars at free edge of slab is not found in BS8110; (xii) 9.3.2 is in relation to shear in slabs which should be identical to that for beams. However it is stated that shears should be avoided in			

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	slabs < 200 mm thick; (xiii) 9.4 in relation to cantilevered projecting structures has incorporated requirements for minimum thickness, minimum steel areas, maximum bar spacing, anchorage length from PNAP173. However, other requirements such as design with reduced stresses are not found; (xiv) 9.5.2.3 contains more stringent requirements of links in circular columns than that in BS8110 Pt. 1 3.12.7.3 as the termination of links should be at 135^{o} hook; (xv) There is an extra requirement in the maximum spacing of traverse reinforcement in wall in 9.6.3 which is 400 mm, as in comparison with BS8110 Pt. 1 3.12.7.4; (xvi) 9.6.5 in relation to reinforcement provisions to plain walls include BS8110 Pt. 1 3.9.4.19 to 22. However, 3.9.4.23 in relation to plain walls with more than 1/10 in tension to resist flexure is not included. Anyhow, this is not important as the wall should not be regarded as plain wall; (xvii) In 9.7.1 and 9.7.2 in relation to pile caps and footings, it is stipulated that the minimum steel percentage should refer to Table 9.1 though Table 9.1 is under the heading “Beam”. So the minimum steel percentage of 0.13% for HY bars should be observed. There is no explicit provision in BS8110 for minimum steel in pile caps and footings; (xviii) 9.7.3 in relation to tie beams has included a requirement that the tie beam should be designed for a load of 10 kN/m if the action of compaction machinery can cause effects to the			

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	tie beams. This is not found in BS8110; (xix) 9.8 in relation to design of corbels carries the same content as 6.5.2.2 to 6.5.2.4 and is identical to BS8110 Pt. 1 5.2.7; (xx) 9.9 contains requirements more stringent than BS8110 in detailing with the aim to enhance ductility. The followings are highlighted : (a) 9.9.1.1(a) requires steel percentage in beam > 0.3% and percentage of tensile steel < 2.5%; (b) 9.9.1.1(c) requires anchorage of beam bar into exterior column to commence beyond centre line of column or 8ϕ instead of column face unless the moment plastic hinge can be formed at 500 mm or half beam depth from column face; (c) 9.9.1.1(d) imposes restriction in locations of laps and mechanical couplers – (i) not within column/beam joints, (ii) not within 1 m from potential plastic hinge locations; (iii) reversing stresses exceeding $0.6f_y$ unless with specified confinement by links. In addition, bars be terminated by a 90^o bend or equivalent to the far face of column; (d) 9.9.1.2(e) requires distribution and curtailment of flexural re-bars be attained in critical sections (potential plastic hinge regions); (e) 9.9.1.2(a) states the link spacing in beam < the lesser of 16ϕ and beam width or depth and corner and alternate compression re-bars be anchored by links; (f) 9.9.1.2(b) states that links be adequately			

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	anchored by means of 135° or 180° hooks. Anchorage by 90° hooks or welded cross bars not permitted; (g) 9.9.2.1(a) states min. (0.8%) and max. steel% (4% with increase to 5.2% at lap)in column; (h) 9.9.2.1(a) requires the smallest dia. of any bars in a row > 2/3 of the largest bar; (i) 9.9.2.1(a) limits max. dia. of column re-bar through beam by (eqn 9.7) dependent on beam depth, with increase by 25% if not forming plastic hinge; (j) 9.9.2.1(b) requires spacing of links to longitudinal bars not be spaced further than 1/4 of the adjacent column dimension or 200 mm; (k) 9.9.2.1(c) requires anchorage of column bar into exterior beam or foundation to commence beyond centre line of beam or foundation or 8ϕ instead of interface unless the moment plastic hinge can be formed at 500 mm or half beam depth from column face; (l) 9.9.2.1(d) states restrictions in locations of laps; (m) 9.9.2.2 describes the establishment of "critical regions" in columns where there are extra requirements on links – (i) link spacing in column < the lesser of 6ϕ and least 1/4 of column lateral dimension; (ii) each longitudinal bar be laterally supported by a link passing around the bar and having an included angle < 135°. (Regions other than "critical regions" fallow 9.5.2)			

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
Section 10	The followings are highlighted : (i) 10.2 lists figures for construction tolerances whilst BS8110 refers most of the requirements to other BS; (ii) 10.3.4 in relation to sampling, testing and compliance criteria of concrete. They are extracted from HKB(C)R but with incorporation of 100 mm test cubes. Such provision is not found in BS8110; (iii) The sub-clause on “Concreting in cold weather” in BS8110 is not incorporated. 10.3.7 on “Concreting in hot weather” is modified from BS8110 Pt. 1 6.2.5 (reference to BS deleted); (iv) Table 10.4 is similar to BS8110 Pt. 1 Table 6.1. However the parameter t (temperature) is deleted and the categorization of cement is OPC and “others” instead of the few types of cement in BS8110; (v) 10.3.8.1 contains general requirements for “Formwork and falsework” similar (but not identical) to BS8110 Pt. 1 6.2.6.1; (vi) 10.3.8.2 lists criteria for striking of formwork identical to that in BS8110 Pt. 1 6.2.6.3.1. In addition, provisions for using longer or shorter striking time for PFA concrete and climbing formwork are included; (vii) Minimum striking time in 10.3.8.2 are in accordance with HKB(C)R Table 10 (with the addition of props to cantilever requiring 10 days striking time) instead of BS8110 Pt. 1 Table 6.2. Furthermore, BS8110 Pt. 1 Table 6.2 gives temperature dependent striking time	Pt. 1 2.3, Section 6, Section 7, Section 8	The followings are highlighted : (i) Pt. 1 2.3 lists general requirements for inspection of construction; (ii) References to other BS are often stated in Section 6 and 7; (iii) Provisions of works in extreme temperatures are given which are deleted in CoPConc2004.	The followings are highlighted : (i) The first part of Section 10 of CoPConc2004 mainly stems from HKB(C)R, CS1, CS2 whilst the second part incorporates workmanship requirements listed in BS8110 Pt. 1 Section 6; (ii) (iii)

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	whilst the striking times in CoPConc2004 are not temperature dependent; (viii) The contents of 10.3.9 in relation to surface finish are extracted from BS8110 Pt. 1 6.2.7. However, the general requirements are differently written and the "classes of finish" have been deleted; (ix) 10.3.10 and 10.3.11 in relation to joints are identical to BS8110 Pt. 1 6.2.9 and 6.2.10 though the wordings are different, except the last sentence of 6.2.9 last para. in relation to treating vertical joint as movement joint; (x) 10.4.1 contains general requirements on re-bars to standards CS2 and other acceptable standards whilst BS8110 Pt. 1 7.1 requires conformance to other BS; (xi) 10.4.2 in relation to cutting and bending of re-bars is identical to BS8110 Pt. 1 7.2 except (a) conformance is not restricted to BS but to acceptable standards; and (b) the requirement of pre-heating re-bars at temperatures below 5^{o}C is deleted; (xii) 10.4.3 is effectively identical to BS8110 Pt. 1 7.3 except that the requirement for spacer blocks be of concrete of small aggregates of equal strength to the parental concrete is replaced by spacer blocks to acceptable standards; (xiii) 10.4.6 is effectively identical to BS8110 Pt. 1 7.6 except (a) conformance to BS changed to acceptable standards; (b) detailed descriptions of the types of welding omitted; and (c) requirement to avoid welding in re-bar bends omitted;			

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	(xiv) 10.5.1 is identical to BS8110 Pt. 1 8.1 except conformance to BS is changed to acceptable standards; (xv) 10.5.5.3 in relation to tensioning apparatus of prestressing tendons is effectively identical to BS8110 Pt. 1 8.7.3 except that CoPConc2004 has an additional requirements that apparatus be calibrated within 6 months; (xvi) 10.5.5.4 in relation to pre-tensioning of deflected tendons, compressive and tensile stresses should be ensured not to exceed permissible limits during transfer of prestressing force to the concrete with the release of holding-up and down forces. BS8110 Pt. 1 8.7.4.3 has omitted the compressive forces; (xvii) 10.5.5.5(b) requires anchorage of post tensioning wires to conform to acceptable standards whilst BS8110 Pt. 1 8.7.5.2 requires compliance with BS4447 (xviii) 10.5.5.5(d) in relation to tensioning procedures which is identical to BS8110 Pt. 1 8.7.5.4, the requirement of not carrying out tensioning below 0^{o}C is omitted. Further, the paragraph in BS8110 stipulating that when full force cannot be developed in an element due to breakage, slip during the case when a large no. of tendons is being stressed is omitted in CoPConc2004; (xix) 10.5.7 contains detailed provisions for grouting of prestressed tendons whilst BS8110 Pt. 1 8.9 requires compliance to BS EN 445, 446.			
Section 11 –	This section outlines measures and procedures in	–	No similar provisions in BS8110.	The control in CoPConc2004

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
Quality Assurance and Control	general quality assurance and control, with reference to local practice. The followings are highlighted : (i) Control are on design, construction and completed products; (ii) Control can be by independent organization; (iii) Concrete must be from supplier certified under the Quality Scheme for the Production and Supply of Concrete (QSPSC); (iv) Control on construction includes surveillance measures.			are summaries of local good practice.
Section 12 – Prestressed Concrete	This section is basically identical to Section 4 of BS8110 Pt. 1. The followings are highlighted : (i) 12.1.5 in relation to durability and fire resistance makes reference to previous recommendations in Sections 4 and 10 whilst BS8110 makes reference also to Part 2 of BS8110; (ii) 12.2.3.1 in relation to redistribution of moments is restricted to concrete grade C70, as in consistency with reinforced concrete. BS8110 Pt. 1 4.2.3.1 does not have this limitation. But the BS covers grades up to 40; (iii) The first loading arrangement in 12.3.3 for continuous beam is not found BS8110 Pt. 1 4.3.3. The loading arrangement is in consistency with 5.1.3.2 for reinforced concrete beams. Though not truly adequate (per similar argument as above), CoPConc2004 is more conclusive than BS8110; (iv) 12.3.8.2 gives ultimate concrete stress 7.0 MPa, as similar to r.c. works; (v) 12.8.2.2 in relation to 1000 h relaxation value which is identical to BS8110 Pt. 1 4.8.2.2, "UK"	Pt. 1 Section 4	The provisions are general.	CoPConc2004 follows quite closely the provisions in BS8110 except for minor changes.

HK CoP Structural Use of Concrete 2004		BS8110:1997 (and 1985)		Remark
Clause No.	Contents	Clause No.	Contents	
	has been deleted in description of manufacturer's appropriate certificate; (vi) 12.8.4 and 12.8.5 in relation to shrinkage and creep of concrete make reference to 3.1.8 and 3.1.7 whilst BS8110 Pt 1. 4.8.4 and 4.8.5 list UK data; (vii) 12.10 makes reference to 8.10.2.2 for transmission lengths in pre-stressed members which is titled "transfer of prestress" which is identical to BS8110 Pt. 1 4.10.1 except that the 2nd paragraph of the BS in relation to the difficulty of determination of transmission length has been deleted; (viii) 12.12.3.1(a) is identical to BS8110 Pt. 1 4.12.3.1.1 except that not only protection against corrosion is added. In 12.12.3.1(c), reference for protection against fire is not identical to BS8110;			
Section 13 – Load Tests of Structures or parts of structures	This section contains testing of structures during construction stage under circumstances such as sub-standard works are suspected and visible defects are identified.	–	No similar provisions in BS8110.	

Appendix B

Assessment of Building Accelerations

Assessment of "along wind" acceleration of Buildings (at top residential floor)

Underlying principles :

Two Approaches are outlined in this Appendix :

(i) The first one is based on the assumption that the building will undergo simple harmonic motion under wind loads. Thus the equation of governing simple harmonic motion which is $\ddot{x} = -\omega^2 x$ where $\ddot{x}$ is the acceleration, x is the displacement of the motion, ω is the circular frequency of the building equal to $2\pi f$ (f is the natural frequency of the building) can be used. However, generally only the "dynamic resonant component" of the motion is needed for calculating the acceleration. So if the G factor which is equal to $1+2I_h\sqrt{g_v{}^2 B + \frac{g_f{}^2 SE}{\zeta}}$ in Appendix F of the Wind Code 2004 is used to arrive at a total displacement which can be considered to be made of up of three components : (a) the static part which is 1 in the equation; (b) the dynamic background component which is $2I_h\sqrt{g_v{}^2 B}$; and (c) the dynamic resonant component $2I_h\sqrt{\frac{g_f{}^2 SE}{\zeta}}$, it is the last component that should be multiplied to ω^2 to arrive at the acceleration causing discomfort. So it is only necessary to calculate the displacement due to the dynamic resonant component by multiplying the total displacement by the factor $2I_h\sqrt{\frac{g_f{}^2 SE}{\zeta}} \Bigg/ \left(1+2I_h\sqrt{g_v{}^2 B + \frac{g_f{}^2 SE}{\zeta}}\right)$. Alternatively, the same result can be obtained by multiplying the factor $2I_h\sqrt{\frac{g_f{}^2 SE}{\zeta}}$ to the static wind pressure, i.e. Table 2 of the Hong Kong Wind Code 2004. The circular frequency, ω of the building can either be obtained by detailed dynamic analysis or by some empirical formula such as 460/*h*.

(ii) The second approach is that listed in Australian Wind Code AS/NSZ

1170.2:2002 Appendix G2. It is based on the simple formula $a = \frac{3}{m_0 h^2}\hat{M}_b$ where m_0 is the average mass per unit height of the building, h is the average roof height of the building above ground $\hat{M}_b$ is the resonant component of peak base bending moment. By the "resonant component", the approach is also based on the same principle by using only the dynamic resonant component in deriving acceleration as the factor $g_R\sqrt{\frac{SE}{\zeta}}$ is multiplied to the overturning moment for assessment of acceleration. The parameters comprising m_0 and h are used for assessment of the dynamic properties of the building. In addition, there is a denominator of $1+2g_v I_h$ in the expression for $\hat{M}_b$ in the Australian Code as different from Hong Kong Wind Code, the reason being that the Australian Code is based on $V_{des,\theta}$ which is 3 second gust whilst Hong Kong Code is based on hourly mean wind speed. So this factor should be ignored when using Hong Kong Code which is based on hourly mean speed.

Furthermore, two aspects should also be noted :

(i) The Concrete Code requires the wind load for assessment of acceleration to be 1-in-10 year return period of 10 minutes duration whilst the wind load arrived for structural design in the Hong Kong Wind Code is based on 1-in-50 year return period of hourly duration. For conversion, the formula listed in Appendix B of the Wind Code can be used (as confirmed by some experts that the formula can be used for downward conversion from 1-in-50 year to 1-in-10 year return periods). The 10 minutes mean speed can also be taken as identical to that of hourly mean speed (also confirmed by the experts.) Or alternatively, as a conservative approach, the factor $1-0.62I^{1.27}\ln(t/3600)$ can be applied where I is the turbulence intensity $I = 0.087\left(\frac{h}{500}\right)^{-0.11}$ taken at top of the building and $t = 600\,\text{sec}$;

(ii) The damping ratio recommended in the Wind Code which is 2% is for

ultimate design. A lower ratio may need to be considered for serviceability check including acceleration. Nevertheless, a 10-year return period at damping ratio 2% should be accepted which is the general practice by the Americans. The worked examples follow are therefore based on damping ratio of 2%, though the readers can easily work out the same for damping ratio of 1% under the same principle.

The procedures for estimation of acceleration are demonstrated by 3 worked examples that follow :

<u>Worked Example B-1</u>

For the 40-storey building shown in Figure B-1 which has been analyzed by ETABS, the acceleration of the top residential floor in the for wind in X-direction is to be computed.

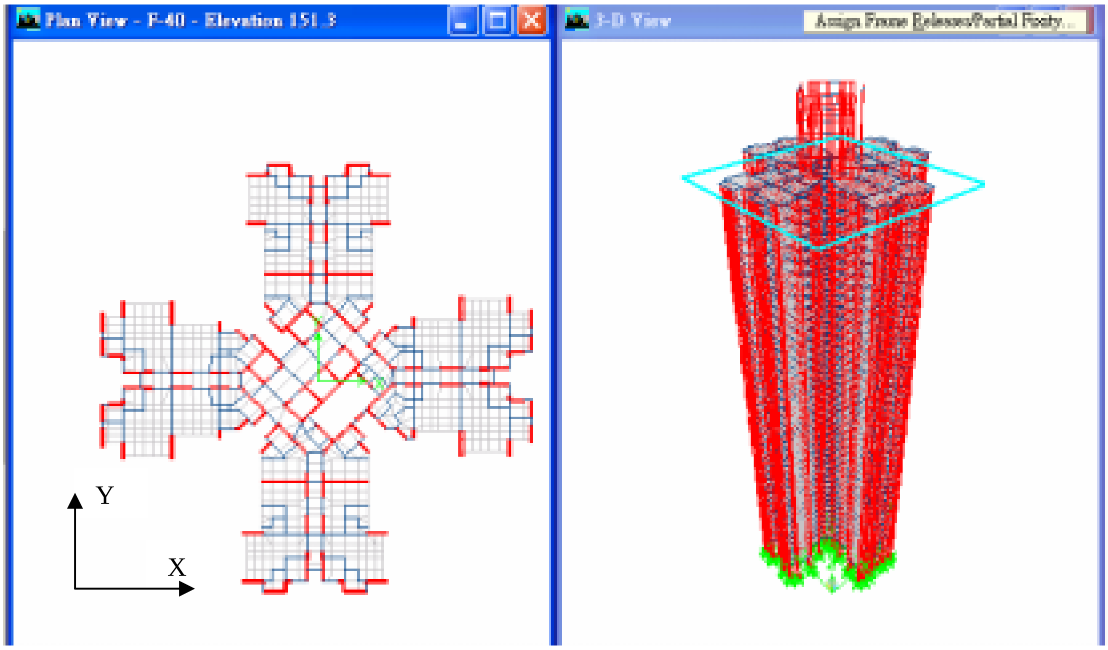


Figure B-1 – 40 storeys building for Worked Example B-1

Data : Building height $h = 121.05\,\text{m}$;

Building plan width and depth are $b = d = 43\,\text{m}$;

Lowest building natural frequencies for the respective motion can be obtained with reference to the modal participating mass ratios as revealed by dynamic analysis in ETABS or other softwares:

$n_{a1} = 0.297\,\text{Hz}$ for rotation about Z axis (torsional)

$n_{a2} = 0.3605\,\text{Hz}$ for translation along Y-direction

$n_{a1} = 0.3892\,\text{Hz}$ for translation along X-direction

For wind in X-direction :

$$I_h = 0.1055\left(\frac{h}{90}\right)^{-0.11} = 0.1055\left(\frac{121.05}{90}\right)^{-0.11} = 0.1021; \quad g_v = 3.7$$

$$g_f = \sqrt{2\ln(3600 n_a)} = \sqrt{2\ln(3600\times 0.3892)} = 3.8066\,;$$

$$L_h = 1000\left(\frac{h}{10}\right)^{0.25} = 1000\left(\frac{121.05}{10}\right)^{0.25} = 1865.35\,;$$

$$B = \frac{1}{1+\dfrac{\sqrt{36h^2+64b^2}}{L_h}} = \frac{1}{1+\dfrac{\sqrt{36\times 121.05^2+64\times 43^2}}{1865.35}} = 0.6989\,;$$

$$\bar{V}_h = \bar{V}_g\left(\frac{h}{500}\right)^{0.11} = 59.5\left(\frac{121.05}{500}\right)^{0.11} = 50.905\,\text{m/sec};$$

$$N = \frac{n_a L_h}{\bar{V}_h} = \frac{0.3892\times 1865.35}{50.905} = 14.262\,;$$

$$S = \frac{1}{\left[1+\dfrac{3.5 n_a h}{\bar{V}_h}\right]\left[1+\dfrac{4 n_a b}{\bar{V}_h}\right]} = \frac{1}{\left[1+\dfrac{3.5\times 0.3892\times 121.05}{50.905}\right]\left[1+\dfrac{4\times 0.3892\times 43}{50.905}\right]} = 0.1019$$

$$E = \frac{0.47N}{\left(2+N^2\right)^{5/6}} = \frac{0.47\times 14.262}{\left(2+14.262^2\right)^{5/6}} = 0.0793\,;$$

$$G = 1+2I_h\sqrt{g_v{}^2 B + \frac{g_f{}^2 SE}{\zeta}} = 1+2\times .1021\sqrt{3.7^2\times 0.6989+\frac{3.8066^2\times 0.1019\times 0.0793}{0.02}}$$

=1.8155;

$$G_{res} = 2I_h\sqrt{\frac{g_f{}^2 SE}{\zeta}} = 2\times .1021\sqrt{\frac{3.8066^2\times 0.1019\times 0.0793}{0.02}} = 0.494\,;$$

$$\therefore G_{res}/G = 0.272$$

Deflection (translation and rotation) of the centre of the top floor calculated in accordance with Appendix G of HKWC2004 is

X-direction : 0.069m

Y-direction : 0.00061m

Z-direction : 0.000154rad

For this symmetrical layout, the Y-deflection and Z-rotation are small and can be ignored.

Procedures :

(i) Conversion from 50 years return period to 10 years return period is by the factor listed in Appendix B of HKWC2004. The factor is

$$\left(\frac{5+\ln(R)}{5+\ln 50}\right)^2=\left(\frac{5+\ln 10}{5+\ln 50}\right)^2=0.6714$$

(ii) Conversion from hourly mean wind speed to 10 minutes mean wind speed is by the factor
$1-0.62I^{1.27}\ln(t/3600)=1-0.62\times 0.1021^{1.27}\ln(600/3600)=1.061$

(iii) So the displacements converted to contain only the dynamic resonant component and to 10 years return period, 10 minutes wind speed can be obtained by multiplying the deflections obtained in accordance with Appendix G of HKWC2004 by the aggregate factor of $0.272\times 0.6714\times 1.061=0.1938$

(iv) The X-deflections for calculation of accelerations is therefore $0.1938\times 0.069=0.0134$ m;

(v) The acceleration of the centre of the block in X-direction is therefore

$(2\pi n_{a3})^2\times 0.0134=(2\pi\times 0.3892)^2\times 0.0134=0.801\,\text{m/sec}^2$ as the fundamental frequency for X-translation is $n_{a3}=0.3892$ Hz listed in the data.

Worked Example B-2

The acceleration of the block in Worked Example B-1 is redone by the Australian Code AS/NSZ 1170.2:2002 Appendix G2 :
Total dead load is 539693 kN and total live load is 160810 kN
Using full dead load and 40% live load for mass computation :
Mass per unit height is

$$m_0=\frac{(539693+160810\times 0.4)\div 9.8}{121.05}\times 10^3=509.165\times 10^3\,\text{kg/m}$$

Overturning moment at 50 years return period is 1114040 kNm when wind is blocing in the X-direction. When the moment is converted to contain only the dynamic resonant component and to 10 years return period, 10 minutes wind speed, it becomes

$\hat{M}_b=0.272\times 0.6714\times 1.061\times 1114040=215857$ kNm, the factors are quoted from Worked Example 1.

So the acceleration in the X-direction is

$$a=\frac{3}{m_0h^2}\hat{M}_b=\frac{3\times 215857\times 10^3}{506.165\times 10^3\times 121.05^2}=0.087\ \text{m/sec}^2.$$

which is greater than that in Worked Example B-1.

Worked Example B-3

Another worked example for finding acceleration in Y-direction is demonstrated for a building shown in Figure B-2 where torsional effect is significant. The building suffers significant torsion where the displacement and acceleration of Point A (at distance 30m in the X-direction and 1.6m in the Y-direction from the centre of rigidity of the building) is most severe. The provision in the Australian Code should be quite limited in this case and therefore not used.

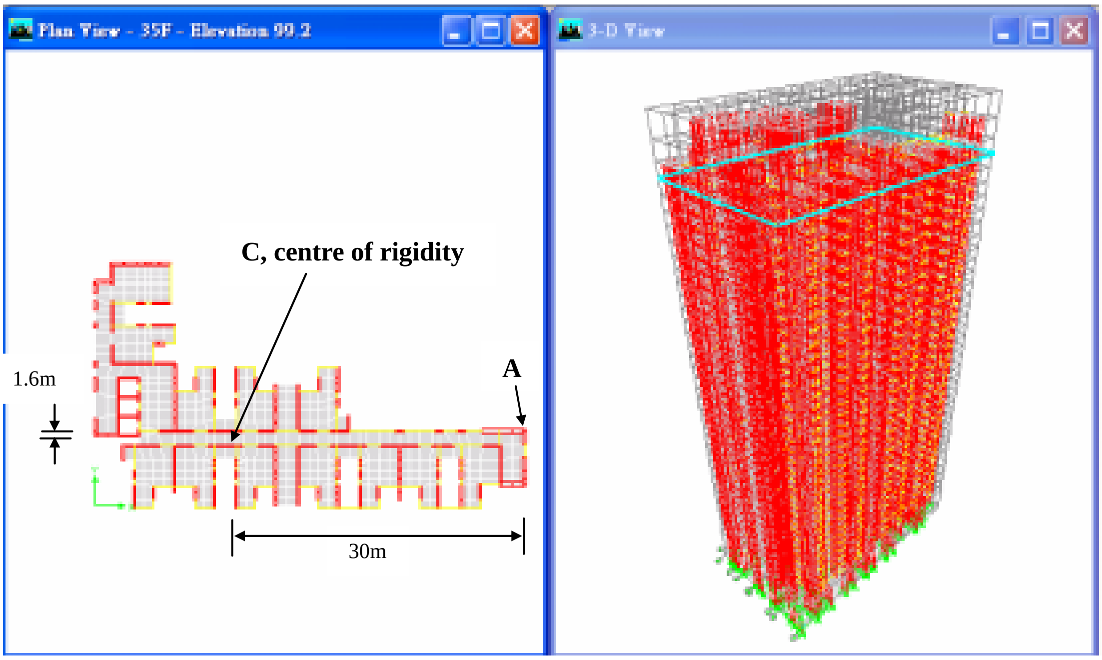


Figure B-2 – Layout of Building where torsional effect is significant

The first 3 fundamental frequencies are listed as follows. They can be read from dynamic analysis of the building by ETABS with reference to the modal participating mass ratios or other softwares. The dynamic resonant component factor $2I_h\sqrt{g_f^{\,2}SE/\zeta}$ are also calculated for the respective direction of motion whilst the dynamic magnification factor G for wind in Y-direction is calculated to be 1.8227.

Direction	Fundamental Periods (sec)	Frequency f (Hz)	Circular frequency $\omega = 2\pi f$ (Hz)	$G_{res} = 2I_h\sqrt{\frac{g_f^{\,2}SE}{\zeta}}$	$\frac{G_{res}}{G}$
Y-direction	2.6598	$n_a = 0.376$	$\omega_1 = 2.3623$	0.512	0.2809
Z-rotation	1.8712	$n_a = 0.5344$	$\omega_2 = 3.3578$	0.3185	0.1747
X-direction	1.5652	$n_a = 0.6389$	$\omega_3 = 4.014$	0.3036	0.1665

Table B-1 – Fundamental Frequencies of Worked Example B-2

The displacements of centre of rigidity of the building at the top residential floor as per analysis in accordance with Appendix G of the Wind Code 2004 after application of the dynamic magnification factor, G is as follows in Table B-2. The corrected values after discount for (i) $G_{resonant}/G$; (ii) 10 minutes duration (factor 1.06); and (iii) 10 years return period (0.6714) are also listed.

	Displacement at Centre of Rigidity	
	before adjustment (read from ETABS output)	after adjustment for (i), (ii), (iii)
X-displacement	0.0262m	0.00311m
Y-displacement	0.119m	0.0238m
Z-rotation	0.00196rad	0.000244rad

Table B-2 – Displacement of Worked Example B-2

The acceleration of the building at its centre of rigidity in X-direction, Y-direction and The acceleration of the respective directions are :

X-direction : $\omega_3^{\,2}\Delta_x = 4.014^2 \times 0.00311 = 0.0501\,\text{m/sec}^2$;

Y-direction : $\omega_1^{\,2}\Delta_y = 2.3623^2 \times 0.0238 = 0.1328\,\text{m/sec}^2$;

Z-direction : $\omega_2^{\,2}\theta_z = 3.3578^2 \times 0.000244 = 0.00275\,\text{rad/sec}^2$;

The linear acceleration at point A will be the vector sum of that in the X and Y-directions, each of which in turn comprises linear component equal to that in the centre of rigidity and a component being magnified by the torsional effect.

Linear acceleration due to Z-rotation acceleration is $0.00275 \times 30 = 0.0825\,\text{m/sec}^2$. Total acceleration in Y-direction is taken as the square root of sum of squares of the direct linear Y acceleration and that induced by rotation. The reason why the acceleration is not taken as algebraic sum of both is because they do not occur at the same frequency. So the total acceleration in Y-direction is

$\sqrt{0.1328^2 + 0.0825^2} = 0.1563\,\text{m/sec}^2$.

Similarly, linear acceleration in the X-direction due to Z-rotation acceleration is $0.00275 \times 1.6 = 0.0044\,\text{m/sec}^2$;

Total acceleration in X-direction is $\sqrt{0.0501^2 + 0.0044^2} = 0.0503\,\text{m/sec}^2$.

The vector sum of the acceleration of point A is therefore

$\sqrt{0.1563^2 + 0.0503^2} = 0.1642 > 0.15\,\text{m/sec}^2$ as required in Cl. 7.3.2 of the Code.

So provisions should be made to reduce the acceleration.

Thus it can be seen that, though the deflection complies with the limit of H/500, the acceleration exceeds the limit of 0.15m/sec^2. However, compliance with the former may be adequate as per Code requirements.

Limitations of the two approaches

It should be borne in mind that the approaches described above are simplified ones. As the approaches are very much based on the assumed natural frequency of the building or arrival of such value by empirical method (the Australian Code), it follows that they should be used in care when the dynamic behaviour of the building is complicated such as having significant cross wind effects or coupling of building modes is significant.

Appendix C

Derivation of Basic Design Formulae of R.C. Beam sections against Flexure

Derivation of Basic Design Formulae of R.C. Beam sections against Bending

The stress strain relationship of a R.C. beam section is illustrated in Figure C-1.

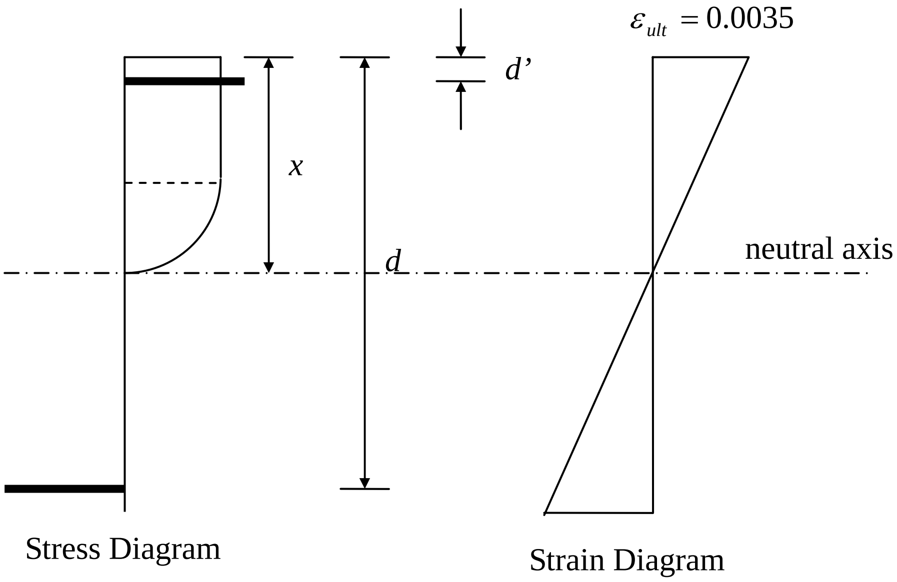


Figure C-1 – Stress Strain diagram for Beam

In Figure C-1 above, the symbols for the neutral axis depth, effective depth, cover to compressive reinforcements are *x*, *d*, and *d'*, as used in BS8110 and the Code.

To derive the contribution of force and moment by the concrete stress block, assume the parabolic portion of the concrete stress block be represented by the equation

$\sigma = A\varepsilon^2 + B\varepsilon$ (where A and B are constants) (Eqn C-1)

So $\dfrac{d\sigma}{d\varepsilon} = 2A\varepsilon + B$ (Eqn C-2)

As $\left.\dfrac{d\sigma}{d\varepsilon}\right|_{\varepsilon=0} = E_c \Rightarrow B = E_c$ where E_c is the tangential Young's Modulus of concrete listed in Table 3.2 of the Code.

Also $\left.\dfrac{d\sigma}{d\varepsilon}\right|_{\varepsilon=\varepsilon_0} = 0 \Rightarrow 2A\varepsilon_0 + B = 0 \Rightarrow A = -\dfrac{B}{2\varepsilon_0} = -\dfrac{E_c}{2\varepsilon_0}$ (Eqn C-3)

As $\sigma = 0.67\dfrac{f_{cu}}{\gamma_m}$ when $\varepsilon = \varepsilon_0$

$\therefore 0.67\dfrac{f_{cu}}{\gamma_m \varepsilon_0{}^2} - \dfrac{E_c}{\varepsilon_0} = -\dfrac{E_c}{2\varepsilon_0} \Rightarrow \dfrac{0.67 f_{cu}}{\gamma_m \varepsilon_0} = \dfrac{E_c}{2} \Rightarrow \varepsilon_0 = \dfrac{1.34 f_{cu}}{E_c \gamma_m}$ (Eqn (C-4)

(accords with 3.14 of the Concrete Code Handbook)

$A = -\frac{E_c}{2\varepsilon_0}$ where $\varepsilon_0 = \frac{1.34 f_{cu}}{E_c \gamma_m}$

So the equation of the parabola is $\sigma = -\frac{E_c}{2\varepsilon_0}\varepsilon^2 + E_c \varepsilon$ for $\varepsilon \le \varepsilon_0$

Consider the linear strain distribution

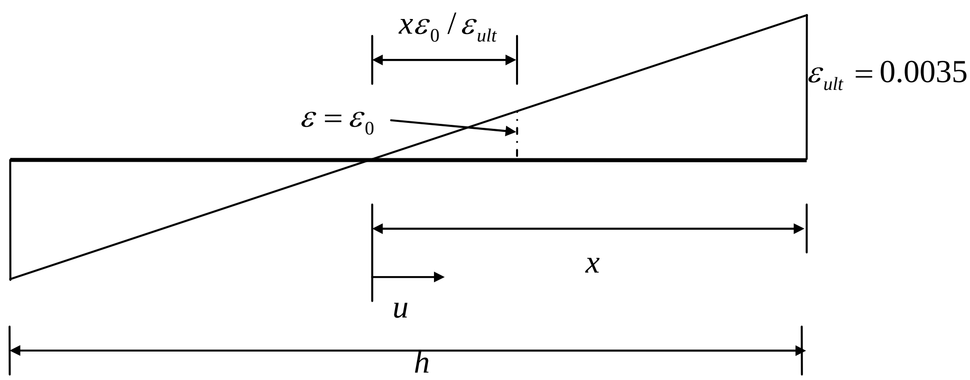


Figure C-2 – Strain diagram across concrete section

At distance u from the neutral axis, $\varepsilon = \varepsilon_{ult}\frac{u}{x}$

So stress at u from the neutral axis up to $x\frac{\varepsilon_0}{\varepsilon_{ult}}$ is

$$\sigma = -\frac{E_c}{2\varepsilon_0}\varepsilon^2 + E_c\varepsilon = -\frac{E_c}{2\varepsilon_0}\left(\varepsilon_{ult}\frac{u}{x}\right)^2 + E_c\left(\varepsilon_{ult}\frac{u}{x}\right) = -\frac{E_c\varepsilon_{ult}^2}{2\varepsilon_0 x^2}u^2 + \frac{E_c\varepsilon_{ult}}{x}u \qquad \text{(Eqn C-5)}$$

Based on (Eqn C-5), the stress strain profiles can be determined. A plot for grade 35 is included for illustration :

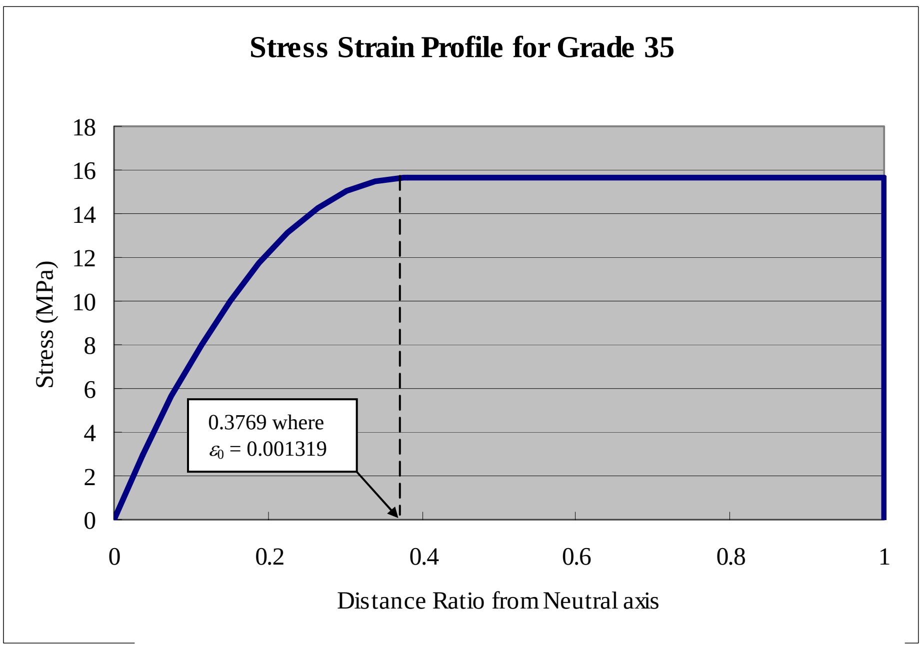


Figure C-3 – Stress strain profile of grades 35

Sectional Design of rectangular Section to rigorous stress strain profile

Making use of the properties of parabola in Figure C-4 offered by the parabolic section as F_{c1} given by

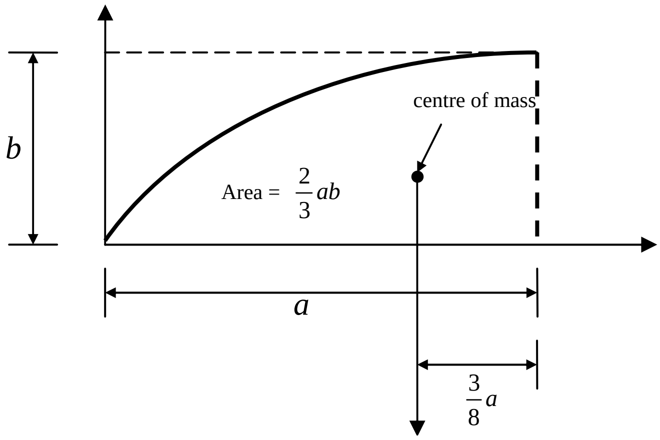


Figure C-4 – Geometrical Properties of Parabola

$$F_{c1} = b\frac{2}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}x0.67\frac{f_{cu}}{\gamma_m} = \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m\varepsilon_{ult}}bx \qquad \text{(Eqn C-6)}$$

and the moment exerted by F_{c1} about centre line of the whole section

$$M_{c1} = F_{c1}\left[\frac{h}{2} - x\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right) - \frac{3}{8}x\frac{\varepsilon_0}{\varepsilon_{ult}}\right] = F_{c1}\left[\frac{h}{2} - x\left(1 - \frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right] \qquad \text{(Eqn C-7)}$$

The force by the straight portion is

$$F_{c2} = \frac{0.67f_{cu}}{\gamma_m}\left(x - x\frac{\varepsilon_0}{\varepsilon_{ult}}\right)b = \frac{0.67f_{cu}bx}{\gamma_m}\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right) \qquad \text{(Eqn C-8)}$$

The moment offered by the constant part about the centre line of the whole section is

$$M_{c2} = F_{c2}\left[\frac{h}{2} - \left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{2}\right] \qquad \text{(Eqn C-9)}$$

The compressive force by concrete as stipulated in (Eqn C-6) and (Eqn C-8) is

$$F_c = F_{c1} + F_{c2} = \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m\varepsilon_{ult}}bx + \frac{0.67f_{cu}bx}{\gamma_m}\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right) = \frac{0.67f_{cu}bx}{3\gamma_m}\left(3 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)$$

For singly reinforcing sections, moment by concrete about the level of the tensile steel is, by (Eqn C-7) and (C-9)

$$M = M_{c1} + M_{c2} = F_{c1}\left[d - x\left(1 - \frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right] + F_{c2}\left[d - \left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{2}\right]$$

$$= \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m \varepsilon_{ult}} bx\left[d - x\left(1 - \frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right] + \frac{0.67 f_{cu} bx}{\gamma_m}\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\left[d - \left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{2}\right]$$

$$\Rightarrow \frac{M}{bd^2} = \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m \varepsilon_{ult}}\frac{x}{d}\left[1 - \frac{x}{d}\left(1 - \frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right] + \frac{0.67 f_{cu}}{\gamma_m}\frac{x}{d}\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\left[1 - \frac{1}{2}\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{d}\right]$$

$$\Rightarrow \frac{M}{bd^2} = \frac{0.67 f_{cu}}{\gamma_m}\frac{x}{d}\left\{\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right) + \left[-\frac{1}{2} + \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\frac{x}{d}\right\}$$

$$\Rightarrow \frac{0.67 f_{cu}}{\gamma_m}\left[-\frac{1}{2} + \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{d}\right)^2 + \frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{d} - \frac{M}{bd^2} = 0$$

(Eqn C-10)

which is a quadratic equation in $\frac{x}{d}$.

As $\frac{x}{d}$ is limited to 0.5 for singly reinforcing sections for grades up to 45 under moment distribution not greater than 10% (Clause 6.1.2.4 of the Code), by (Eqn C-10), $\frac{M}{bd^2 f_{cu}}$ will be limited to K' values listed as

$K' = 0.154$ for grade 30
$K' = 0.152$ for grade 35
$K' = 0.151$ for grade 40
$K' = 0.150$ for grade 45

which are all smaller than 0.156 under the simplified stress block.

However, for $45 < f_{cu} \le 70$ where $\frac{x}{d}$ is limited to 0.4 for singly reinforcing sections under moment distribution not greater than 10% (Clause 6.1.2.4 of the Code), again by (Eqn 3-1) $\frac{M}{bd^2 f_{cu}}$ will be limited to

$K' = 0.125$ for grade 50
$K' = 0.123$ for grade 60
$K' = 0.121$ for grade 70

which are instead greater than 0.120 under the simplified stress block. This is because at concrete grade > 45, the Code has limited the rectangular stress block to 0.8 times of the neutral axis depth.

With the $\frac{x}{d}$ analyzed by (Eqn C-9), the forces in concrete

$$F_c = F_{c1} + F_{c2} = \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m \varepsilon_{ult}} bx + \frac{0.67 f_{cu} bx}{\gamma_m}\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right) \Rightarrow \frac{F_c}{bd} = \frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{d}$$

can be calculated which will be equal to the required force to be provided by steel, thus

$$0.87 f_y \frac{A_{st}}{bd} = \frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{d} \Rightarrow \frac{A_{st}}{bd} = \frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{d}$$

(Eqn C-11)

When $\frac{M}{bd^2 f_{cu}}$ exceeds the limited value for single reinforcement. Compression reinforcements at d' from the surface of the compression side should be added. The compression reinforcements will take up the difference between the applied moment and $K'bd^2$

$$0.87 f_y \frac{A_{sc}}{bd}\left(1 - \frac{d'}{d}\right) = \left(\frac{M}{bd^2 f_{cu}} - K'\right) \Rightarrow \frac{A_{sc}}{bd} = \frac{\left(\frac{M}{bd^2 f_{cu}} - K'\right) f_{cu}}{0.87 f_y \left(1 - \frac{d'}{d}\right)} \qquad \text{(Eqn C-12)}$$

And the same amount of steel will be added to the tensile steel.

$$\frac{A_{st}}{bd} = \frac{1}{0.87 f_y}\frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\eta + \frac{\left(\frac{M}{bd^2 f_{cu}} - K'\right) f_{cu}}{0.87 f_y \left(1 - \frac{d'}{d}\right)} \qquad \text{(Eqn C-13)}$$

where η is the limit of $\frac{x}{d}$ ratio which is 0.5 for grade 45 and below and 0.4 for grades up to and including 70.

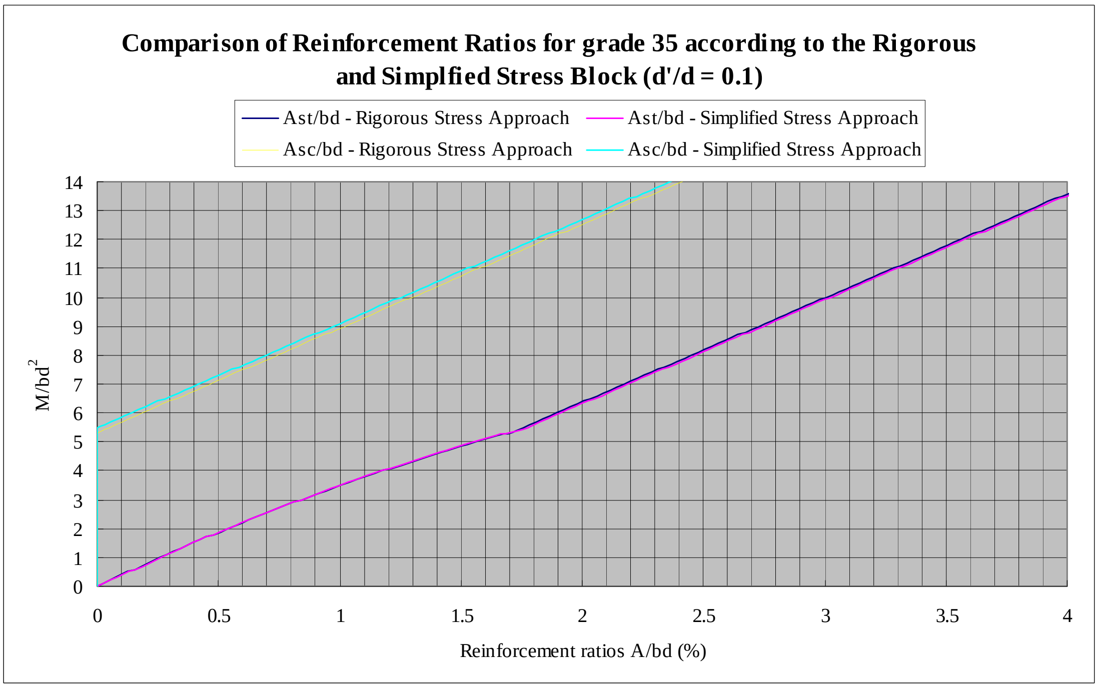
Furthermore, there is a limitation of lever arm ratio not to exceed 0.95 which requires

$$\frac{\frac{0.67 f_{cu}}{\gamma_m}\left[-\frac{1}{2} + \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{d}\right)^2 + \frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{d}}{\frac{0.67 f_{cu}}{\gamma_n}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{d}} \le 0.95$$

$$\Rightarrow \frac{x}{d} \geq \frac{0.05\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)}{\left[\frac{1}{2}-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}+\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]} \qquad \text{(Eqn C-14)}$$

Thus the lower limits for the neutral axis depth ratios are 0.112, 0.113, 0.114 and 0.115 for grades 30, 35, 40, 45 respectively. Thus for small moments acting on beam with $\frac{x}{d}$ not fulfilling (Eqn C-14), $A_{st} = \frac{M}{0.87 f_y \times 0.95d}$ (Eqn C-15)

As illustration for comparison between the rigorous and simplified stress block approaches, plots of $\frac{M}{bd^2}$ against steel percentages for grade 35 is plotted as



It can be seen that the differences are very small, maximum error is 1%.

However, for high grade concrete where the K' values are significantly reduced in the rigorous stress block approach (mainly due to the switching of upper limits of the neutral axis depth ratios from 0.5 to 0.4 and 0.33 for high grade concrete), the differences are much more significant for doubly reinforced sections, as can be seen from the Design Charts enclosed in this Appendix that compressive steel ratios increased when concrete grade switches from grade 45 to 50 as the neutral axis depth ratio changes from 0.5 to 0.4.

Determination of reinforcements for Flanged Beam Section – T- or L-Sections

For simplicity, only the simplified stress block in accordance with Figure 6.1 of the Code is adopted in the following derivation. The symbol η is used to denote the ratio of the length of the simplified stress block to the neutral axis depth. Thus $\eta = 0.9$ for $f_{cu} \le 45$; $\eta = 0.8$ for $45 < f_{cu} \le 70$; $\eta = 0.72$ for $70 < f_{cu} \le 100$.

The exercise is first carried out by treating the width of the beam as b_{eff} and analyze the beam as if it is a rectangular section. If η of neutral axis depth is within the depth of the flange, i.e. $\eta \frac{x}{d} \le \frac{h_f}{d}$, the reinforcement so arrived is adequate for the section. The requirement for $\eta \frac{x}{d} \le \frac{h_f}{d}$ is

$$\eta \frac{x}{d} = 1 - \sqrt{1 - \frac{K}{0.225}} \le \frac{h_f}{d} \qquad \text{(Eqn C-15)}$$

The lever arm $z = d - \frac{1}{2}\eta x$

$$\Rightarrow \frac{z}{d} = 1 - \frac{1}{2}\eta \frac{x}{d} = 1 - \frac{1}{2}\left(1 - \sqrt{1 - \frac{K}{0.225}}\right) = 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \qquad \text{(Eqn C-16)}$$

If, however, $\eta \frac{x}{d} > \frac{h_f}{d}$, the section has to be reconsidered with reference to Figure C-5.

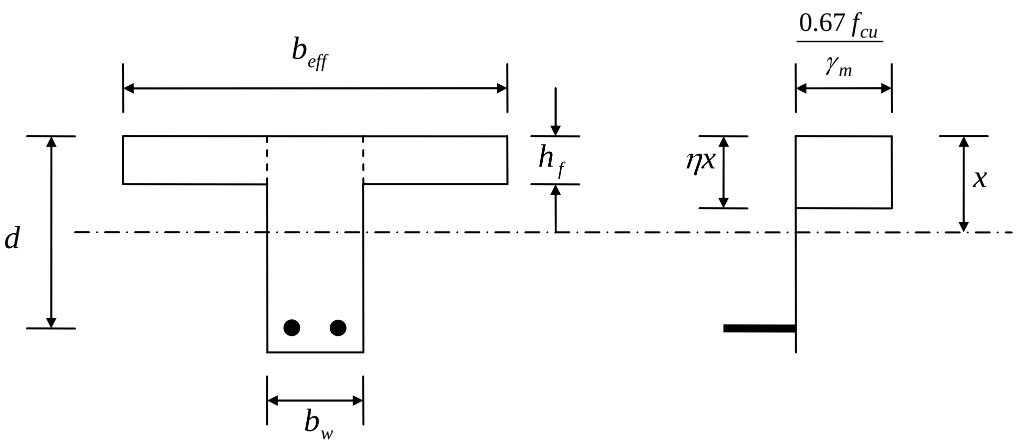


Figure C-5 – Analysis of a T or L beam section

For singly reinforced sections, taking moment about the level of the reinforcing steel,

$$M = \frac{0.67 f_{cu}}{\gamma_m}(b_{eff} - b_w)h_f\left(d - \frac{h_f}{2}\right) + \frac{0.67 f_{cu}}{\gamma_m} b_w(\eta x)\left(d - \frac{\eta}{2}x\right)$$

$$\Rightarrow \frac{M}{b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m}\left(\frac{b_{eff}}{b_w} - 1\right)\frac{h_f}{d}\left(1 - \frac{1}{2}\frac{h_f}{d}\right) + \frac{0.67 f_{cu}}{\gamma_m}\left(\eta\frac{x}{d}\right)\left(1 - \frac{\eta}{2}\frac{x}{d}\right) \quad \text{(Eqn C-17)}$$

Putting $\frac{M_f}{b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m}\frac{h_f}{d}\left(\frac{b_{eff}}{b_w} - 1\right)\left(1 - \frac{1}{2}\frac{h_f}{d}\right)$ (Eqn C-18)

The equation is in fact the contribution of the moment of resistance of the section by the flange, (Eqn C-17) becomes

$$\frac{0.67 f_{cu}}{\gamma_m}\frac{\eta^2}{2}\left(\frac{x}{d}\right)^2 - \frac{0.67 f_{cu}}{\gamma_m}\eta\frac{x}{d} + \frac{M - M_f}{b_w d^2} = 0 \quad \text{(Eqn C-19)}$$

which is a quadratic equation for solution of $\frac{x}{d}$ where M_f can be predetermined by (Eqn C-18). Provided $\frac{x}{d} \le \varphi$ where $\varphi = 0.5$ for $f_{cu} > 45$; 0.4 for $f_{cu} > 70$ and 0.33 for $f_{cu} > 100$, single reinforcement be provided by the following equation which is derived by balancing the steel force and the concrete force.

$$0.87 f_y A_{st} = \frac{0.67 f_{cu}}{\gamma_m}\left[(b_{eff} - b_w)h_f + b_w \eta x\right] \Rightarrow \frac{A_{st}}{b_w d} = \frac{0.67 f_{cu}}{\gamma_m 0.87 f_y}\left[\left(\frac{b_{eff}}{b_w} - 1\right)\frac{h_f}{d} + \eta\frac{x}{d}\right]$$

(Eqn C-20)

If $\frac{x}{d} = \varphi$, the maximum moment of resistance by concrete is reached which is (by taking moment about the tensile steel level)

$$M_{c\max} = M_{f\max} + M_{b\max} = \frac{0.67 f_{cu}}{\gamma_m}\left[(b_{eff} - b_w)h_f\left(d - \frac{h_f}{2}\right) + b_w \eta\varphi d\left(d - \frac{1}{2}\varphi d\right)\right]$$

$$\Rightarrow K' = \frac{M_c}{b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m}\left[\left(\frac{b_{eff}}{b_w} - 1\right)\frac{h_f}{d}\left(1 - \frac{1}{2}\frac{h_f}{d}\right) + \eta\varphi\left(1 - \frac{1}{2}\varphi\right)\right] \quad \text{(Eqn C-21)}$$

and tensile steel required will be, by (Eqn C-20)

$$\frac{A_{st,bal}}{b_w d} = \frac{0.67 f_{cu}}{\gamma_m 0.87 f_y}\left[\left(\frac{b_{eff}}{b_w} - 1\right)\frac{h_f}{d} + \eta\varphi\right] \quad \text{(Eqn C-22)}$$

If the applied moment exceeds M_c, the “excess moment” will be taken up compressive steel A_{sc} with cover to reinforcement c'.

$$0.87 f_y A_{sc}(d - d') = M - M_c$$

$$\Rightarrow \frac{A_{sc}}{b_w d} = \frac{M - M_c}{b_w d^2 (1 - d'/d) 0.87 f_y} = \frac{1}{0.87 f_y (1 - d'/d)} \left[\frac{M}{b_w d^2} - \frac{M_c}{b_w d^2} \right]$$

$$= \frac{1}{0.87 f_y (1 - d'/d)} \left[\frac{M}{b_w d^2} - \frac{0.67 f_{cu}}{\gamma_m} \left[\left(\frac{b_{eff}}{b_w} - 1 \right) \frac{h_f}{d} \left(1 - \frac{1}{2} \frac{h_f}{d} \right) + \eta\varphi \left(1 - \frac{1}{2}\varphi \right) \right] \right]$$

(Eqn C-23)

The total tensile steel will be

$$\frac{A_{st}}{b_w d} = \frac{A_{st,bal}}{b_w d} + \frac{A_{sc}}{b_w d}$$

The followings are stated for $f_{cu} \le 45$ where $\eta = 0.9$ and $\varphi = 0.5$ which is most commonly used in flexural members:

For $\eta \times$neutral axis depth below flange, (Eqn C-19) can be written as :

$$0.1809 f_{cu} \left(\frac{x}{d} \right)^2 - 0.402 f_{cu} \frac{x}{d} + \frac{M - M_f}{b_w d^2} = 0 \qquad \text{(Eqn C-24)}$$

where $0.9x \ge h_f$ and $\frac{M_f}{b_w d^2} = \frac{0.67 f_{cu}}{\gamma_m} \frac{h_f}{d} \left(\frac{b_{eff}}{b_w} - 1 \right) \left(1 - \frac{1}{2} \frac{h_f}{d} \right)$

$$\frac{A_{st}}{b_w d} = \frac{0.67 f_{cu}}{\gamma_m 0.87 f_y} \left[\left(\frac{b_{eff}}{b_w} - 1 \right) \frac{h_f}{d} + 0.9 \frac{x}{d} \right] \qquad \text{(Eqn C-24)}$$

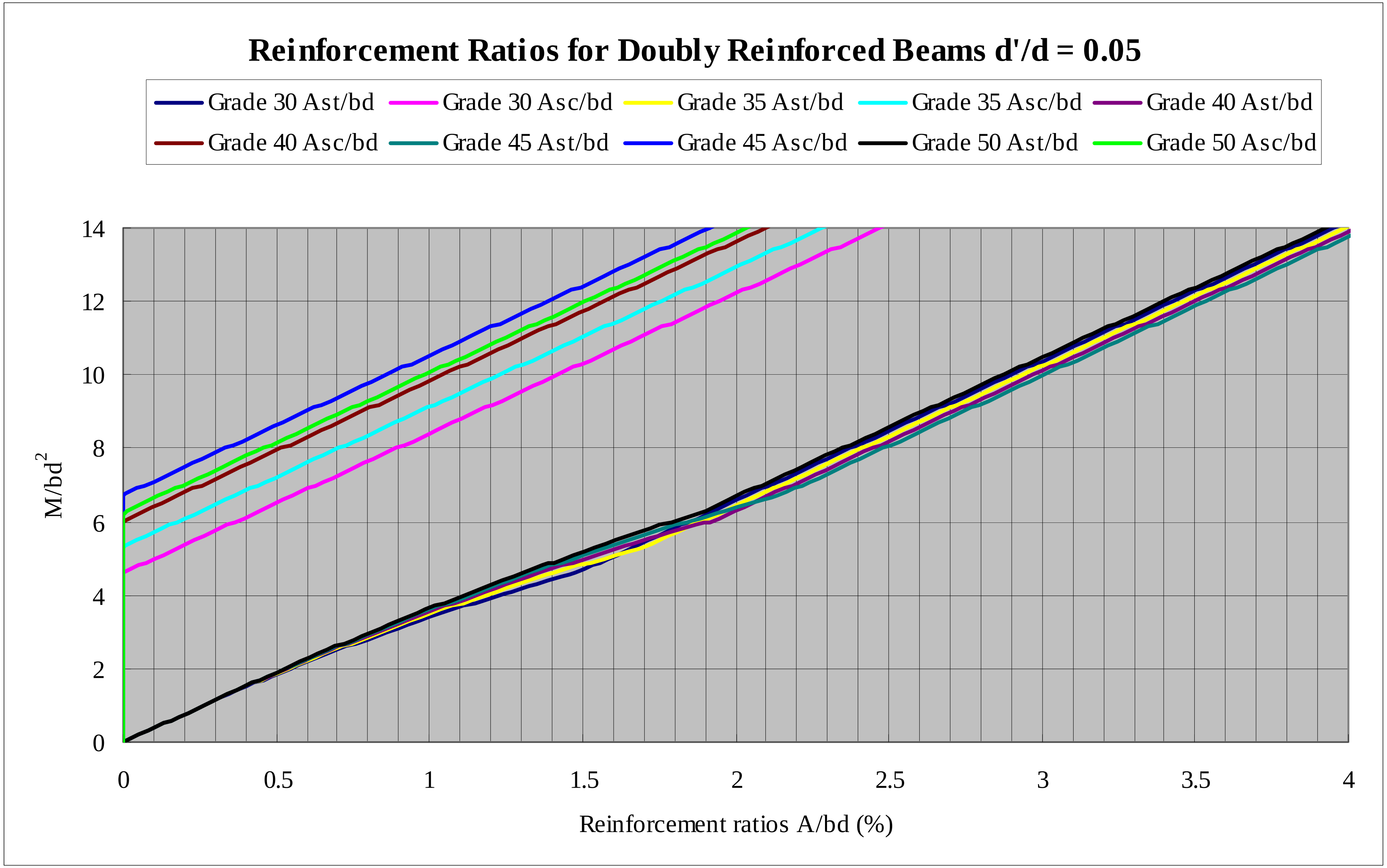
For double reinforcements where $x > 0.5d$ by (Eqn C-24), substituting $\eta = 0.9$ and $\varphi = 0.5$ into (Eqn C-23)

$$\frac{A_{sc}}{b_w d} = \frac{1}{0.87 f_y (1 - d'/d)} \left[\frac{M}{b_w d^2} - \frac{0.67 f_{cu}}{\gamma_m} \left[\left(\frac{b_{eff}}{b_w} - 1 \right) \frac{h_f}{d} \left(1 - \frac{1}{2} \frac{h_f}{d} \right) + 0.3375 \right] \right]$$

(Eqn C-25)

By (Eqn C-22),

$$\frac{A_{st}}{b_w d} = \frac{A_{st,bal}}{b_w d} + \frac{A_{sc}}{b_w d} = \frac{0.67 f_{cu}}{\gamma_m 0.87 f_y} \left[\left(\frac{b_{eff}}{b_w} - 1 \right) \frac{h_f}{d} + 0.45 \right] + \frac{A_{sc}}{b_w d} \qquad \text{(Eqn C-26)}$$



Reinforcement Ratios for Doubly Reinforced Beams d'/d = 0.05
Grade 30 Ast/bd
Grade 30 Asc/bd
Grade 35 Ast/bd
Grade 35 Asc/bd
Grade 40 Ast/bd
Grade 40 Asc/bd
Grade 45 Ast/bd
Grade 45 Asc/bd
Grade 50 Ast/bd
Grade 50 Asc/bd
M/bd^2
0
2
4
6
8
10
12
14
0
0.5
1
1.5
2
2.5
3
3.5
4
Reinforcement ratios A/bd (%)

Chart C-1

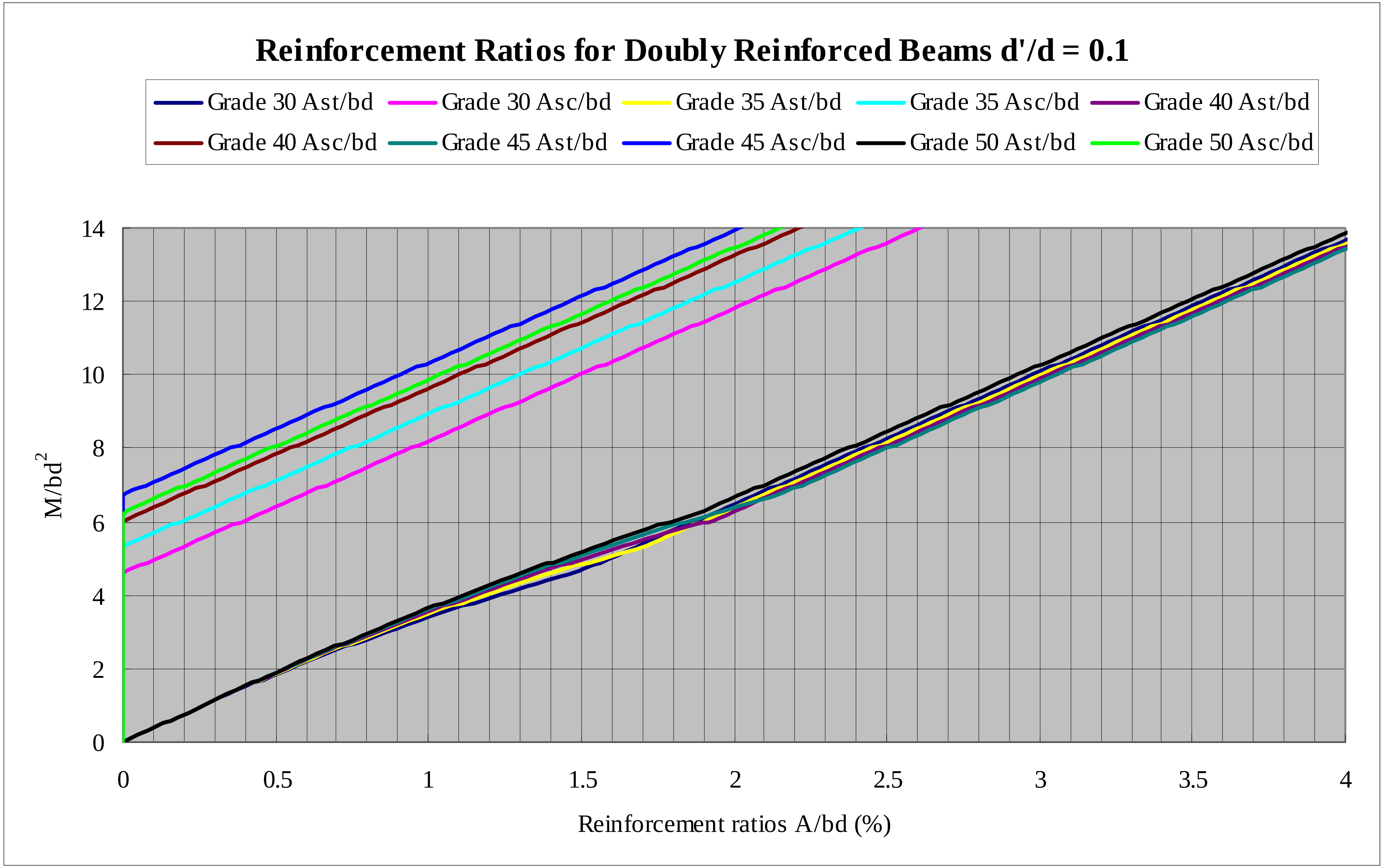

Reinforcement Ratios for Doubly Reinforced Beams d'/d = 0.1
Grade 30 Ast/bd
Grade 30 Asc/bd
Grade 35 Ast/bd
Grade 35 Asc/bd
Grade 40 Ast/bd
Grade 40 Asc/bd
Grade 45 Ast/bd
Grade 45 Asc/bd
Grade 50 Ast/bd
Grade 50 Asc/bd
M/bd²
0
2
4
6
8
10
12
14
0
0.5
1
1.5
2
2.5
3
3.5
4
Reinforcement ratios A/bd (%)

Chart C-2

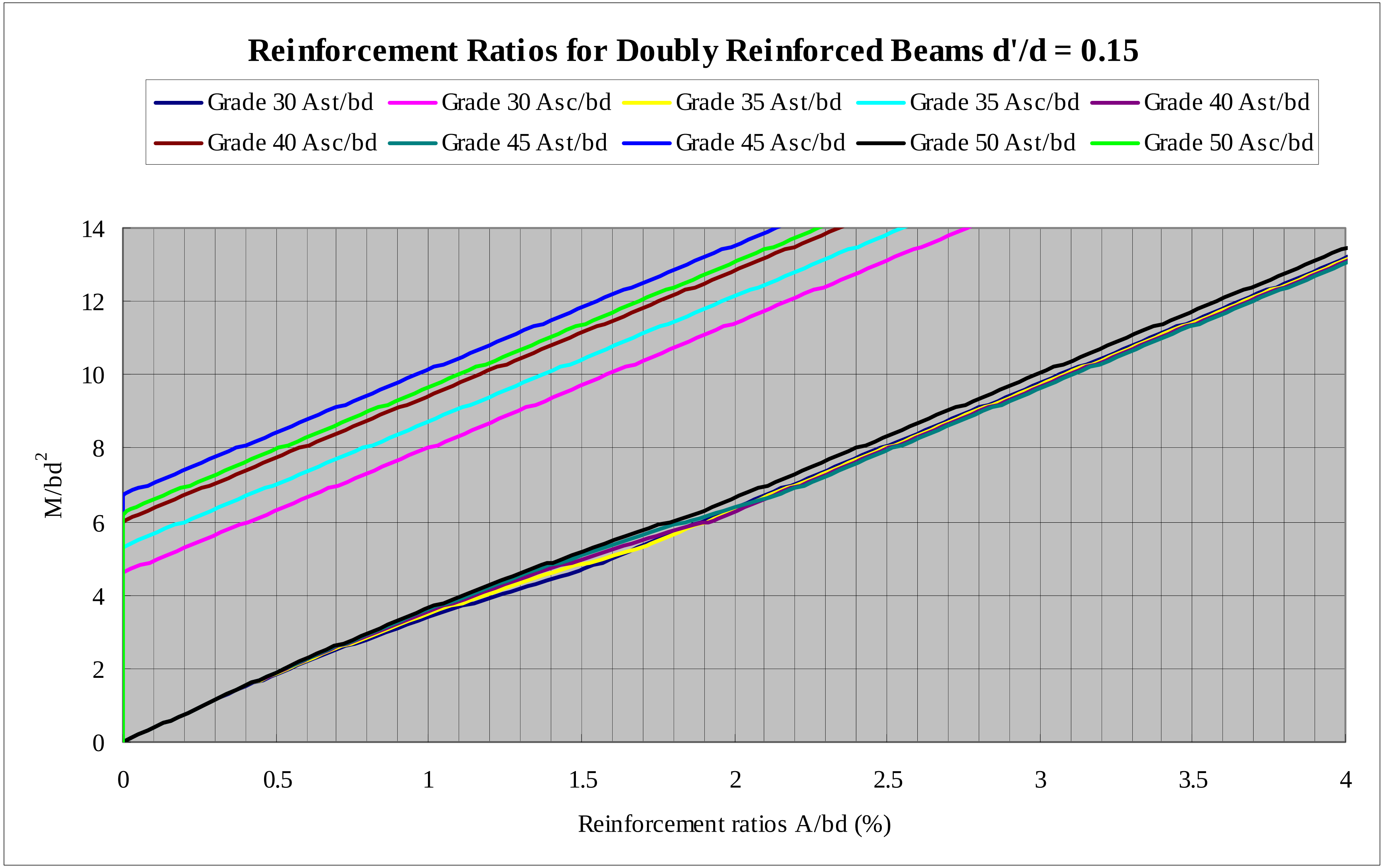
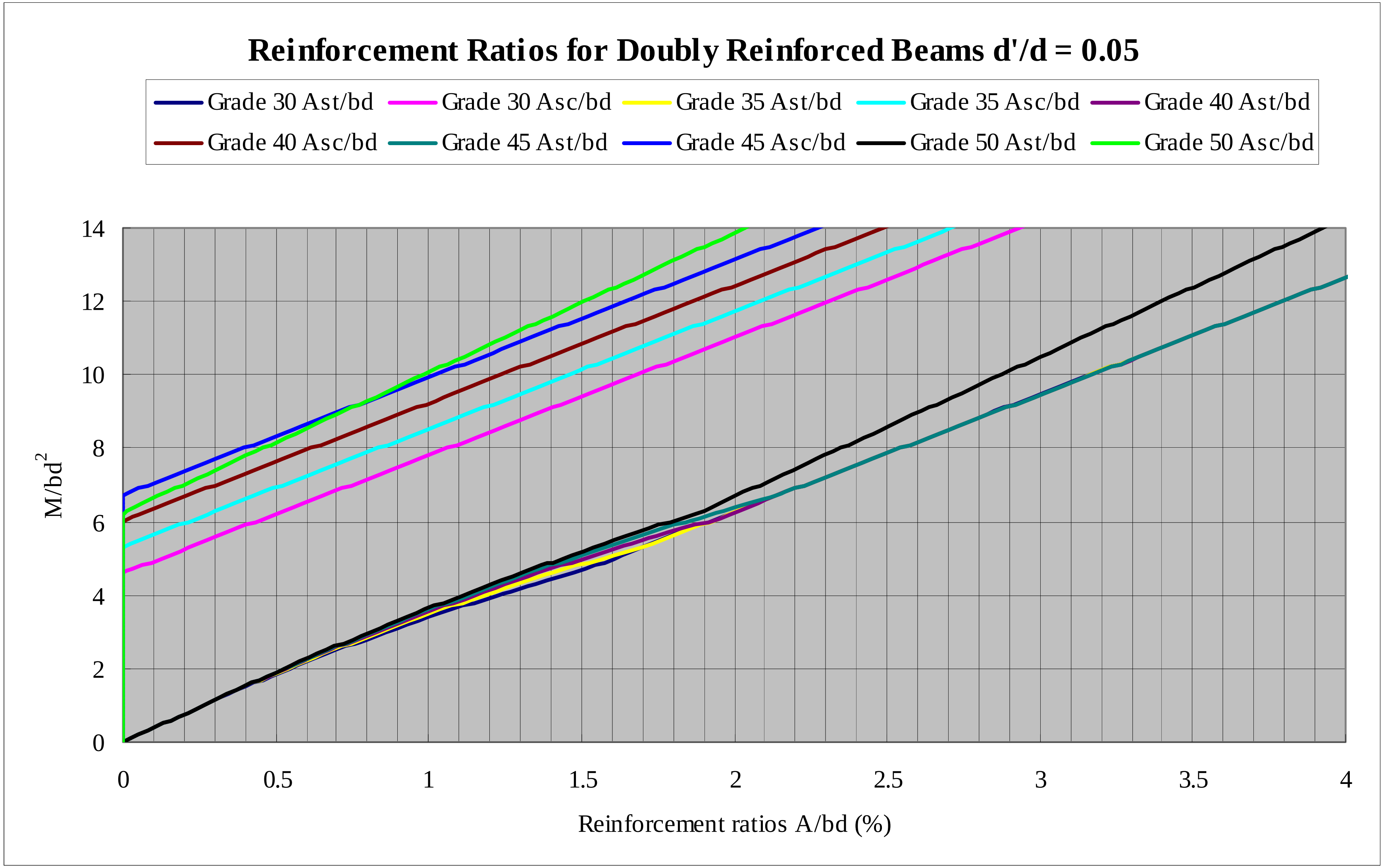

Reinforcement Ratios for Doubly Reinforced Beams d'/d = 0.15
Grade 30 Ast/bd
Grade 30 Asc/bd
Grade 35 Ast/bd
Grade 35 Asc/bd
Grade 40 Ast/bd
Grade 40 Asc/bd
Grade 45 Ast/bd
Grade 45 Asc/bd
Grade 50 Ast/bd
Grade 50 Asc/bd
M/bd2
0
2
4
6
8
10
12
14
0
0.5
1
1.5
2
2.5
3
3.5
4
Reinforcement ratios A/bd (%)

Chart C-3



Reinforcement Ratios for Doubly Reinforced Beams d'/d = 0.05
Grade 30 Ast/bd
Grade 30 Asc/bd
Grade 35 Ast/bd
Grade 35 Asc/bd
Grade 40 Ast/bd
Grade 40 Asc/bd
Grade 45 Ast/bd
Grade 45 Asc/bd
Grade 50 Ast/bd
Grade 50 Asc/bd
M/bd2
Reinforcement ratios A/bd (%)

Chart C-4

Appendix D

Underlying Theory and Design Principles for Plate Bending Element

Underlying Theory and Design Principles for Plate Bending Element

By the finite element method, a plate bending structure is idealized as an assembly of discrete elements joined at nodes. Through the analysis, "node forces" at each node of an element, each of which comprises two bending moments and a shear force can be obtained, the summation of which will balance the applied load at the node. Figures D-1a and D-1b illustrates the phenomena.

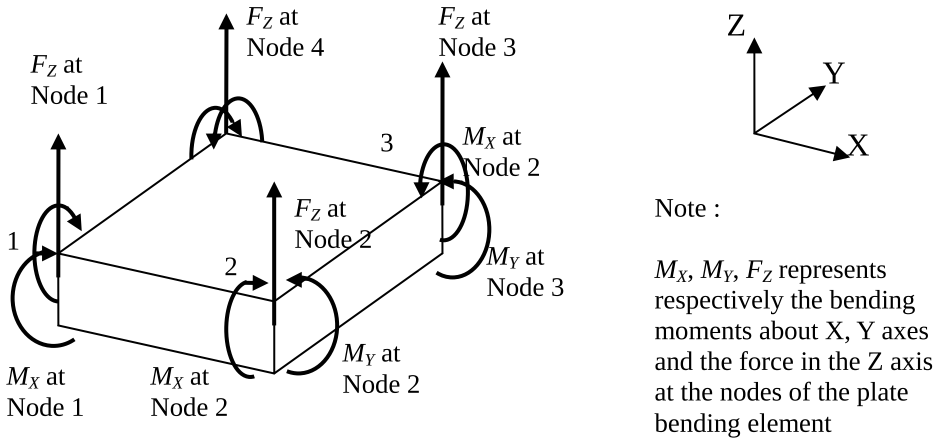


Figure D-1a – Diagrammatic illustration of the Node Forces at the four Nodes of a Plate Bending Element 1234.

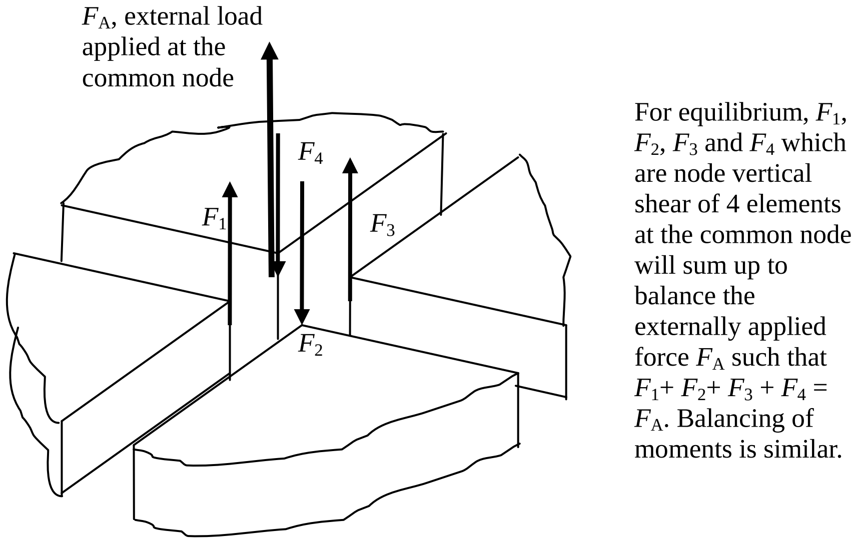


Figure D-1b – Diagrammatic illustration of balancing of Node shear forces at a common node to 2 or more adjoining elements. The four elements joined at the common node are displaced diagrammatically for clarity.

The finite element method goes further to analyze the “stresses” within the discrete elements. It should be noted that “stresss” is a terminology of the finite element method which refer to bending moments, twisting moments and shear forces per unit width in plate bending element. They represent the actual internal forces within the plate structure in accordance with the plate bending theory. R.H. Woods (1968) has developed the famous Wood-Armer Equations to convert the bending moments and twisting moments (both are moments per unit width) at any point to “design moments” in two directions for structural design purpose.

Outline of the plate bending theory

Apart from bending moment in two mutually perpendicular directions as well known by engineers, a twisting moment can be proved to be in existence by the plate bending theory. The bending and twisting moments constitutes a “moment field” which represents the actual structural behaviour of a plate bending structure. The existence of the twisting moment and its nature are discussed in the followings. Consider a triangular element in a plate bending structure with two of its sides aligning with the global X and Y directions as shown in Figure D-2 where moments M_X and M_Y (both in kNm per m width) are acting respectively about X and Y. A moment M_B will generally be acting on the hypotenuse making an angle of θ with the X-axis as shown to achieve equilibrium. However, as the resultant of M_X and M_Y does not necessarily align with M_B, so there will generally be a moment acting in the perpendicular direction of M_B to achieve equilibrium which is denoted as M_T. The vector direction of M_T is normal to the face of the hypotenuse. So instead of “bending” the element like M_X, M_Y and M_B which produces flexural stresses, it “twists” the element and produce shear stress in the in-plane direction. The shear stress will follow a triangular pattern as shown in Figure D-2 for stress-strain compatibility. M_T is therefore termed the “twisting moment”. Furthermore, in order to achieve rotational equilibrium about an axis out of plane, the shear stress will have to be “complementary”. As the hypotenuse can be in any directions of the plate structure, it follows that at any point in the plate bending structure, there will generally be two bending moments, say M_X and M_Y in two mutually perpendicular directions coupled with a complementary twisting moment M_{XY} as indicated in Figure 11a. The phenomenon is in exact analogy to the in-plane stress problem where generally two direct stresses coupled with a shear stress exist and these components vary with directions. The equations relating M_B, M_T with M_X, M_Y, M_{XY} and θ derived from equilibrium conditions are stated as follows:

$$M_B = \frac{1}{2}\left(M_X + M_Y\right) + \frac{1}{2}\left(M_X - M_Y\right)\cos 2\theta + M_{XY}\sin 2\theta$$

$$M_T = \frac{1}{2}\left(M_X - M_Y\right)\sin 2\theta - M_{XY}\sin 2\theta$$

(Eqn D-1)

In addition, if θ is so varied that M_T vanishes when $\theta = \phi$, then the element will be having pure bending in the direction. The moments will be termed the “principal moments” and denoted as M_1, M_2, again in exact analogy with the in-plane stress problem having principal stresses at orientations where shear stresses are zero. The angle ϕ can be worked out by $\phi = \frac{1}{2}\tan^{-1}\frac{2M_{XY}}{(M_X - M_Y)}$ (Eqn D-2)

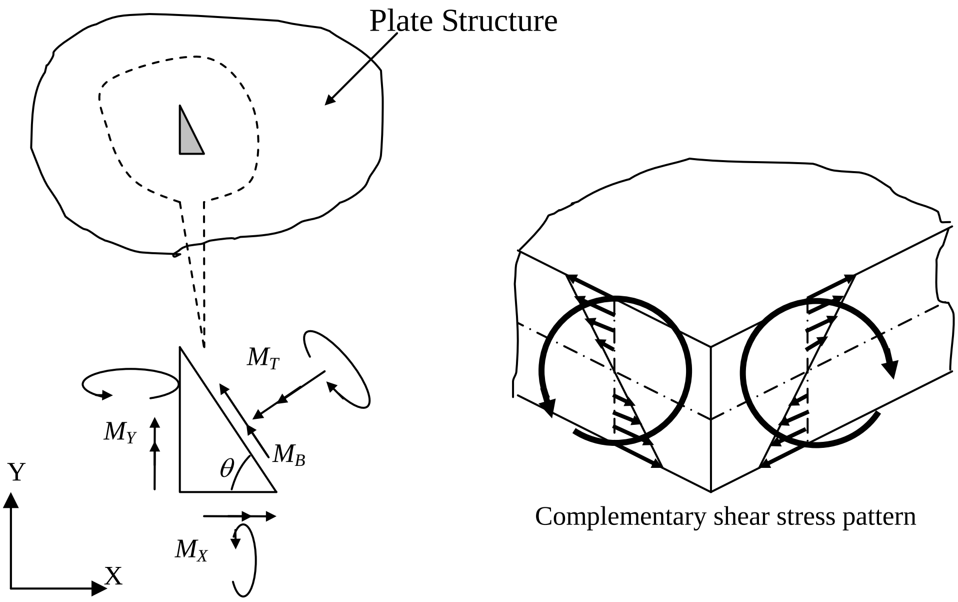


Figure D-2 – Derivation and nature of the “Twisting Moment”

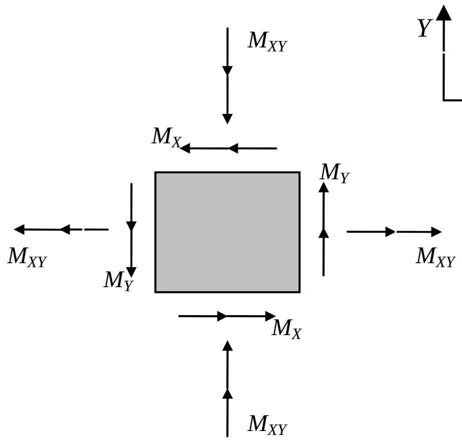


Figure D-3a – General co-existence of bending moments and twisting moment in a plate bending structure

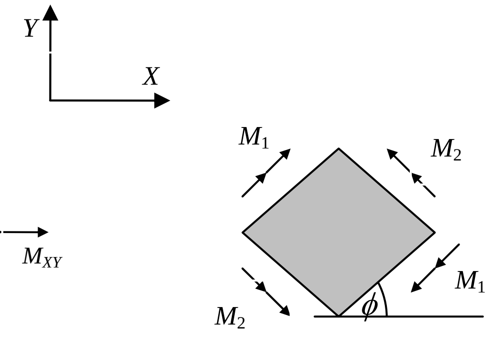


Figure D-3b – Principal moment in a plate bending structure

Again, as similar to the in-plane stress problem, one may view that the plate bending structure is actually having principal moments "bending" in the principal directions which are free of "twisting". Theoretically, it will be adequate if the designer designs for these principal moments in the principal directions which generally vary from point to point. However, practically this is not achievable for reinforced concrete structures as we cannot vary the directions of the reinforcing steels from point to point and from load case to load case.

The "stress" approach for design against flexure would therefore involve formulae for providing reinforcing steels in two directions (mostly in orthogonal directions) adequate to resist the "moment field" comprising the bending moments and twisting moments. The most popular one is the "Wood Armer" Equations by Woods (1968), the derivation of which is based on the "normal yield criterion" which requires the provided reinforcing steels at any point to be adequate to resist the normal moment which is the bending moment M_B in any directions as calculated from (Eqn D-1). The effects of the twisting moments have been taken into account in the formulae. The Wood Armer Equations are listed as follows.

For bottom steel reinforcement provisions:

Generally $\quad M_X^* = M_X + \left|M_{XY}\right|$; $\quad M_Y^* = M_Y + \left|M_{XY}\right|$;

If $\quad M_X^* < 0$, then $\quad M_X^* = 0 \quad$ and $\quad M_Y^* = M_Y + \left|\dfrac{M_{XY}{}^2}{M_X}\right|$

If $M_Y^* < 0$, then $M_Y^* = 0$ and $M_X^* = M_X + \left|\dfrac{M_{XY}{}^2}{M_Y}\right|$

For top steel reinforcement provisions:

Generally $M_X^* = M_X - |M_{XY}|$; $M_Y^* = M_Y - |M_{XY}|$;

If $M_X^* > 0$, then $M_X^* = 0$ and $M_Y^* = M_Y - \left|\dfrac{M_{XY}{}^2}{M_X}\right|$

If $M_Y^* > 0$, then $M_Y^* = 0$ and $M_X^* = M_X - \left|\dfrac{M_{XY}{}^2}{M_Y}\right|$

(Eqn D-3)

The equations have been incorporated in the New Zealand Standard NZS 3101:Part 2:1995 as solution approach for a general moment field.

The “stress” approach is therefore based on the actual structural behaviour of the plate bending structure which is considered as a direct and realistic approach. The approach is particularly suitable for structures analyzed by the finite element method which produces a complete set of components of the internal forces of the plate bending structures including twisting moments, $Q_{\max}$. Design has to cater for all these components to ensure structural adequacy.

<u>Design against shear</u>

As an alternative to checking or designing punching shear in slab in accordance with 6.1.5.7 of the Code by which the punching shear load created by column (or pile in pile cap) is effectively averaged over a perimeter, more accurate design or checking can be carried out which is based on finite element analysis by which an accurate shear stress distribution in the slab structure can be obtained. The finite element analysis outputs the “shear stresses” (shear force per unit width) in accordance with the general plate bending theory at the “X-face” and “Y-face” of an element which are respectively $Q_{XZ} = -\dfrac{\partial M_Y}{\partial x} + \dfrac{\partial M_{XY}}{\partial y}$ and $Q_{YZ} = \dfrac{\partial M_X}{\partial y} - \dfrac{\partial M_{XY}}{\partial y}$, as diagrammatically illustrated in Figure D-4. It can be easily shown that the maximum shear after “compounding” these two components will occur in a plane at an orientation $\theta = \tan^{-1}\left(\dfrac{Q_{XZ}}{Q_{YZ}}\right)$ on plan and the value of the maximum shear is

$Q_{\max} = \sqrt{Q_{XZ}^2 + Q_{YZ}^2}$ as per the illustration in the same Figure. Thus one can view that both Q_{XZ} and Q_{YZ} are components of the actual shears in a pre-set global axis system. The actual shear stress is $Q_{\max}$, the action of which tends to produce shear failure at the angle θ on plan as shown in Figure D-3. So the designer needs to check or design for $Q_{\max}$ at the spot. There is no necessity to design for Q_{XZ} and Q_{YZ} separately.

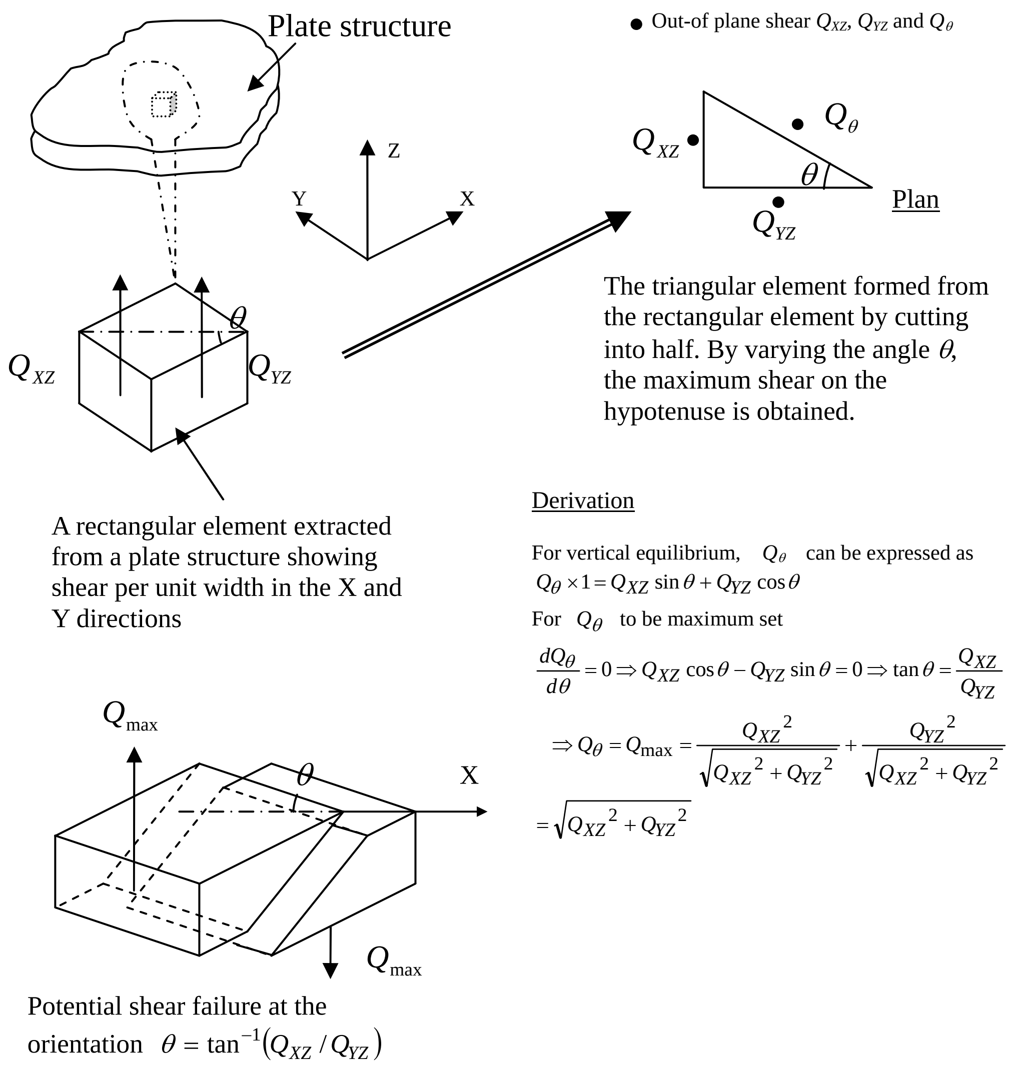


Figure D-4 – Diagrammatic illustration of shear "stresses" in the X and Y faces of an element in Plate bending structure, potential shear failure and Derivation of the magnitude and orientation of the design shear stress

Following the usual practice of designing against shear in accordance with the Code, if the $Q_{\max}$ does not exceed allowable shear strength of concrete based on v_c (the

design concrete shear stress with enhancement as appropriate) no shear reinforcements will be required. Otherwise, reinforcements will be required to cater for the difference.

The "stress" approach for shear design based on Q_{max} can best be carried out by graphical method, so as to avoid handling the large quantity of data obtainable in the finite element analysis. An illustration of the method for a raft footing is indicated in Figure D-5 as :

(i) an enveloped shear stress (shear force per unit width) contour map of a structure due to applied loads is first plotted as shown in Figure D-5(a);

(ii) the concrete shear strength contour of the structure which is a contour map indicating the shear strength of the concrete structure after enhancement of the design concrete shear stress (v_c) to closeness of supports in accordance with established code requirements (say BS8110) is plotted as shown in Figure D-5(b);

(iii) locations where the stresses exceed the enhanced strengths be reinforced by shear links as appropriate in accordance with established code requirements as shown in Figure D-5(c).

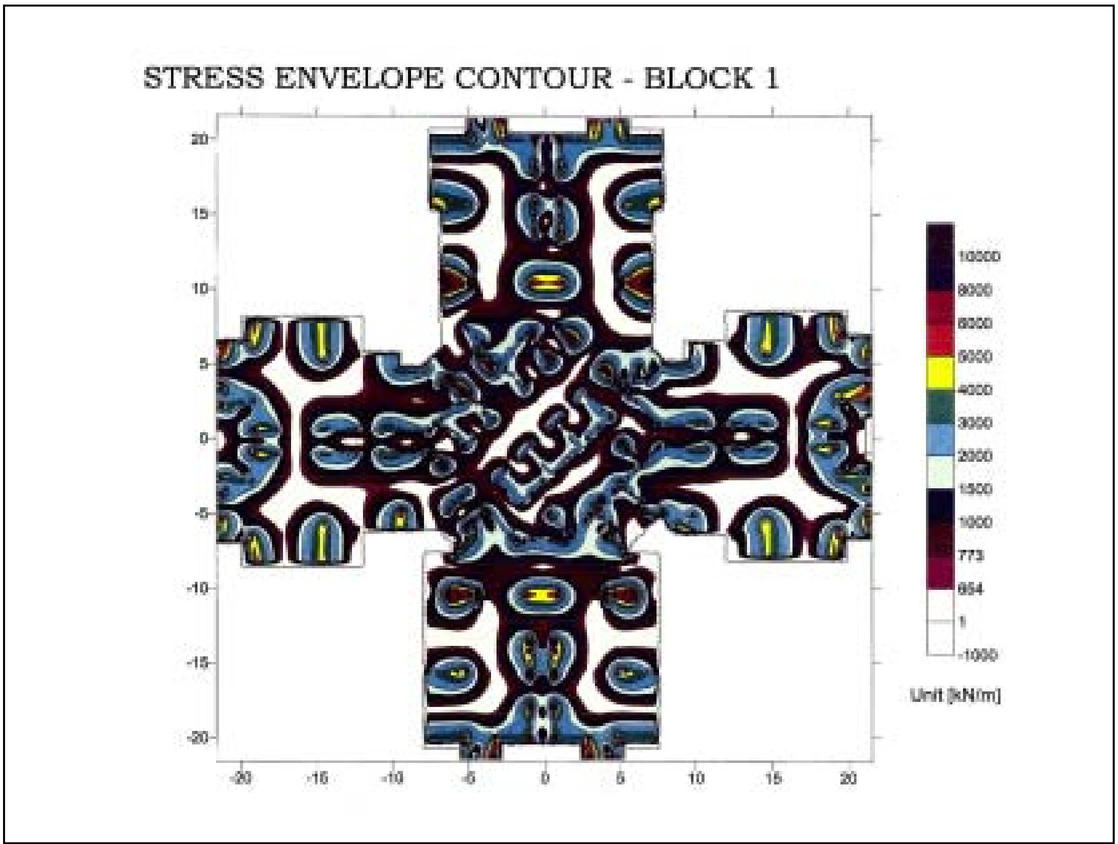


Figure D-5a – Stress contour of enveloped shear "stresses" of a raft footing due to applied load

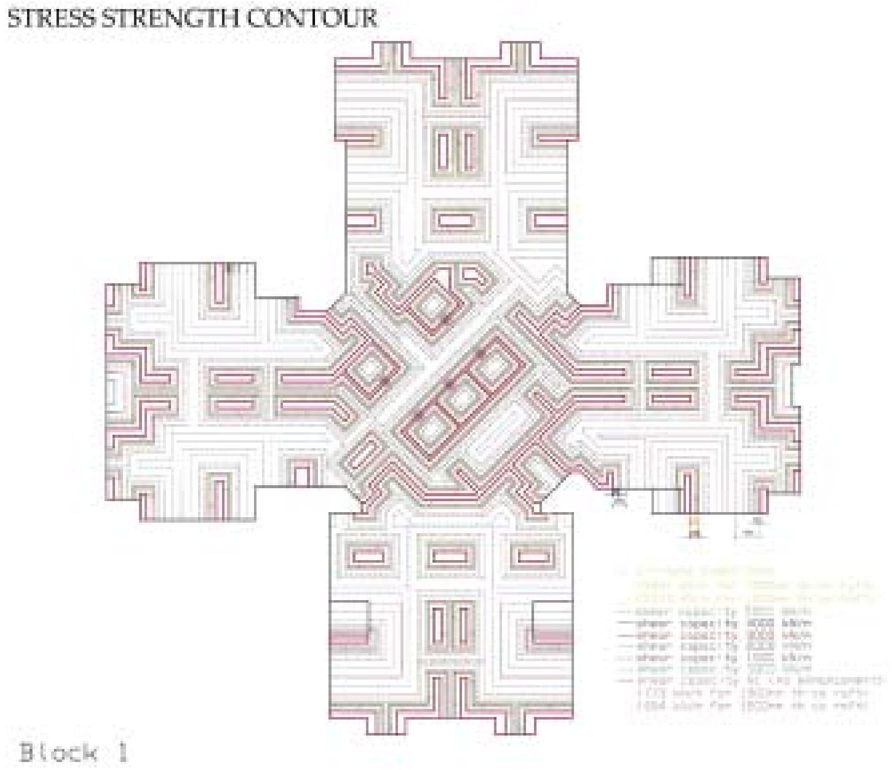


Figure D-5b – Strength contour of the raft footing with enhancement

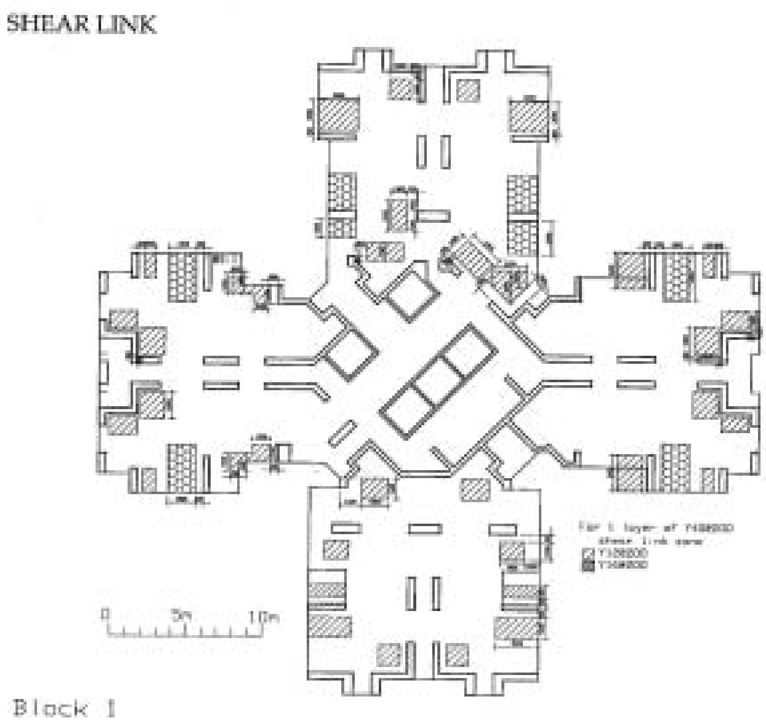


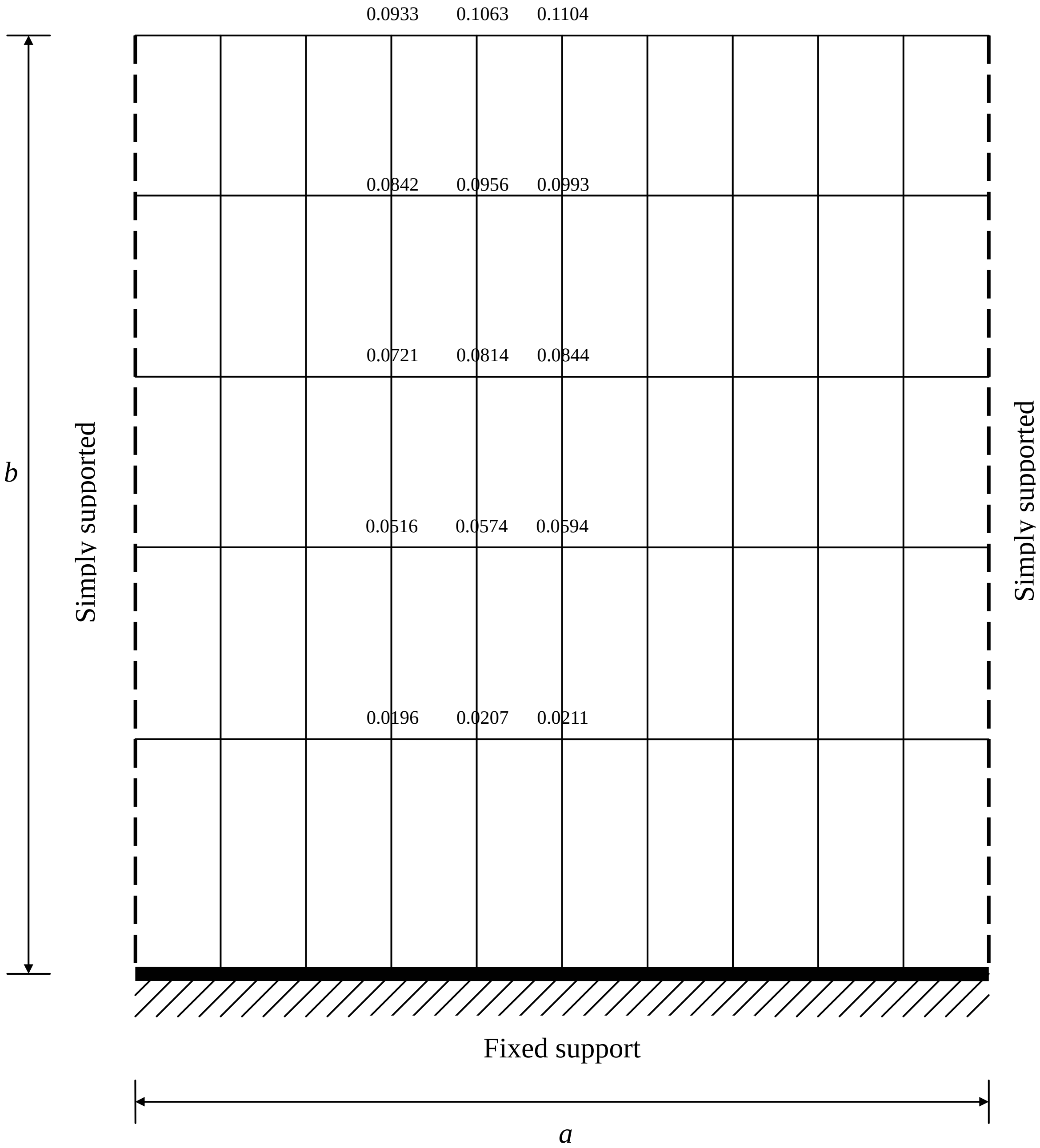
Figure D-5c – Arrangement of shear reinforcements

Appendix E

Moment Coefficients for three side supported Slabs

Bending Coefficients in the plate of the indicated support conditions and length breadth ratio are interpolated from Table 1.38 of "Tables for the Analysis of Plates, Slabs and Diaphragms based on Elastic Theory"

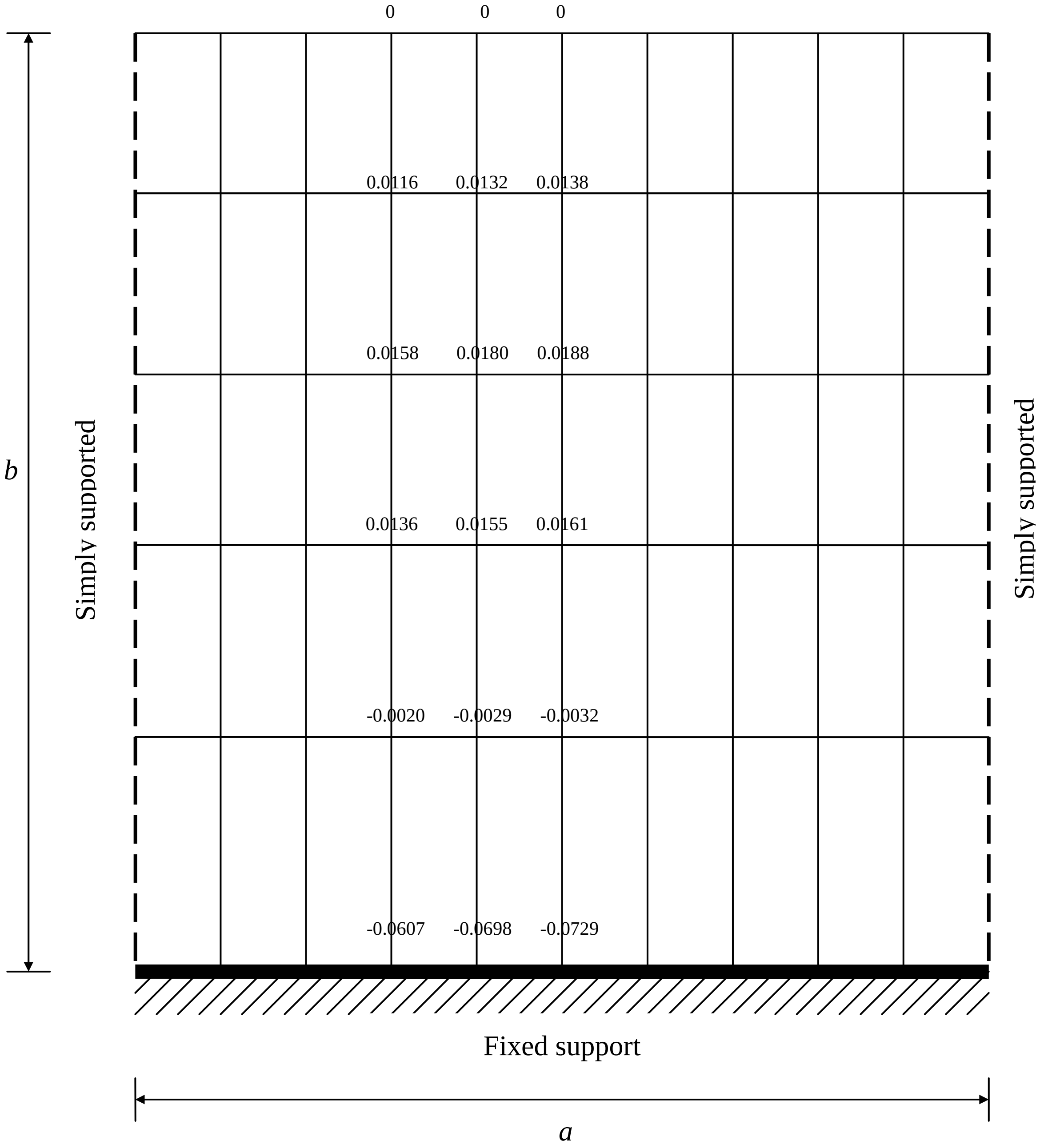
$\frac{b}{a} = 2$



Coefficients for bending along X direction (+ve : sagging; –ve : hogging)

Moments at various points is $c.f.\times q\times a^2$ where q is the u.d.l.

$\frac{b}{a} = 2$



Coefficients for bending along Y direction (+ve : sagging; –ve : hogging)

Moments at various points is $c.f.\times q\times b^2$ where q is the u.d.l.

Appendix F

Derivation of Design Formulae for Rectangular Columns to Rigorous Stress Strain Curve of Concrete

Appendix F

Derivation of Design Formulae for Rectangular Columns to Rigorous Stress Strain Curve of Concrete

(I) Computing stress / force contribution of concrete stress block

Assuming the parabolic portion of the concrete stress block as indicated in Fig. 3.8 of HKCoP2004 be represented by the equation

$\sigma = A\varepsilon^2 + B\varepsilon$ (where A and B are constants) (Eqn F-1)

So $\dfrac{d\sigma}{d\varepsilon} = 2A\varepsilon + B$ (Eqn F-2)

As $\left.\dfrac{d\sigma}{d\varepsilon}\right|_{\varepsilon=0} = E_c \Rightarrow B = E_c$ where E_c is the tangential Young's Modulus of concrete listed in Table 3.2 of the Code.

Also $\left.\dfrac{d\sigma}{d\varepsilon}\right|_{\varepsilon=\varepsilon_0} = 0 \Rightarrow 2A\varepsilon_0 + B = 0 \Rightarrow A = -\dfrac{B}{2\varepsilon_0} = -\dfrac{E_c}{2\varepsilon_0}$ (Eqn F-3)

As $\sigma = 0.67\dfrac{f_{cu}}{\gamma_m}$ when $\varepsilon = \varepsilon_0$

$\therefore 0.67\dfrac{f_{cu}}{\gamma_m \varepsilon_0^2} - \dfrac{E_c}{\varepsilon_0} = -\dfrac{E_c}{2\varepsilon_0} \Rightarrow \dfrac{0.67 f_{cu}}{\gamma_m \varepsilon_0} = \dfrac{E_c}{2} \Rightarrow \varepsilon_0 = \dfrac{1.34 f_{cu}}{E_c \gamma_m}$ (Eqn (F-4)

(accords with 3.14 of the Concrete Code Handbook)

$A = -\dfrac{E_c}{2\varepsilon_0}$ where $\varepsilon_0 = \dfrac{1.34 f_{cu}}{E_c \gamma_m}$

So the equation of the parabola is $\sigma = -\dfrac{E_c}{2\varepsilon_0}\varepsilon^2 + E_c\varepsilon$ for $\varepsilon \le \varepsilon_0$

Consider the linear strain distribution

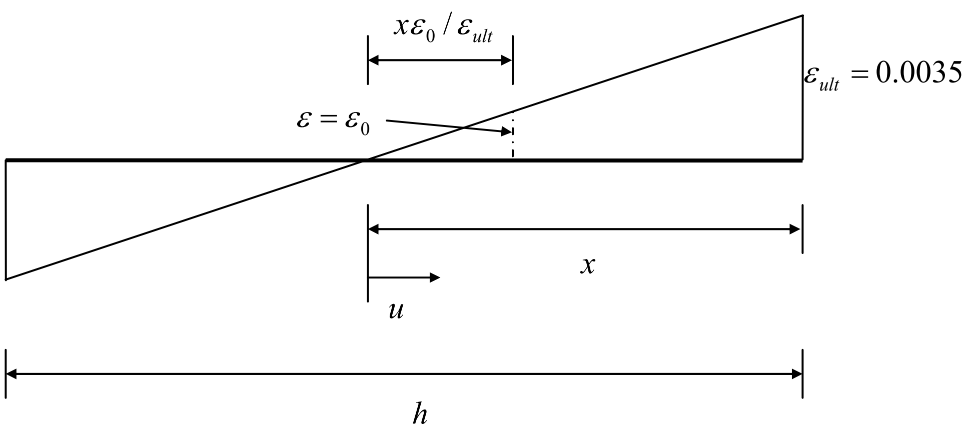


Figure F-1 – Strain diagram across concrete section

At distance u from the neutral axis, $\varepsilon = \varepsilon_{ult}\dfrac{u}{x}$

So stress at u from the neutral axis up to $x\dfrac{\varepsilon_0}{\varepsilon_{ult}}$ is

$$\sigma = -\frac{E_c}{2\varepsilon_0}\varepsilon^2 + E_c\varepsilon = -\frac{E_c}{2\varepsilon_0}\left(\varepsilon_{ult}\frac{u}{x}\right)^2 + E_c\left(\varepsilon_{ult}\frac{u}{x}\right) = -\frac{E_c\varepsilon_{ult}{}^2}{2\varepsilon_0 x^2}u^2 + \frac{E_c\varepsilon_{ult}}{x}u \qquad \text{(Eqn F-5)}$$

Based on (Eqn F-5), the stress strain profiles for grade 35 within the concrete compression section are plotted in Figure F2 for illustration.

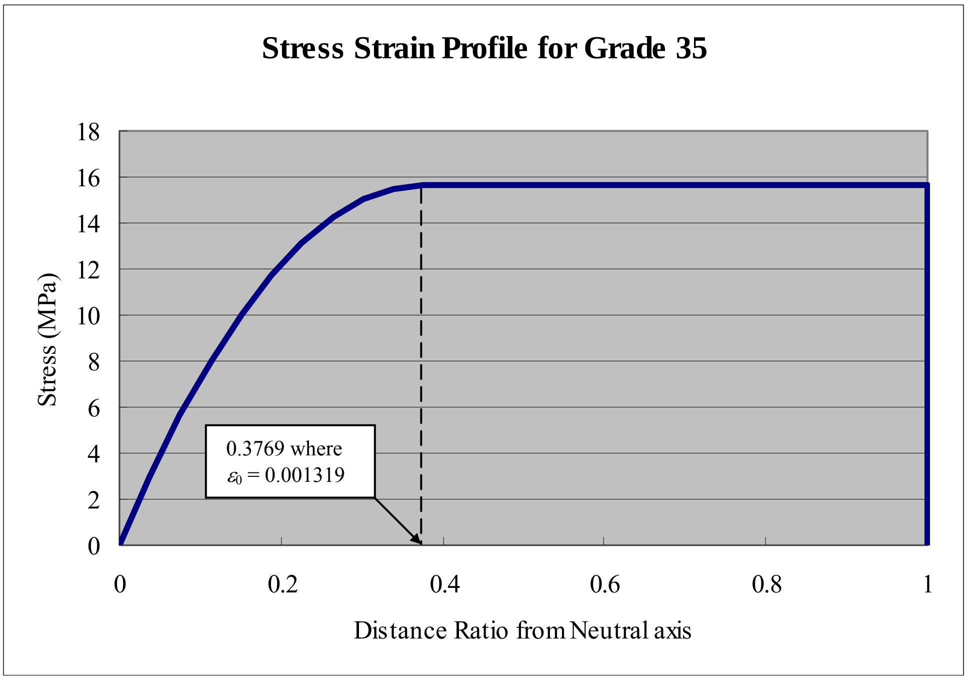


Figure F-2 – Stress strain profile of grades 35

By the properties of parabola as shown in Figure F-3, we can formulate total force offered by the parabolic section as F_{c1} given by

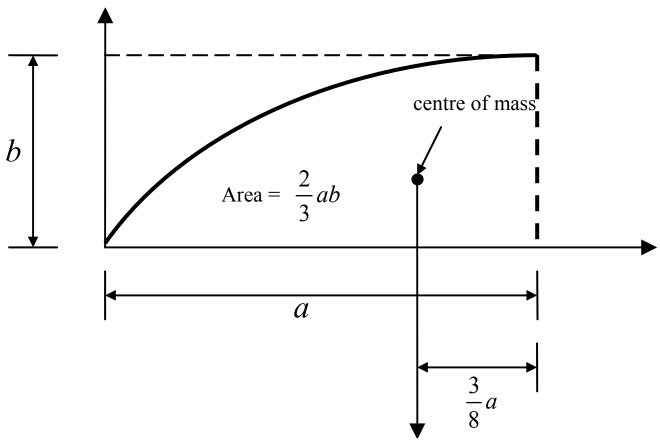


Figure F-3 – Geometrical Properties of Parabola

$$F_{c1} = b\frac{2}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} x 0.67\frac{f_{cu}}{\gamma_m} = \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m \varepsilon_{ult}} bx \qquad \text{(Eqn F-6)}$$

and the moment exerted by F_{c1} about centre line of the whole section

$$M_{c1} = F_{c1}\left[\frac{h}{2} - x\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right) - \frac{3}{8}x\frac{\varepsilon_0}{\varepsilon_{ult}}\right] = F_{c1}\left[\frac{h}{2} - x\left(1 - \frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right] \qquad \text{(Eqn F-7)}$$

The force by the straight portion is

$$F_{c2} = \frac{0.67 f_{cu}}{\gamma_m}\left(x - x\frac{\varepsilon_0}{\varepsilon_{ult}}\right)b = \frac{0.67 f_{cu} bx}{\gamma_m}\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right) \qquad \text{(Eqn F-8)}$$

The moment offered by the constant part about the centre line of the whole section is

$$M_{c2} = F_{c2}\left[\frac{h}{2} - \left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{2}\right] \qquad \text{(Eqn F-9)}$$

Thus if full section of concrete in compression exists in the column section

$$\frac{F_c}{bh} = \frac{F_{c1}}{bh} + \frac{F_{c2}}{bh} = \frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h} \qquad \text{(Eqn F-10)}$$

$$\frac{M_{c1}}{bh^2} = \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m \varepsilon_{ult}} bx\left[\frac{h}{2} - x\left(1 - \frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right]\frac{1}{bh^2} = \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m \varepsilon_{ult}}\left(\frac{x}{h}\right)\left[\frac{1}{2} - \left(\frac{x}{h}\right)\left(1 - \frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right]$$

$$\frac{M_{c2}}{bh^2} = \frac{0.67 f_{cu} bx}{\gamma_m}\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\left[\frac{h}{2} - \left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{2}\right]\frac{1}{bh^2} = \frac{0.67 f_{cu}}{2\gamma_m}\left(\frac{x}{h}\right)\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\left[1 - \left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}\right]$$

$$\frac{M_c}{bh^2} = \frac{M_{c1} + M_{c2}}{bh^2} = \frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m \varepsilon_{ult}}\left(\frac{x}{h}\right)\left[\frac{1}{2} - \left(\frac{x}{h}\right)\left(1 - \frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right] + \frac{0.67 f_{cu}}{2\gamma_m}\left(\frac{x}{h}\right)\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\left[1 - \left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}\right]$$

$$= \frac{0.67 f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left\{\frac{1}{2} - \frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}} + \left[-\frac{1}{2} + \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\frac{x}{h}\right\} \qquad \text{(Eqn F-11)}$$

(II) Derivation of Basic Design Formulae of R.C. column sections

Cases 1 to 7 with different stress / strain profile of concrete and steel across the column section due to the differences in the neutral axis depth ratios, $\frac{x}{h}$, are investigated. The section is reinforced by continuous reinforcements A_{sh} along its length h idealized as continuum and reinforcements at its end faces A_{sb} with cover d'.

Pursuant to the derivation of the stress strain relationship of concrete and steel, the

stress strain diagram of concrete and steel for Cases 1 to 7 are as follows, under the definition of symbols as :

b :	width of the column
h :	length of the column
x :	neutral axis depth of the column
A_{sb} :	total steel area at the end faces of the column
d' :	concrete cover to the centre of the end face steel
A_{sh} :	total steel area along the length of the column

Case 1 (a) – where (i) $x/h < 7/3(d'/h)$ for $d'/h \le 3/14$; and (ii) $x/h < 7/11(1 - d'/h)$ for $d'/h > 3/14$

Pursuant to the derivation of the stress strain relationship of concrete and steel, the stress strain diagram of concrete and steel for Case 1(a) is as indicated in Figure F-1(a) :

It should be noted that F_{sc1} is in elastic whilst F_{st1} is in plastic range as $d'/h < 3/14$

Steel compressive force in the portion steel elastic zone by A_{sb} is

$$F_{sc1} = \left(\frac{x-d'}{4x/7}\right) \times 0.87 f_y \times 0.5 A_{sb} = \frac{7}{4}\left(1-\frac{d'}{x}\right) \times 0.87 f_y \times 0.5 A_{sb} \qquad \text{(Eqn F-12)}$$

Steel compressive force in the portion steel plastic zone by A_{sh} is

$$F_{sc2} = 0.87 f_y \times \frac{A_{sh}}{h}\left(\frac{3x}{7}\right) = 0.87 f_y \times A_{sh}\left(\frac{3}{7}\frac{x}{h}\right) \qquad \text{(Eqn F-13)}$$

Steel compressive force in the portion steel elastic zone by A_{sh} is

$$F_{sc3} = 0.87 f_y \times \frac{A_{sh}}{h}\left(\frac{4x}{7}\right) \times \frac{1}{2} = 0.87 f_y \times A_{sh}\left(\frac{2}{7}\frac{x}{h}\right) \qquad \text{(Eqn F-14)}$$

Steel tensile force in the portion steel plastic zone by A_{sb} is

$$F_{st1} = 0.87 f_y \times 0.5 A_{sb} \qquad \text{(Eqn F-15)}$$

Steel tensile force in the portion steel plastic zone by A_{sh} is

$$F_{st2} = 0.87 f_y \times \frac{A_{sh}}{h}\left(h - \frac{11x}{7}\right) = 0.87 f_y \times A_{sh}\left(1 - \frac{11}{7}\frac{x}{h}\right) \qquad \text{(Eqn F-16)}$$

Steel tensile force in the portion steel elastic zone by A_{sh} is

$$F_{st3} = 0.87 f_y \times \frac{A_{sh}}{h}\left(\frac{4x}{7}\right) \times \frac{1}{2} = 0.87 f_y \times A_{sh}\left(\frac{2}{7}\frac{x}{h}\right) \qquad \text{(Eqn F-17)}$$

To balance the external load N_u

$$F_{c1} + F_{c2} + F_{sc1} + F_{sc2} + F_{sc3} - F_{st1} - F_{st2} - F_{st3} = N_u$$

$$\Rightarrow F_{c1} + F_{c2} + F_{sc1} + F_{sc2} - F_{st1} - F_{st2} = N_u$$

$$\Rightarrow \frac{N_u}{bh} = \frac{0.67 f_{cu}}{3\gamma_m}\left(3 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h} + 0.87 f_y \left(2\frac{x}{h} - 1\right)\frac{A_{sh}}{bh} + \left(\frac{3}{8} - \frac{7}{8}\frac{d'}{h}\frac{h}{x}\right)0.87 f_y \frac{A_{sb}}{bh}$$

(Eqn F-18)

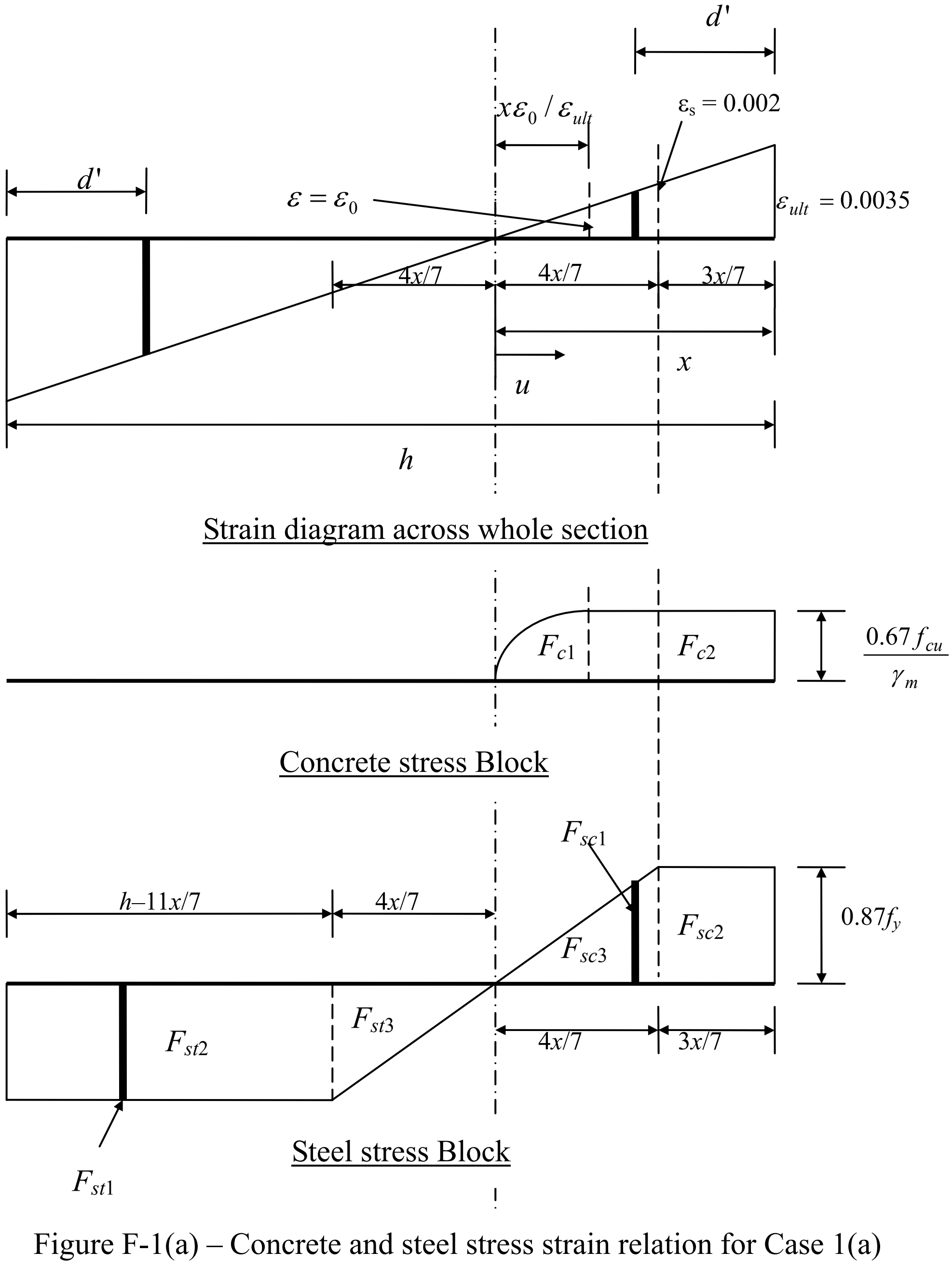


Figure F-1(a) – Concrete and steel stress strain relation for Case 1(a)

Re-arranging (F-18)

$$\left[\frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right) + 2 \times 0.87 f_y \frac{A_{sh}}{bh}\right]\left(\frac{x}{h}\right)^2 - \left[\frac{N_u}{bh} + 0.87 f_y \left(\frac{A_{sh}}{bh} - \frac{3}{8}\frac{A_{sb}}{bh}\right)\right]\frac{x}{h}$$

$$-\frac{7}{8}0.87 f_y \frac{A_{sb}}{bh}\frac{d'}{h} = 0$$

(Eqn F-19)

(Eqn F-19) can be used for solve for $\dfrac{x}{h}$

To balance the external load M_u

$$M_{c1}+M_{c2}+M_{sc1}+M_{sc2}+M_{sc3}+M_{st1}+M_{st2}+M_{st3}=M_u$$

$$M_{c1}+M_{c2}+F_{sc1}\left(\frac{h}{2}-d'\right)+F_{sc2}\left(\frac{h}{2}-\frac{3x}{14}\right)+F_{sc3}\left[\frac{h}{2}-\frac{3x}{7}-\frac{4x}{21}\right]+F_{st1}\left(\frac{h}{2}-d'\right)$$

$$+F_{st2}\left[\frac{h}{2}-\frac{1}{2}\left(h-\frac{11x}{7}\right)\right]+F_{st3}\left[\frac{h}{2}-\left(h-\frac{11x}{7}\right)-\frac{4x}{21}\right]=M_u$$

$$\Leftrightarrow M_u=M_{c1}+M_{c2}+F_{sc1}\left(\frac{h}{2}-d'\right)+F_{sc2}\left(\frac{h}{2}-\frac{3x}{14}\right)+F_{sc3}\left(\frac{h}{2}-\frac{13x}{21}\right)+F_{st1}\left(\frac{h}{2}-d'\right)$$

$$+F_{st2}\left(\frac{11x}{14}\right)+F_{st3}\left(\frac{29x}{21}-\frac{h}{2}\right) \quad \text{(Eqn F-20)}$$

where

$$\frac{M_{c1}}{bh^2}=\frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m\varepsilon_{ult}}bx\left[\frac{h}{2}-x\left(1-\frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right]\frac{1}{bh^2}=\frac{1.34\varepsilon_0 f_{cu}}{3\gamma_m\varepsilon_{ult}}\left(\frac{x}{h}\right)\left[\frac{1}{2}-\left(\frac{x}{h}\right)\left(1-\frac{5}{8}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right]$$

(Eqn F-21)

$$\frac{M_{c2}}{bh^2}=\frac{0.67f_{cu}bx}{\gamma_m}\left(1-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\left[\frac{h}{2}-\left(1-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{2}\right]\frac{1}{bh^2}=\frac{0.67f_{cu}}{2\gamma_m}\left(\frac{x}{h}\right)\left(1-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\left[1-\left(1-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}\right]$$

(Eqn F-22)

$$\frac{M_{sc1}}{bh^2}=\frac{7}{4}\left(1-\frac{d'}{x}\right)\times 0.87f_y\times 0.5\frac{A_{sb}}{bh^2}\left(\frac{h}{2}-d'\right)=\frac{7}{4}\left(1-\frac{d'}{h}\frac{h}{x}\right)\times 0.87f_y\times 0.5\frac{A_{sb}}{bh}\left(\frac{1}{2}-\frac{d'}{h}\right)$$

(Eqn F-23)

$$\frac{M_{sc2}}{bh^2}=0.87f_y\times A_{sh}\left(\frac{3}{7}\frac{x}{h}\right)\left(\frac{h}{2}-\frac{3x}{14}\right)\frac{1}{bh^2}=0.87f_y\frac{A_{sh}}{bh}\left(\frac{3}{7}\frac{x}{h}\right)\left(\frac{1}{2}-\frac{3}{14}\frac{x}{h}\right) \quad \text{(Eqn F-24)}$$

$$\frac{M_{sc3}}{bh^2}=0.87f_y\times A_{sh}\left(\frac{2}{7}\frac{x}{h}\right)\left(\frac{h}{2}-\frac{13x}{21}\right)\frac{1}{bh^2}=0.87f_y\frac{A_{sh}}{bh}\left(\frac{2}{7}\frac{x}{h}\right)\left(\frac{1}{2}-\frac{13}{21}\frac{x}{h}\right) \quad \text{(Eqn F-25)}$$

$$\frac{M_{st1}}{bh^2}=\frac{0.87f_y\times 0.5A_{sb}}{bh^2}\left(\frac{h}{2}-d'\right)=0.87f_y\times 0.5\frac{A_{sb}}{bh}\left(\frac{1}{2}-\frac{d'}{h}\right) \quad \text{(Eqn F-26)}$$

$$\frac{M_{st2}}{bh^2}=0.87f_y\times\frac{A_{sh}}{bh^2}\left(1-\frac{11}{7}\frac{x}{h}\right)\left(\frac{11x}{14}\right)=0.87f_y\times\frac{A_{sh}}{bh}\left(1-\frac{11}{7}\frac{x}{h}\right)\left(\frac{11}{14}\frac{x}{h}\right) \quad \text{(Eqn F-27)}$$

$$\frac{M_{st3}}{bh^2}=0.87f_y\times A_{sh}\left(\frac{2}{7}\frac{x}{h}\right)\left(\frac{29x}{21}-\frac{h}{2}\right)\frac{1}{bh^2}=0.87f_y\times\frac{A_{sh}}{bh}\left(\frac{2}{7}\frac{x}{h}\right)\left(\frac{29}{21}\frac{x}{h}-\frac{1}{2}\right)$$

(Eqn F-28)

Summing up

$$\frac{M_{c1}+M_{c2}}{bh^2}=\frac{0.67f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left\{\frac{1}{2}-\frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}}+\left[-\frac{1}{2}+\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\frac{x}{h}\right\} \quad \text{(Eqn F-29)}$$

$$\frac{M_{sc1}+M_{st1}}{bh^2}=\frac{7}{4}\left(1-\frac{d'}{h}\frac{h}{x}\right)\times 0.87f_y\times 0.5\frac{A_{sb}}{bh}\left(\frac{1}{2}-\frac{d'}{h}\right)+0.87f_y\times 0.5\frac{A_{sb}}{bh}\left(\frac{1}{2}-\frac{d'}{h}\right)$$

$$= 0.87 f_y \frac{A_{sb}}{bh}\left(\frac{1}{2}-\frac{d'}{h}\right)\left(\frac{11}{8}-\frac{7}{8}\frac{d'}{h}\frac{h}{x}\right)$$

(Eqn F-30)

$$\frac{M_{sc2}+M_{st2}}{bh^2} = 0.87 f_y \frac{A_{sh}}{bh}\left(\frac{3}{7}\frac{x}{h}\right)\left(\frac{1}{2}-\frac{3}{14}\frac{x}{h}\right)+0.87 f_y \times \frac{A_{sh}}{bh}\left(1-\frac{11}{7}\frac{x}{h}\right)\left(\frac{11}{14}\frac{x}{h}\right)$$

$$= 0.87 f_y \frac{A_{sh}}{bh}\left[\left(\frac{x}{h}\right)-\frac{65}{49}\left(\frac{x}{h}\right)^2\right] \quad \text{(Eqn F-31)}$$

$$\frac{M_{sc3}+M_{st3}}{bh^2} = 0.87 f_y \frac{A_{sh}}{bh}\left(\frac{2}{7}\frac{x}{h}\right)\left(\frac{1}{2}-\frac{13}{21}\frac{x}{h}\right)+0.87 f_y \times \frac{A_{sh}}{bh}\left(\frac{2}{7}\frac{x}{h}\right)\left(\frac{29}{21}\frac{x}{h}-\frac{1}{2}\right)$$

$$= 0.87 f_y \frac{A_{sh}}{bh}\frac{32}{147}\left(\frac{x}{h}\right)^2 \quad \text{(Eqn F-32)}$$

$$\text{Total } \frac{M_s}{bh^2} = 0.87 f_y \left\{\frac{A_{sb}}{bh}\left(\frac{1}{2}-\frac{d'}{h}\right)\left(\frac{11}{8}-\frac{7}{8}\frac{d'}{h}\frac{h}{x}\right)+\frac{A_{sh}}{bh}\left[\left(\frac{x}{h}\right)-\frac{163}{147}\left(\frac{x}{h}\right)^2\right]\right\} \quad \text{(Eqn F-33)}$$

$$\therefore \frac{M_u}{bh^2} = \frac{0.67 f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left\{\frac{1}{2}-\frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}}+\left[-\frac{1}{2}+\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\frac{x}{h}\right\}$$

$$+0.87 f_y \left\{\frac{A_{sb}}{bh}\left(\frac{1}{2}-\frac{d'}{h}\right)\left(\frac{11}{8}-\frac{7}{8}\frac{d'}{h}\frac{h}{x}\right)+\frac{A_{sh}}{bh}\left[\left(\frac{x}{h}\right)-\frac{163}{147}\left(\frac{x}{h}\right)^2\right]\right\} \quad \text{(Eqn F-34)}$$

<u>Case 1 (b) – 7/11(1 – *d'/h*) ≤ *x/h* < 7/3(*d'/h*) where *d'/h* > 3/14</u>

Case 1(b) is similar to Case 1(a) except that both F_{sc1} and F_{st1} are in the elastic range as *d'/h* > 3/14.

Re Figure F-1(b), the various components of stresses in concrete and in steel are identical to that of Case 1(b) except that by F_{st1}, the stress of which is

$$\frac{h-x-d'}{4x/7}0.87 f_y$$

So the $\frac{F_{st1}}{bh} = -\frac{h-x-d'}{4x/7}0.87 f_y \times 0.5\frac{A_{sb}}{bh} = -\frac{7}{8}\times 0.87 f_y\left(\frac{h}{x}-1-\frac{d'}{h}\frac{h}{x}\right)\frac{A_{sb}}{bh}$

$$\frac{F_{st1}}{bh}+\frac{F_{sc1}}{bh} = \frac{7}{8}\times 0.87 f_y\left(2-\frac{h}{x}\right)\frac{A_{sb}}{bh}$$

$$\therefore \frac{N_u}{bh} = \frac{0.67 f_{cu}}{3\gamma_m}\left(3-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}+0.87 f_y\left(2\frac{x}{h}-1\right)\frac{A_{sh}}{bh}+\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}\right)0.87 f_y \frac{A_{sb}}{bh} \quad \text{(Eqn F-35)}$$

$$\Rightarrow \left[\frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)+0.87 f_y \times 2\frac{A_{sh}}{bh}\right]\left(\frac{x}{h}\right)^2+\left[0.87 f_y\left(\frac{7}{4}\frac{A_{sb}}{bh}-\frac{A_{sh}}{bh}\right)-\frac{N_u}{bh}\right]\frac{x}{h}$$

$$-0.87\times\frac{7}{8}\frac{A_{sb}}{bh} = 0 \quad \text{(Eqn F-36)}$$

$$\frac{M_{sc1}+M_{st1}}{bh^2}=\frac{7}{8}\left(1-\frac{d'}{h}\frac{h}{x}\right)\times 0.87f_y\frac{A_{sb}}{bh}\left(\frac{1}{2}-\frac{d'}{h}\right)+0.87f_y\frac{7}{8}\times\left(\frac{h}{x}-1-\frac{d'}{h}\frac{h}{x}\right)\frac{A_{sb}}{bh}\left(\frac{1}{2}-\frac{d'}{h}\right)$$

$$=0.87f_y\frac{7}{8}\times\frac{A_{sb}}{bh}\left(\frac{1}{2}-\frac{d'}{h}\right)\left(1-2\frac{d'}{h}\right)\frac{h}{x} \qquad \text{(Eqn F-37)}$$

$$\therefore\frac{M_u}{bh^2}=\frac{0.67f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left\{\left(\frac{1}{2}-\frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)+\left[\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{1}{2}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{h}\right)\right\}$$

$$+0.87f_y\left\{\frac{7}{8}\frac{A_{sb}}{bh}\left(\frac{1}{2}-\frac{d'}{h}\right)\left(1-2\frac{d'}{h}\right)\frac{h}{x}+\frac{A_{sh}}{bh}\left[\left(\frac{x}{h}\right)-\frac{163}{147}\left(\frac{x}{h}\right)^2\right]\right\} \qquad \text{(Eqn F-38)}$$

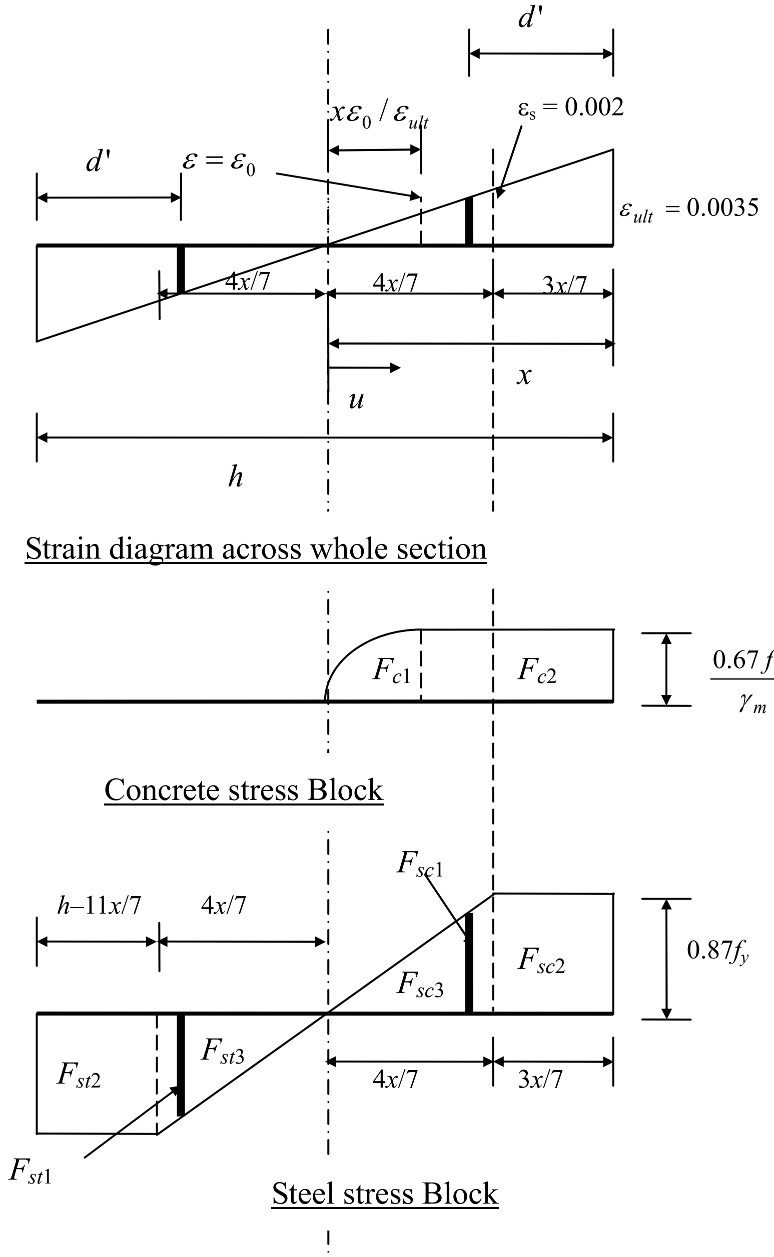


Figure F-1(b) – Concrete and steel stress strain relation for Case 1(b)

Case 2 – $7/3(d'/h) \le x/h < 7/11(1 - d'/h)$

There are two sub-cases to be considered in Case 2,

i.e. Case 2(a) – $\frac{d'}{h} \ge \frac{3}{14}$ and Case 2(b) – $\frac{d'}{h} < \frac{3}{14}$

For Case 2(a), where $\frac{d'}{h} \ge \frac{3}{14}$. However, $\frac{7}{3}\frac{d'}{h} \ge \frac{1}{2}$ and $\frac{7}{11}\left(1 - \frac{d'}{h}\right) < \frac{1}{2}$. So this case doesn't exist. For Case 2(b), where $\frac{d'}{h} < \frac{3}{14}$ both A_{sc1} and A_{st1} are in the plastic zone as shown in Figure F-2.

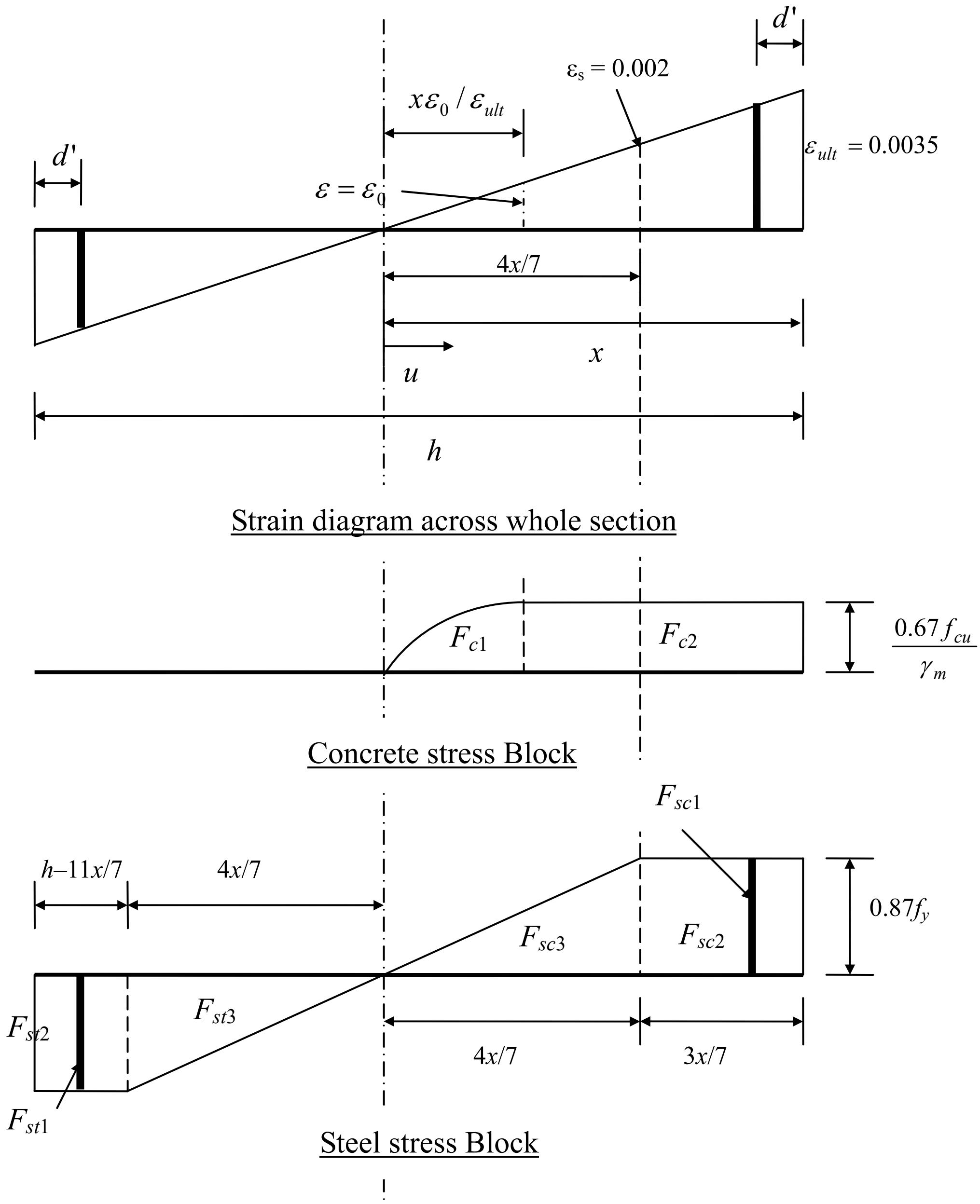


Figure F-2 – Concrete and steel stress strain relation for Case 2(b)

The various components of stresses in concrete and steel are identical to that of Case 2(a) except that of A_{sc1} where

$F_{sc1} = 0.87 f_y \times 0.5 A_{sb}$ and $M_{sc1} = 0.87 f_y \times 0.5 A_{sb}\left(\frac{1}{2} - \frac{d'}{h}\right)$

It can be seen that F_{st1} and F_{sc1} are identical but opposite in direction, so cancel out. By formulation similar to the above,

$$\frac{N_u}{bh} = \frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h} + 0.87 f_y\left(2\frac{x}{h} - 1\right)\frac{A_{sh}}{bh} \qquad \text{(Eqn F-39)}$$

$$\Rightarrow \frac{x}{h} = \frac{\frac{N_u}{bh} + 0.87 f_y \frac{A_{sh}}{bh}}{\left[\frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right) + 2 \times 0.87 f_y \frac{A_{sh}}{bh}\right]} \qquad \text{(Eqn F-40)}$$

$$\frac{M_u}{bh^2} = \frac{0.67 f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left\{\frac{1}{2} - \frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}} + \left[-\frac{1}{2} + \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\frac{x}{h}\right\}$$

$$+ 0.87 f_y\left\{\frac{A_{sb}}{bh}\left(\frac{1}{2} - \frac{d'}{h}\right) + \frac{A_{sh}}{bh}\left[\left(\frac{x}{h}\right) - \frac{163}{147}\left(\frac{x}{h}\right)^2\right]\right\} \qquad \text{(Eqn F-41)}$$

<u>Case 3 – where 7/3(d'/h) ≤ x/h < 7/11 for d'/h > 3/14 and 7/11(1 – d'/h) ≤ x/h < 7/11 for d'/h < 3/14</u>

The concrete / steel stress / strain diagram is worked out as indicated in Figure F-3 :

The components of stresses are identical to Case 2 except that F_{st1} become elastic which is

$$F_{st1} = \frac{0.87 f_y (h - x - d')}{4x/7} \times 0.5 A_{sb} = 0.87 f_y \frac{(h - x - d')}{x}\frac{7}{8}A_{sb} = 0.87 f_y\left(\frac{h}{x} - \frac{d'}{x} - 1\right)\frac{7}{8}A_{sb}$$

$$\Rightarrow \frac{F_{st1}}{bh} = 0.87 f_y\left(\frac{h}{x} - \frac{d'}{h}\frac{h}{x} - 1\right)\frac{7}{8}\frac{A_{sb}}{bh}$$

$$\therefore \frac{F_{sc1}}{bh} + \frac{F_{st1}}{bh} = 0.87 f_y \times 0.5\frac{A_{sb}}{bh} - 0.87 f_y\left(\frac{h}{x} - \frac{d'}{h}\frac{h}{x} - 1\right)\frac{7}{8}\frac{A_{sb}}{bh} = 0.87 f_y \frac{A_{sb}}{bh}\left(\frac{11}{8} - \frac{7}{8}\frac{h}{x} + \frac{7}{8}\frac{d'}{x}\right)$$

(Eqn F-42)

$$\therefore \frac{N_u}{bh} = \frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h} + 0.87 f_y \frac{A_{sb}}{bh}\left(\frac{11}{8} - \frac{7}{8}\frac{h}{x} + \frac{7}{8}\frac{d'}{x}\right) + 0.87 f_y\left(2\frac{x}{h} - 1\right)\frac{A_{sh}}{bh}$$

(Eqn F-43)

Re-arranging (Eqn F-43)

$$\left[\frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right) + 2 \times 0.87 f_y \frac{A_{sh}}{bh}\right]\left(\frac{x}{h}\right)^2 + \left[0.87 f_y\left(\frac{A_{sb}}{bh}\frac{11}{8} - \frac{A_{sh}}{bh}\right) - \frac{N_u}{bh}\right]\left(\frac{x}{h}\right)$$

$$+0.87 f_y \frac{7}{8}\frac{A_{sb}}{bh}\left(\frac{d'}{h}-1\right)=0 \qquad \text{(Eqn F-44)}$$

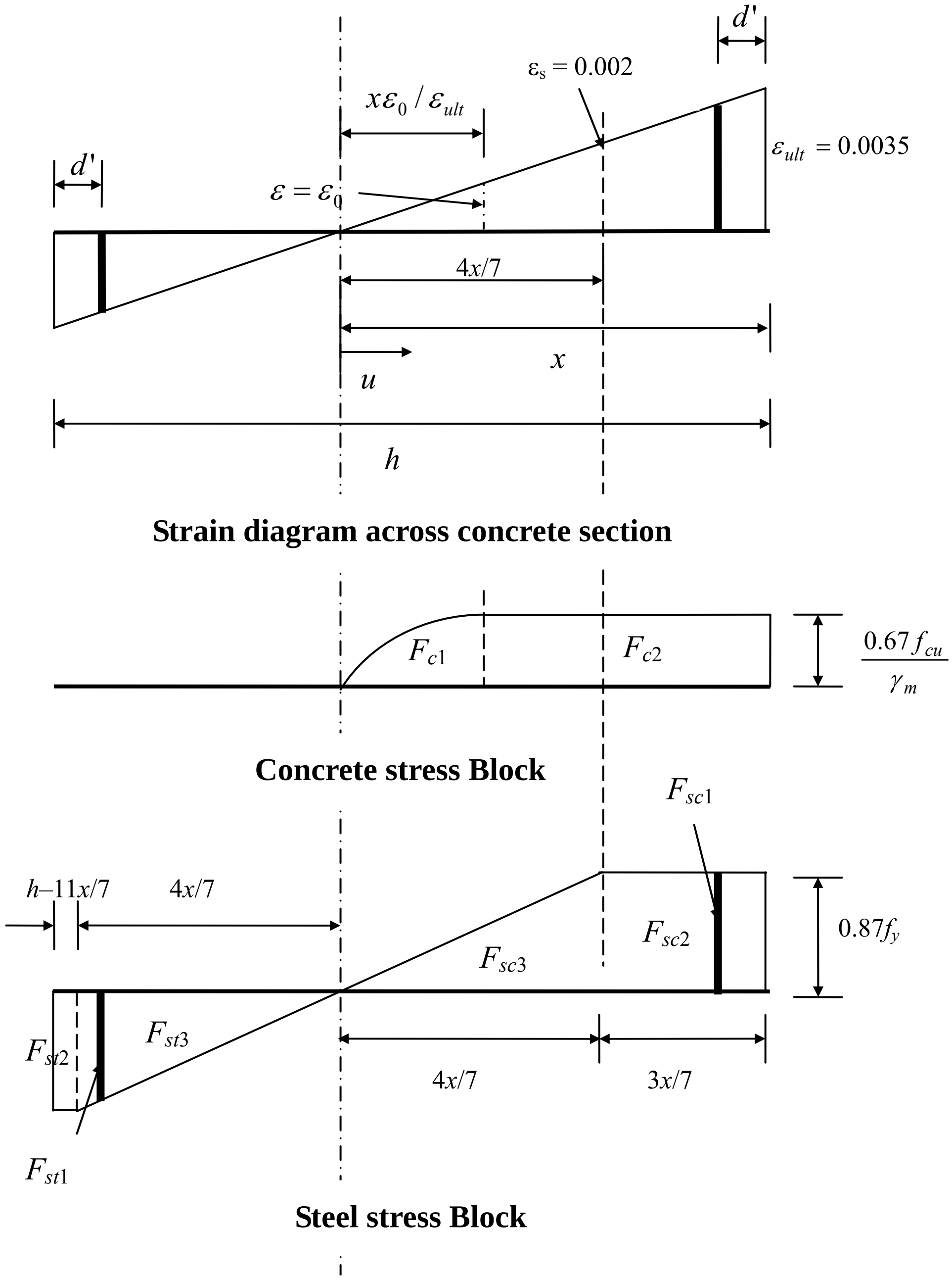


Figure F-3 – Concrete and steel stress strain relation for Case 3

To balance the external moment M_u, all components are identical to Case 2 except that by F_{st1} which is

$$\frac{M_{st1}}{bh^2}=0.87 f_y A_{sb}\frac{7}{8}\left(\frac{h}{x}-\frac{d'}{x}-1\right)\left(\frac{h}{2}-d'\right)\frac{1}{bh^2}=0.87 f_y\frac{A_{sb}}{bh}\frac{7}{8}\left(\frac{h}{x}-\frac{d'}{x}-1\right)\left(\frac{1}{2}-\frac{d'}{h}\right)$$

(Eqn F-45)

$$\therefore \frac{M_u}{bh^2} = \frac{0.67 f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left\{\left(\frac{1}{2}-\frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)+\left[\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{1}{2}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{h}\right)\right\}$$

$$+0.87 f_y\left\{\left(\frac{1}{2}-\frac{d'}{h}\right)\left(\frac{7}{8}\frac{h}{x}-\frac{7}{8}\frac{d'}{h}\frac{h}{x}-\frac{3}{8}\right)\frac{A_{sb}}{bh}+\left[\left(\frac{x}{h}\right)-\frac{163}{147}\left(\frac{x}{h}\right)^2\right]\frac{A_{sh}}{bh}\right\} \quad \text{(Eqn F-46)}$$

So, by pre-determining the steel ratios for $\frac{A_{sb}}{bh}$ and $\frac{A_{sh}}{bh}$, we can solve for $\frac{x}{h}$ by (Eqn F-44) under the applied load N_u. The moment of resistance M_u can then be obtained by (Eqn F-46). The section is adequate if M_u is greater than the applied moment.

<u>Case 4 – where $x \le h < 11x/7$, i.e. $7/11 \le x/h < 1$</u>

The concrete / steel stress / strain diagram is worked out as in Figure 3-4. The stress components are identical to Case 3 except that F_{st2} vanishes and F_{st3} reduces as indicated in Figure F-4 :

Steel tensile force in the portion steel elastic zone by A_{sh} is

$$F_{st3} = 0.87 f_y \times \frac{A_{sh}}{h}(h-x)\times\frac{1}{2}\frac{h-x}{4x/7} = 0.87 f_y \times \frac{7}{8}\frac{(h-x)^2}{hx}A_{sh} = 0.87 f_y \times \frac{7}{8}\left(\frac{h}{x}+\frac{x}{h}-2\right)A_{sh}$$

(Eqn F-47)

To balance the external load N_u

$$\Rightarrow \frac{N_u}{bh} = \frac{0.67 f_{cu} bx}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)+0.87 f_y\frac{A_{sb}}{bh}\left(\frac{11}{8}-\frac{7}{8}\frac{h}{x}+\frac{7}{8}\frac{d'}{x}\right)$$

$$+0.87 f_y \times A_{sh}\frac{5}{7}\frac{x}{h}-0.87 f_y \times \frac{7}{8}\left(\frac{h}{x}+\frac{x}{h}-2\right)A_{sh}$$

$$\Rightarrow \frac{N_u}{bh} = \frac{0.67 f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}+0.87 f_y\frac{A_{sb}}{bh}\left(\frac{11}{8}-\frac{7}{8}\frac{h}{x}+\frac{7}{8}\frac{d'}{x}\right)+0.87 f_y\frac{A_{sh}}{bh}\left(-\frac{9}{56}\frac{x}{h}-\frac{7}{8}\frac{h}{x}+\frac{7}{4}\right)$$

(Eqn F-48)

$$\Leftrightarrow \left[\frac{0.67 f_{cu}}{3\gamma_m}\left(3-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)-\frac{9}{56}\times 0.87 f_y\frac{A_{sh}}{bh}\right]\left(\frac{x}{h}\right)^2+\left[0.87 f_y\left(\frac{A_{sb}}{bh}\frac{11}{8}+\frac{A_{sh}}{bh}\frac{7}{4}\right)-\frac{N_u}{bh}\right]\left(\frac{x}{h}\right)$$

$$+0.87 f_y\frac{7}{8}\left[\frac{A_{sb}}{bh}\left(\frac{d'}{h}-1\right)-\frac{A_{sh}}{bh}\right]=0 \quad \text{(Eqn F-49)}$$

To balance the external load M_u about the centre of the column section

$$\frac{M_u}{bh^2} = \frac{M_c}{bh^2}+\frac{M_{sc1}+M_{st1}}{bh^2}+\frac{M_{sc2}}{bh^2}+\frac{M_{sc3}}{bh^2}+\frac{M_{st3}}{bh^2}$$

$$\Rightarrow \frac{M_u}{bh^2} = \frac{M_c}{bh^2}+\frac{M_{sc1}+M_{st1}}{bh^2}+\frac{F_{sc2}}{bh^2}\left(\frac{h}{2}-\frac{3x}{14}\right)+\frac{F_{sc3}}{bh^2}\left(\frac{h}{2}-\frac{3x}{7}-\frac{4x}{21}\right)+\frac{F_{st3}}{bh^2}\left[\frac{h}{2}-\left(\frac{h-x}{3}\right)\right]$$

(Eqn F-50)

Total Moment $\dfrac{M}{bh^2} = \dfrac{M_c}{bh^2} + \dfrac{M_s}{bh^2}$, i.e.

$$\frac{M_u}{bh^2} = \frac{0.67 f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left\{\left(\frac{1}{2} - \frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}}\right) + \left[\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{2} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{h}\right)\right\}$$

$$+0.87 f_y\left\{\left(\frac{1}{2} - \frac{d'}{h}\right)\left[\frac{7}{8}\left(\frac{h}{x}\right) - \frac{7}{8}\left(\frac{d'}{h}\right)\left(\frac{h}{x}\right) - \frac{3}{8}\right]\frac{A_{sb}}{bh} + \left[\frac{7}{48}\frac{h}{x} - \frac{9}{112}\frac{x}{h} + \frac{9}{392}\left(\frac{x}{h}\right)^2\right]\frac{A_{sh}}{bh}\right\}$$

(Eqn F-51)

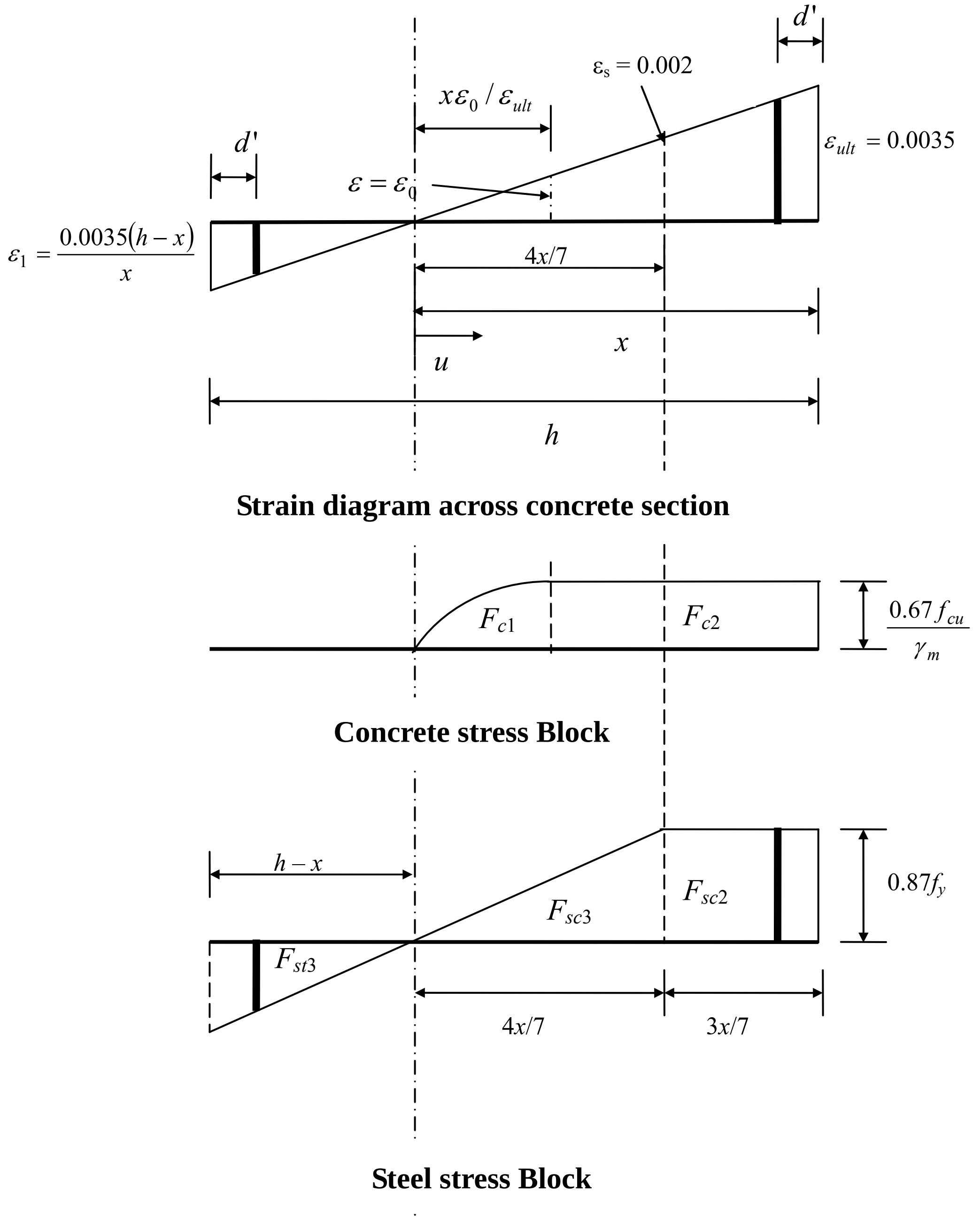


Figure 3-4 – Concrete and steel stress strain relation for Case 4

Case 5 – where $x > h > (1 - \varepsilon_0/\varepsilon_{ult})x$, i.e $1 \le x/h < 1/(1 - \varepsilon_0/\varepsilon_{ult})$

The concrete / steel stress / strain diagram is worked out as follows. It should be noted that the neutral axis depth ratio is greater than unity and hence becomes a hypothetical concept :

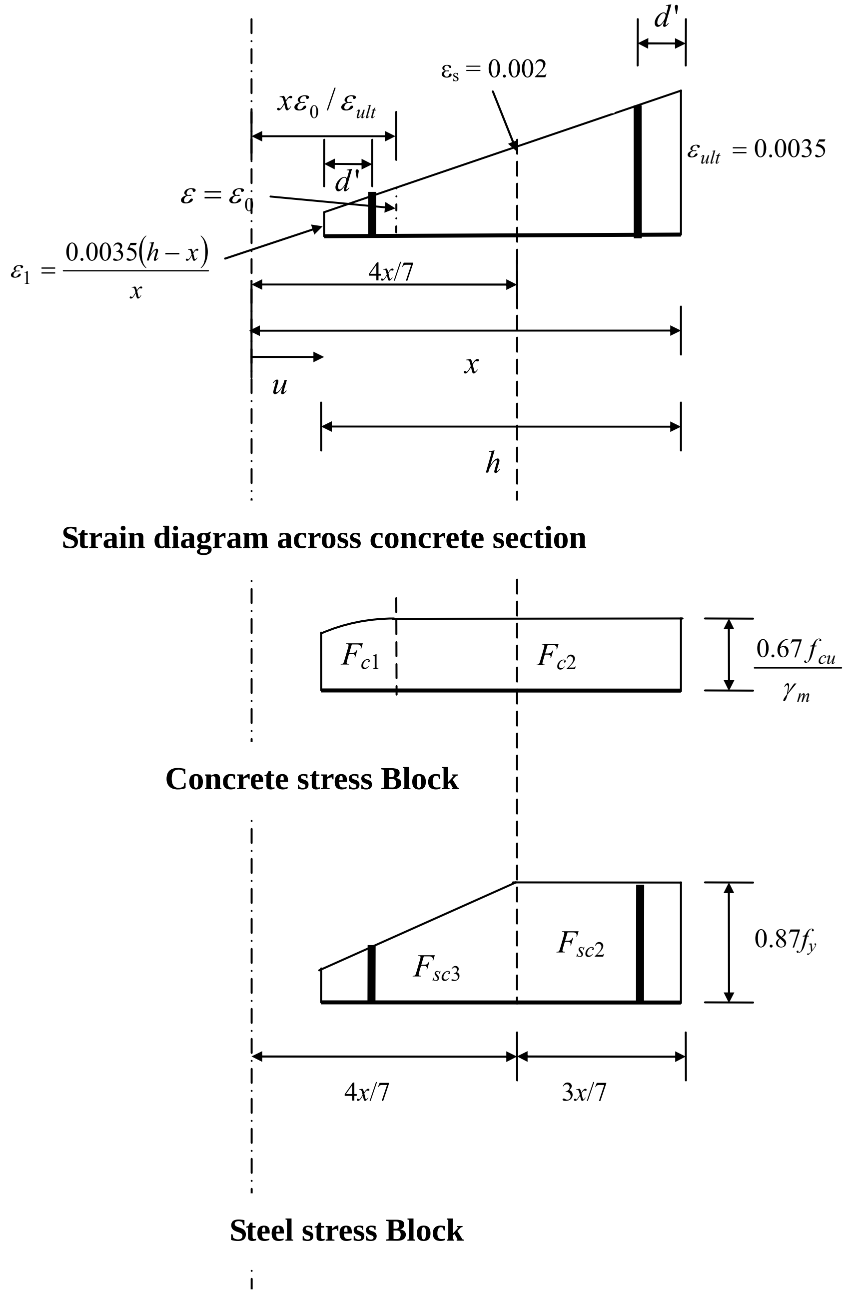


Figure F-5 – Concrete and steel stress strain relation for Case 5

Concrete compressive stresses and forces

$$F_{c1} = \int_{x-h}^{x\varepsilon_0/\varepsilon_{ult}} \sigma b du \quad \text{where} \quad \sigma = -\frac{E_c \varepsilon_{ult}^{\ 2}}{2\varepsilon_0 x^2} u^2 + \frac{E_c \varepsilon_{ult}}{x} u \qquad \text{(Eqn F-52)}$$

$$F_{c1}=\int_{x-h}^{x\varepsilon_0/\varepsilon_{ult}}\left[-\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0x^2}u^2+\frac{E_c\varepsilon_{ult}}{x}u\right]bdu=-\frac{E_c\varepsilon_{ult}^{\ 2}b}{2\varepsilon_0x^2}\int_{x-h}^{x\varepsilon_0/\varepsilon_{ult}}u^2du+\frac{E_c\varepsilon_{ult}b}{x}\int_{x-h}^{x\varepsilon_0/\varepsilon_{ult}}u\,du$$

$$\Rightarrow\frac{F_{c1}}{bh}=\left[\frac{E_c\varepsilon_{ult}}{2}\left(\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}-1\right)-\frac{E_c\varepsilon_{ult}^{\ 2}}{6\varepsilon_0}\left(\frac{\varepsilon_0^{\ 3}}{\varepsilon_{ult}^{\ 3}}-1\right)\right]\frac{x}{h}+\left(E_c\varepsilon_{ult}-\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0}\right)$$
$$+\left(\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0}-\frac{E_c\varepsilon_{ult}}{2}\right)\frac{h}{x}-\frac{E_c\varepsilon_{ult}^{\ 2}}{6\varepsilon_0}\left(\frac{h}{x}\right)^2 \qquad \text{(Eqn F-53)}$$

$$F_{c2}=\frac{0.67f_{cu}bx}{\gamma_m}\left(1-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\Leftrightarrow\frac{F_{c2}}{bh}=\frac{0.67f_{cu}}{\gamma_m}\left(1-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h} \qquad \text{(Eqn F-54)}$$

$$\frac{F_c}{bh}=\frac{F_{c1}}{bh}+\frac{F_{c2}}{bh}=\left[\frac{E_c\varepsilon_{ult}}{2}\left(\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}-1\right)-\frac{E_c\varepsilon_{ult}^{\ 2}}{6\varepsilon_0}\left(\frac{\varepsilon_0^{\ 3}}{\varepsilon_{ult}^{\ 3}}-1\right)\right]\frac{x}{h}+\left(E_c\varepsilon_{ult}-\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0}\right)$$
$$+\left(\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0}-\frac{E_c\varepsilon_{ult}}{2}\right)\frac{h}{x}-\frac{E_c\varepsilon_{ult}^{\ 2}}{6\varepsilon_0}\left(\frac{h}{x}\right)^2+\frac{0.67f_{cu}}{\gamma_m}\left(1-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}$$

$$\frac{F_c}{bh}=\left[\frac{E_c\varepsilon_{ult}}{2}\left(\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}-1\right)-\frac{E_c\varepsilon_{ult}^{\ 2}}{6\varepsilon_0}\left(\frac{\varepsilon_0^{\ 3}}{\varepsilon_{ult}^{\ 3}}-1\right)+\frac{E_c\varepsilon_0}{2}\left(1-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right]\frac{x}{h}+\left(E_c\varepsilon_{ult}-\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0}\right)$$
$$+\left(\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0}-\frac{E_c\varepsilon_{ult}}{2}\right)\frac{h}{x}-\frac{E_c\varepsilon_{ult}^{\ 2}}{6\varepsilon_0}\left(\frac{h}{x}\right)^2 \qquad \text{(Eqn F-55)}$$

Steel compressive force in the portion steel plastic zone by A_{sb} is

$$F_{sc1}=0.87f_y\times0.5A_{sb}\Leftrightarrow\frac{F_{sc1}}{bh}=0.87f_y\times0.5\frac{A_{sb}}{bh} \qquad \text{(Eqn F-56)}$$

$$F_{sc1'}=\frac{0.87f_y\left(x-h+d'\right)}{4x/7}\times0.5A_{sb}=0.87f_y\frac{\left(x-h+d'\right)}{x}\frac{7}{8}A_{sb}=0.87f_y\left(1-\frac{h}{x}+\frac{d'}{x}\right)\frac{7}{8}A_{sb}$$

$$\Rightarrow\frac{F_{sc1'}}{bh}=0.87f_y\left(1-\frac{h}{x}+\frac{d'}{x}\right)\frac{7}{8}\frac{A_{sb}}{bh}=0.87f_y\left(1-\frac{h}{x}+\frac{d'}{h}\frac{h}{x}\right)\frac{7}{8}\frac{A_{sb}}{bh}=0.87f_y\left[1-\frac{h}{x}\left(1-\frac{d'}{h}\right)\right]\frac{7}{8}\frac{A_{sb}}{bh}$$

$$\frac{F_{sc1}}{bh}+\frac{F_{sc1'}}{bh}=0.87f_y\times0.5\frac{A_{sb}}{bh}+0.87f_y\left[1-\frac{h}{x}\left(1-\frac{d'}{h}\right)\right]\frac{7}{8}\frac{A_{sb}}{bh}=0.87f_y\left[\frac{11}{8}-\frac{7}{8}\frac{h}{x}\left(1-\frac{d'}{h}\right)\right]\frac{A_{sb}}{bh}$$
$$\text{(Eqn F-57)}$$

Steel compressive force in the portion steel plastic zone by A_{sh} is

$$F_{sc2}=0.87f_y\times\frac{A_{sh}}{h}\left(\frac{3x}{7}\right)=0.87f_y\times A_{sh}\left(\frac{3}{7}\frac{x}{h}\right)\Leftrightarrow\frac{F_{sc2}}{bh}=0.87f_y\left(\frac{3}{7}\frac{x}{h}\right)\frac{A_{sh}}{bh} \qquad \text{(Eqn F-58)}$$

Steel compressive force in the portion steel elastic zone by A_{sh} is

$$F_{sc3}=0.87f_y\times\frac{x-h}{4x/7}\times\frac{A_{sh}}{h}\left(h-\frac{3x}{7}\right)+0.87f_y\left(1-\frac{x-h}{4x/7}\right)\times\frac{A_{sh}}{h}\left(h-\frac{3x}{7}\right)\times\frac{1}{2}$$

$$\Rightarrow\frac{F_{sc3}}{bh}=0.87f_y\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{33}{56}\frac{x}{h}\right)\frac{A_{sh}}{bh} \qquad \text{(Eqn F-59)}$$

As $N_u = F_c + F_{sc1} + F_{sc1'} + F_{sc2} + F_{sc3} \Leftrightarrow \frac{N_u}{bh} = \frac{F_c}{bh} + \frac{F_{sc1}}{bh} + \frac{F_{sc1'}}{bh} + \frac{F_{sc2}}{bh} + \frac{F_{sc3}}{bh}$

$$\Rightarrow \frac{N_u}{bh} = \left[E_c\varepsilon_{ult}\left(-\frac{1}{6}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}} + \frac{1}{2}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{2} + \frac{1}{6}\frac{\varepsilon_{ult}}{\varepsilon_0}\right) - 0.87f_y\frac{9}{56}\frac{A_{sh}}{bh}\right]\frac{x}{h}$$
$$+\left(E_c\varepsilon_{ult} - \frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0} + 0.87f_y\frac{11}{8}\frac{A_{sb}}{bh} + 0.87f_y\frac{7}{4}\frac{A_{sh}}{bh}\right)$$
$$+\left[\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0} - \frac{E_c\varepsilon_{ult}}{2} + 0.87f_y\frac{7}{8}\frac{A_{sb}}{bh}\left(\frac{d'}{h} - 1\right) - 0.87f_y\frac{7}{8}\frac{A_{sh}}{bh}\right]\frac{h}{x} - \frac{E_c\varepsilon_{ult}^{\ 2}}{6\varepsilon_0}\left(\frac{h}{x}\right)^2 \quad \text{(EqnF-60)}$$

Re-arranging (Eqn F-60)

$$\left[E_c\varepsilon_{ult}\left(-\frac{1}{6}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}} + \frac{1}{2}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{2} + \frac{1}{6}\frac{\varepsilon_{ult}}{\varepsilon_0}\right) - 0.87f_y\frac{9}{56}\frac{A_{sh}}{bh}\right]\left(\frac{x}{h}\right)^3$$
$$+\left(E_c\varepsilon_{ult} - \frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0} + 0.87f_y\frac{11}{8}\frac{A_{sb}}{bh} + 0.87f_y\frac{7}{4}\frac{A_{sh}}{bh} - \frac{N_u}{bh}\right)\left(\frac{x}{h}\right)^2$$
$$+\left[\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0} - \frac{E_c\varepsilon_{ult}}{2} + 0.87f_y\frac{A_{sb}}{bh}\frac{7}{8}\left(\frac{d'}{h} - 1\right) - 0.87f_y\frac{7}{8}\frac{A_{sh}}{bh}\right]\frac{x}{h} - \frac{E_c\varepsilon_{ult}^{\ 2}}{6\varepsilon_0} = 0$$

(Eqn F-61)

which is a cubic equation in $\frac{x}{h}$.

Summing the Moments as follows :

Concrete compressive stresses and moments

$$M_{c1} = \int_{x-h}^{x\varepsilon_0/\varepsilon_{ult}} \sigma b du\left(\frac{h}{2} - x + u\right) \quad \text{where} \quad \sigma = -\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0 x^2}u^2 + \frac{E_c\varepsilon_{ult}}{x}u$$

$$M_{c1} = \int_{x-h}^{x\varepsilon_0/\varepsilon_{ult}} \left[-\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0 x^2}u^2 + \frac{E_c\varepsilon_{ult}}{x}u\right] b\left(\frac{h}{2} - x + u\right)du$$

$$\frac{M_{c1}}{bh^2} = E_c\varepsilon_{ult}\left[\frac{1}{2}\left(\frac{1}{2} - \frac{x}{h}\right)\left[\left(\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}} - 1\right)\frac{x}{h} + 2 - \frac{h}{x}\right] + \frac{1}{3}\left[\left(\frac{\varepsilon_0^{\ 3}}{\varepsilon_{ult}^{\ 3}} - 1\right)\left(\frac{x}{h}\right)^2 + 3\frac{x}{h} - 3 + \frac{h}{x}\right]\right]$$
$$-\frac{E_c\varepsilon_{ult}^{\ 2}}{2\varepsilon_0}\left[\frac{1}{3}\left(\frac{1}{2} - \frac{x}{h}\right)\left[\left(\frac{\varepsilon_0^{\ 3}}{\varepsilon_{ult}^{\ 3}} - 1\right)\frac{x}{h} + 3 - 3\frac{h}{x} + \left(\frac{h}{x}\right)^2\right] + \frac{1}{4}\left[\left(\frac{\varepsilon_0^{\ 4}}{\varepsilon_{ult}^{\ 4}} - 1\right)\left(\frac{x}{h}\right)^2 + 4\frac{x}{h} - 6 + 4\frac{h}{x} - \left(\frac{h}{x}\right)^2\right]\right]$$

$$\frac{M_{c2}}{bh^2} = \frac{0.67f_{cu}bx}{\gamma_m}\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\left[\frac{h}{2} - \left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{2}\right]\frac{1}{bh^2} = \frac{0.67f_{cu}}{2\gamma_m}\left(\frac{x}{h}\right)\left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\left[1 - \left(1 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}\right]$$

$$\frac{M_{sc1}}{bh^2} = \frac{0.87f_y \times 0.5A_{sb}}{bh^2}\left(\frac{h}{2} - d'\right) = 0.87f_y \times 0.5\frac{A_{sb}}{bh}\left(\frac{1}{2} - \frac{d'}{h}\right)$$

$$\frac{M_{sc1'}}{bh^2} = -0.87f_y\left(1 - \frac{h}{x} + \frac{d'}{x}\right)\frac{7}{8}\frac{A_{sb}}{bh}\left(\frac{h}{2} - d'\right)\frac{1}{h} = -0.87f_y\frac{7}{8}\left(1 - \frac{h}{x} + \frac{d'}{h}\frac{h}{x}\right)\left(\frac{1}{2} - \frac{d'}{h}\right)\frac{A_{sb}}{bh}$$

$$= -0.87 f_y \frac{7}{8}\left[1+\left(\frac{d'}{h}-1\right)\frac{h}{x}\right]\left(\frac{1}{2}-\frac{d'}{h}\right)\frac{A_{sb}}{bh}$$

Total Moment $\frac{M_u}{bh^2}=\frac{M_{c1}}{bh^2}+\frac{M_{c2}}{bh^2}+\frac{M_{sc1}}{bh^2}+\frac{M_{sc1'}}{bh^2}+\frac{M_{sc2}}{bh^2}+\frac{M_{sc3}}{bh^2}$

$$\Rightarrow \frac{M_u}{bh^2}=E_c\varepsilon_{ult}\left(-\frac{1}{24}\frac{\varepsilon_0^{\ 3}}{\varepsilon_{ult}^{\ 3}}+\frac{1}{6}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}-\frac{1}{4}\frac{\varepsilon_0}{\varepsilon_{ult}}+\frac{1}{6}-\frac{1}{24}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)\left(\frac{x}{h}\right)^2$$

$$+E_c\varepsilon_{ult}\left[-\frac{1}{12}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}+\frac{1}{4}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{1}{4}+\frac{1}{12}\frac{\varepsilon_{ult}}{\varepsilon_0}\right]\frac{x}{h}+E_c\varepsilon_{ult}\left(\frac{1}{12}-\frac{1}{12}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)\frac{h}{x}+E_c\varepsilon_{ult}\frac{1}{24}\frac{\varepsilon_{ult}}{\varepsilon_0}\left(\frac{h}{x}\right)^2$$

$$+0.87 f_y\frac{A_{sb}}{bh}\times\left(\frac{1}{2}-\frac{d'}{h}\right)\left[\frac{7}{8}\left(1-\frac{d'}{h}\right)\frac{h}{x}-\frac{3}{8}\right]+0.87 f_y\left[\frac{7}{48}\frac{h}{x}-\frac{9}{112}\frac{x}{h}+\frac{9}{392}\left(\frac{x}{h}\right)^2\right]\frac{A_{sh}}{bh}$$

(Eqn F-62)

<u>Case 6 – where $(1-\varepsilon_0/\varepsilon_{ult})x>h>3x/7$, i.e. $1/(1-\varepsilon_0/\varepsilon_{ult})\le x/h<7/3$</u>

Case 6 is similar to Case 5 except that F_{c1} vanishes. The concrete / steel stress / strain diagram is worked out as in Figure F-6 :

Referring to (Eqn F-55) by replacing $F_{c1}+F_{c2}$ by $\frac{0.67 f_{cu}}{\gamma_m}$

$$\frac{N_u}{bh}=\frac{0.67 f_{cu}}{\gamma_m}+0.87 f_y\left[\frac{11}{8}-\frac{7}{8}\frac{h}{x}\left(1-\frac{d'}{h}\right)\right]\frac{A_{sb}}{bh}+0.87 f_y\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{9}{56}\frac{x}{h}\right)\frac{A_{sh}}{bh}$$

(Eqn F-63)

Re-arranging $\Leftrightarrow 0.87 f_y\frac{9}{56}\frac{A_{sh}}{bh}\left(\frac{x}{h}\right)^2+\left[\frac{N_u}{bh}-\frac{0.67 f_{cu}}{\gamma_m}-0.87 f_y\left(\frac{11}{8}\frac{A_{sb}}{bh}+\frac{7}{4}\frac{A_{sh}}{bh}\right)\right]\left(\frac{x}{h}\right)$

$$-0.87 f_y\left[\frac{7}{8}\left(\frac{d'}{h}-1\right)\frac{A_{sb}}{bh}-\frac{7}{8}\frac{A_{sh}}{bh}\right]=0$$

(Eqn F-64)

which is a quadratic equation in $\frac{x}{h}$ which can be solved

For Moment that can be provided by the section, similar to Case 5 except that $M_c=0$. So

$$\frac{M_u}{bh^2}=0.87 f_y\frac{A_{sb}}{bh}\times\left(\frac{1}{2}-\frac{d'}{h}\right)\left[\frac{7}{8}\left(1-\frac{d'}{h}\right)\frac{h}{x}-\frac{3}{8}\right]+0.87 f_y\left[\frac{7}{48}\frac{h}{x}-\frac{9}{112}\frac{x}{h}+\frac{9}{392}\left(\frac{x}{h}\right)^2\right]\frac{A_{sh}}{bh}$$

(Eqn F-65)

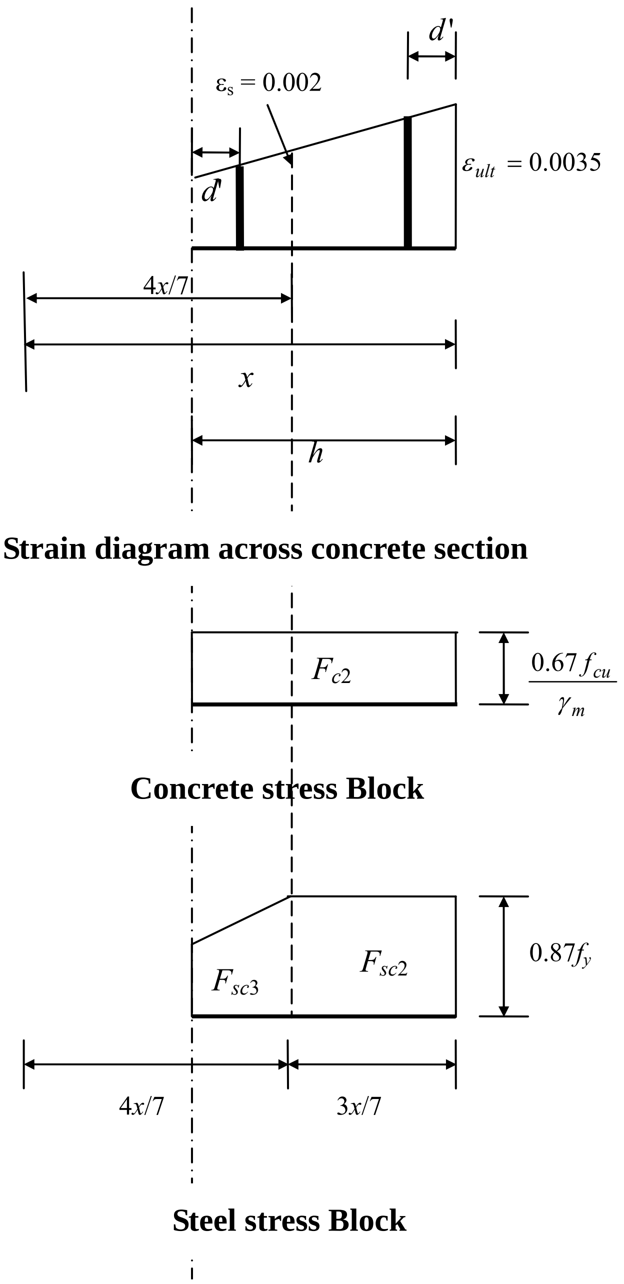


Figure 3-6 – Concrete and steel stress strain relation for Case 6

<u>Case 7 – where $x/h \geq 7/3$</u>

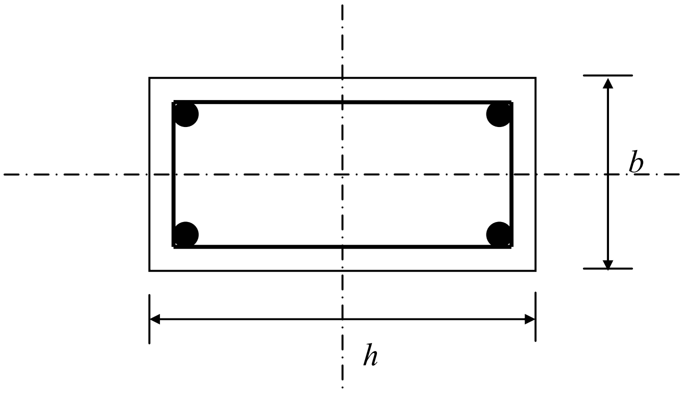
In this case, the concrete and steel in the entire column section are under ultimate stress. The axial load will be simply

$$\frac{N_u}{bh} = \frac{0.67 f_{cu}}{\gamma_m} + 0.87 f_y \left(\frac{A_{sb}}{bd} + \frac{A_{sh}}{bd} \right) \qquad \text{(Eqn F-66)}$$

and the moment is zero.

$$\frac{M_u}{bh^2} = 0 \qquad \text{(Eqn F-67)}$$

(III) Design formulae for 4-bar column sections for determination of reinforcement ratios



It is the aim of the section of the Appendix to derive formulae for the determination of $\frac{A_{sb}}{bh}$ against applied axial load and moment under a pre-determined sectional size. In the following derivations, $\frac{A_{sh}}{bh}$ are set to zero. The process involves :

(i) For the 7 cases discussed in the foregoing, eliminate $\frac{A_{sb}}{bh}$ between equations obtained from balancing $\frac{N_u}{bh}$ and $\frac{M_u}{bh^2}$ by making $\frac{A_{sb}}{bh}$ subject of formulae in the equation for balancing of $\frac{N_u}{bh}$ substitute into the equation for balancing of $\frac{M_u}{bh^2}$. The equation obtained in a polynomial in $\frac{x}{h}$ which can be solved by equations (if quadratic or cubic or even 4th power) or by numerical methods. Solution in $\frac{x}{h}$ will be valid if the value arrived at agree with the pre-determined range of the respective case;

(ii) Back substitute the accepted value of $\frac{x}{h}$ into the equation obtained by balancing $\frac{N_u}{bh}$ to solve for $\frac{A_{sb}}{bh}$.

<u>Case 1 (a) – where (i) $x/h < 7/3(d'/h)$ for $d'/h \le 3/14$; and (ii) $x/h < 7/11(1 - d'/h)$ for $d'/h > 3/14$</u>

Putting $\frac{A_{sh}}{bh} = 0$;

(Eqn F-18) $\Rightarrow \frac{N_u}{bh} = \frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h} + \left(\frac{3}{8} - \frac{7}{8}\frac{d'}{h}\frac{h}{x}\right)0.87 f_y \frac{A_{sb}}{bh}$

$$\Rightarrow 0.87 f_y \frac{A_{sb}}{bh} = \frac{\dfrac{N_u}{bh} - \dfrac{0.67 f_{cu}}{\gamma_m}\left(1 - \dfrac{1}{3}\dfrac{\varepsilon_0}{\varepsilon_{ult}}\right)\dfrac{x}{h}}{\left(\dfrac{3}{8} - \dfrac{7}{8}\dfrac{d'}{h}\dfrac{h}{x}\right)} \qquad \text{(Eqn F-68)}$$

Substituting into (Eqn F-34)

$$\Rightarrow \frac{M_u}{bh^2} = \frac{0.67 f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left\{\frac{1}{2} - \frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}} + \left[-\frac{1}{2} + \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\frac{x}{h}\right\}$$

$$+ 0.87 f_y \frac{A_{sb}}{bh}\left(\frac{1}{2} - \frac{d'}{h}\right)\left(\frac{11}{8} - \frac{7}{8}\frac{d'}{h}\frac{h}{x}\right)$$

$$\Rightarrow \frac{0.67 f_{cu}}{\gamma_m}\frac{3}{8}\left[-\frac{1}{2} + \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{h}\right)^3$$

$$+ \frac{0.67 f_{cu}}{\gamma_m}\left[-\frac{1}{2} + \frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}} + \frac{29}{16}\frac{d'}{h} - \frac{3}{4}\frac{d'}{h}\frac{\varepsilon_0}{\varepsilon_{ult}} + \frac{7}{96}\frac{d'}{h}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{h}\right)^2$$

$$+ \left[\frac{N_u}{bh}\frac{11}{8}\left(\frac{1}{2} - \frac{d'}{h}\right) - \frac{3}{8}\frac{M_u}{bh^2} + \frac{0.67 f_{cu}}{\gamma_m}\frac{7}{8}\left(\frac{d'}{h}\right)^2\left(\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} - 1\right)\right]\frac{x}{h}$$

$$+ \left[\frac{7}{8}\frac{M_u}{bh^2}\frac{d'}{h} - \frac{N_u}{bh}\frac{7}{8}\left(\frac{1}{2} - \frac{d'}{h}\right)\frac{d'}{h}\right] = 0 \qquad \text{(Eqn F-69)}$$

Upon solving (Eqn F-69) for $\dfrac{x}{h}$, back-substitution into (Eqn F-68) to calculate $\dfrac{A_{sb}}{bh}$.

Case 1 (b) – $7/11(1 - d'/h) \le x/h < 7/3(d'/h)$ where $d'/h > 3/14$

Putting $\dfrac{A_{sh}}{bh} = 0$; (Eqn F-35)

$$\Rightarrow \frac{N_u}{bh} = \frac{0.67 f_{cu}}{\gamma_m}\left(1 - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h} + \left(\frac{7}{4} - \frac{7}{8}\frac{h}{x}\right)0.87 f_y \frac{A_{sb}}{bh}$$

$$\Rightarrow 0.87 f_y \frac{A_{sb}}{bh} = \frac{\dfrac{N_u}{bh} - \dfrac{0.67 f_{cu}}{\gamma_m}\left(1 - \dfrac{1}{3}\dfrac{\varepsilon_0}{\varepsilon_{ult}}\right)\dfrac{x}{h}}{\left(\dfrac{7}{4} - \dfrac{7}{8}\dfrac{h}{x}\right)} \qquad \text{(Eqn F-70)}$$

Substituting into (Eqn F-38), again putting $\dfrac{A_{sh}}{bh} = 0$ and simplifying,

$$\Rightarrow \frac{0.67 f_{cu}}{\gamma_m}\frac{7}{4}\left[-\frac{1}{2} + \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{h}\right)^3 + \frac{0.67 f_{cu}}{\gamma_m}\left[\frac{21}{16} - \frac{7}{12}\frac{\varepsilon_0}{\varepsilon_{ult}} + \frac{7}{96}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{h}\right)^2$$

$$- \frac{7}{4}\left\{\frac{M_u}{bh^2} + \frac{0.67 f_{cu}}{\gamma_m}\left(\frac{1}{2} - \frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\left[1 - 2\frac{d'}{h} + 2\left(\frac{d'}{h}\right)^2\right]\right\}\frac{x}{h} + \frac{7}{8}\left[\frac{M_u}{bh^2} + \frac{1}{2}\left(1 - 2\frac{d'}{h}\right)^2\frac{N_u}{bh}\right] = 0$$

(Eqn F-71)

Upon solving (Eqn F-71) for $\frac{x}{h}$, back-substitution into (Eqn F-70) to calculate $\frac{A_{sb}}{bh}$.

Case 2 – 7/3(*d'*/*h*) ≤ *x*/*h* < 7/11 – 7/11(*d'*/*h*) and *d'*/*h* > 3/14

Using the equations summarized in Section 4 and setting $\frac{A_{sh}}{bh}=0$ in (Eqn F-39)

$$\frac{x}{h}=\frac{N_u}{bh}\div\left[\frac{0.67f_{cu}}{\gamma_m}\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right] \qquad \text{(Eqn F-72)}$$

Substituting $\frac{x}{h}$ obtained in (Eqn F-72), substituting into (Eqn F-41) and calculate $\frac{A_{sb}}{bh}$ as

$$\frac{A_{sb}}{bh}=\left\{\frac{M_u}{bh^2}-\frac{0.67f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left[\frac{1}{2}\left(1-\frac{x}{h}\right)-\frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}}+\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\left(\frac{x}{h}\right)-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\left(\frac{x}{h}\right)\right]\right\}\div\left[0.87f_y\left(\frac{1}{2}-\frac{d'}{h}\right)\right]$$

(Eqn F-73)

Case 3 – where 7/3(*d'*/*h*) ≤ *x*/*h* < 7/11 for *d'*/*h* > 3/14 and
7/11(1 – *d'*/*h*) ≤ *x*/*h* < 7/11 for *d'*/*h* < 3/14
and Case 4 – where 7/11 ≤ *x*/*h* < 1

Using the equation relating $\frac{x}{h}$ and $\frac{N_u}{bh}$ in (Eqn F-43) and setting $\frac{A_{sh}}{bh}=0$.

$$\frac{N_u}{bh}=\frac{0.67f_{cu}}{3\gamma_m}\left(3-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}+0.87f_y\frac{A_{sb}}{bh}\left(\frac{11}{8}-\frac{7}{8}\frac{h}{x}+\frac{7}{8}\frac{d'}{x}\right)$$

$$\Leftrightarrow\frac{A_{sb}}{bh}=\frac{\dfrac{N_u}{bh}-\dfrac{0.67f_{cu}}{\gamma_m}\left(1-\dfrac{1}{3}\dfrac{\varepsilon_0}{\varepsilon_{ult}}\right)\dfrac{x}{h}}{0.87f_y\left(\dfrac{11}{8}-\dfrac{7}{8}\dfrac{h}{x}+\dfrac{7}{8}\dfrac{d'}{h}\dfrac{h}{x}\right)} \qquad \text{(Eqn F-74)}$$

Substituting $\frac{A_{sh}}{bh}=0$, (Eqn F-46) and simplifying

$$\Leftrightarrow\frac{0.67f_{cu}}{\gamma_m}\frac{11}{8}\left[\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{1}{2}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{h}\right)^3$$

$$+\frac{0.67f_{cu}}{\gamma_m}\left[\frac{21}{16}-\frac{7}{12}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{13}{16}\frac{d'}{h}+\frac{7}{96}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2+\frac{5}{12}\frac{\varepsilon_0}{\varepsilon_{ult}}\frac{d'}{h}-\frac{7}{96}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\frac{d'}{h}\right]\left(\frac{x}{h}\right)^2$$

$$+\left\{\frac{0.67f_{cu}}{\gamma_m}\frac{7}{8}\left(\frac{d'}{h}-1\right)\left(1-\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{d'}{h}+\frac{1}{3}\frac{d'}{h}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)-\frac{3}{8}\left(\frac{1}{2}-\frac{d'}{h}\right)\frac{N_u}{bh}-\frac{11}{8}\frac{M_u}{bh^2}\right\}\left(\frac{x}{h}\right)$$

$$-\frac{7}{8}\left(\frac{1}{2}-\frac{d'}{h}\right)\left(\frac{d'}{h}-1\right)\frac{N_u}{bh}-\frac{7}{8}\frac{M_u}{bh^2}\left(\frac{d'}{h}-1\right)=0 \qquad \text{(Eqn F-75)}$$

which is a cubic equation in $\left(\frac{x}{h}\right)$

Upon solving $\left(\frac{x}{h}\right)$ lying between $\frac{7}{11}\left(1-\frac{d'}{h}\right)$ and 1, $\frac{A_{sb}}{bh}$ can be obtained by back-substituting into (Eqn 5-7)

$$\frac{A_{sb}}{bh}=\frac{\frac{N_u}{bh}-\frac{0.67f_{cu}}{3\gamma_m}\left(3-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}}{0.87f_y\left(\frac{11}{8}-\frac{7}{8}\frac{h}{x}+\frac{7}{8}\frac{d'}{h}\frac{h}{x}\right)} \qquad \text{(Eqn F-76)}$$

<u>Case 5 – where $1 \le x/h < 1/(1-\varepsilon_0/\varepsilon_{ult})$</u>

Referring to Case 5 of Section 3 and setting $\frac{A_{sh}}{bd}=0$ in (Eqn F-60)

Solving $\frac{A_{sb}}{bd}$ by

$$\frac{N_u}{bh}=\left[\frac{E_c\varepsilon_{ult}}{2}\left(\frac{\varepsilon_0^{\,2}}{\varepsilon_{ult}^{\,2}}-1\right)-\frac{E_c\varepsilon_{ult}^{\,2}}{6\varepsilon_0}\left(\frac{\varepsilon_0^{\,3}}{\varepsilon_{ult}^{\,3}}-1\right)+\frac{E_c\varepsilon_0}{2}\left(1-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right]\frac{x}{h}+\left(E_c\varepsilon_{ult}-\frac{E_c\varepsilon_{ult}^{\,2}}{2\varepsilon_0}\right)$$
$$+\left(\frac{E_c\varepsilon_{ult}^{\,2}}{2\varepsilon_0}-\frac{E_c\varepsilon_{ult}}{2}\right)\frac{h}{x}-\frac{E_c\varepsilon_{ult}^{\,2}}{6\varepsilon_0}\left(\frac{h}{x}\right)^2+0.87f_y\left[\frac{11}{8}-\frac{7}{8}\left(\frac{d'}{h}-1\right)\frac{h}{x}\right]\frac{A_{sb}}{bh}$$

$$\Rightarrow 0.87f_y\frac{A_{sb}}{bh}=\left[\frac{N_u}{bh}-\left(-\frac{1}{2}-\frac{1}{6}\frac{\varepsilon_0^{\,2}}{\varepsilon_{ult}^{\,2}}+\frac{1}{6}\frac{\varepsilon_{ult}}{\varepsilon_0}+\frac{1}{2}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}\right]\div\left[\frac{11}{8}-\frac{7}{8}\left(\frac{d'}{h}-1\right)\frac{h}{x}\right]$$
$$+\left[-\left(E_c\varepsilon_{ult}-\frac{E_c\varepsilon_{ult}^{\,2}}{2\varepsilon_0}\right)-\left(\frac{E_c\varepsilon_{ult}^{\,2}}{2\varepsilon_0}-\frac{E_c\varepsilon_{ult}}{2}\right)\frac{h}{x}+\frac{E_c\varepsilon_{ult}^{\,2}}{6\varepsilon_0}\left(\frac{h}{x}\right)^2\right]\div\left[\frac{11}{8}-\frac{7}{8}\left(\frac{d'}{h}-1\right)\frac{h}{x}\right]$$

(Eqn F-77)

Substituting into (Eqn F-62) and again setting $\frac{A_{sh}}{bd}=0$

$$\frac{M_u}{bh^2}=E_c\varepsilon_{ult}\left\{\frac{1}{2}\left(\frac{1}{2}-\frac{x}{h}\right)\left[\left(\frac{\varepsilon_0^{\,2}}{\varepsilon_{ult}^{\,2}}-1\right)\frac{x}{h}+2-\frac{h}{x}\right]+\frac{1}{3}\left[\left(\frac{\varepsilon_0^{\,3}}{\varepsilon_{ult}^{\,3}}-1\right)\left(\frac{x}{h}\right)^2+3\frac{x}{h}-3+\frac{h}{x}\right]\right\}$$
$$-\frac{E_c\varepsilon_{ult}^{\,2}}{2\varepsilon_0}\left[\frac{1}{3}\left(\frac{1}{2}-\frac{x}{h}\right)\left(\left(\frac{\varepsilon_0^{\,3}}{\varepsilon_{ult}^{\,3}}-1\right)\frac{x}{h}+3-3\frac{h}{x}+\left(\frac{h}{x}\right)^2\right)+\frac{1}{4}\left(\left(\frac{\varepsilon_0^{\,4}}{\varepsilon_{ult}^{\,4}}-1\right)\left(\frac{x}{h}\right)^2+4\frac{x}{h}-6+4\frac{h}{x}-\left(\frac{h}{x}\right)^2\right)\right]$$
$$+\frac{0.67f_{cu}}{2\gamma_m}\left(\frac{x}{h}\right)\left(1-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\left[1-\left(1-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}\right]+0.87f_y\frac{A_{sb}}{bh}\times\left(\frac{1}{2}-\frac{d'}{h}\right)\left[\frac{7}{8}\left(1-\frac{d'}{h}\right)\frac{h}{x}-\frac{3}{8}\right]$$

to solve for $\frac{x}{h}$. (Eqn F-78)

Back-substituting into (Eqn 5-10) to solve for $\dfrac{A_{sb}}{bd}$

Case 6 – where $(1-\varepsilon_0/\varepsilon_{ult})x > h > 3x/7$ i.e. $1/(1-\varepsilon_0/\varepsilon_{ult}) \le x/h < 7/3(1-d'/h)$

Referring to (Eqn F-63) of Case 6 of and setting $\dfrac{A_{sh}}{bd}=0$

$$\frac{N_u}{bh}=\frac{0.67f_{cu}}{\gamma_m}+0.87f_y\left[\frac{11}{8}-\frac{7}{8}\frac{h}{x}\left(1-\frac{d'}{h}\right)\right]\frac{A_{sb}}{bh}$$

$$\Rightarrow 0.87f_y\frac{A_{sb}}{bh}=\left(\frac{N_u}{bh}-\frac{0.67f_{cu}}{\gamma_m}\right)\div\left[\frac{11}{8}-\frac{7}{8}\frac{h}{x}\left(1-\frac{d'}{h}\right)\right]$$

Substituting into (Eqn F-64) of Case 6 and again setting $\dfrac{A_{sh}}{bd}=0$

$$\frac{M}{bh^2}=0.87f_y\frac{A_{sb}}{bh}\times\left(\frac{1}{2}-\frac{d'}{h}\right)\left[\frac{7}{8}\left(1-\frac{d'}{h}\right)\frac{h}{x}-\frac{3}{8}\right]$$

$$\Rightarrow\frac{x}{h}=\frac{\frac{7}{8}\left(1-\frac{d'}{h}\right)\left[\left(\frac{N_u}{bh}-\frac{0.67f_{cu}}{\gamma_m}\right)\left(\frac{1}{2}-\frac{d'}{h}\right)+\frac{M}{bh^2}\right]}{\left[\frac{11}{8}\frac{M}{bh^2}+\left(\frac{N_u}{bh}-\frac{0.67f_{cu}}{\gamma_m}\right)\left(\frac{1}{2}-\frac{d'}{h}\right)\frac{3}{8}\right]}$$

With $\dfrac{x}{h}$ determined, calculate $\dfrac{A_{sb}}{bh}$ by

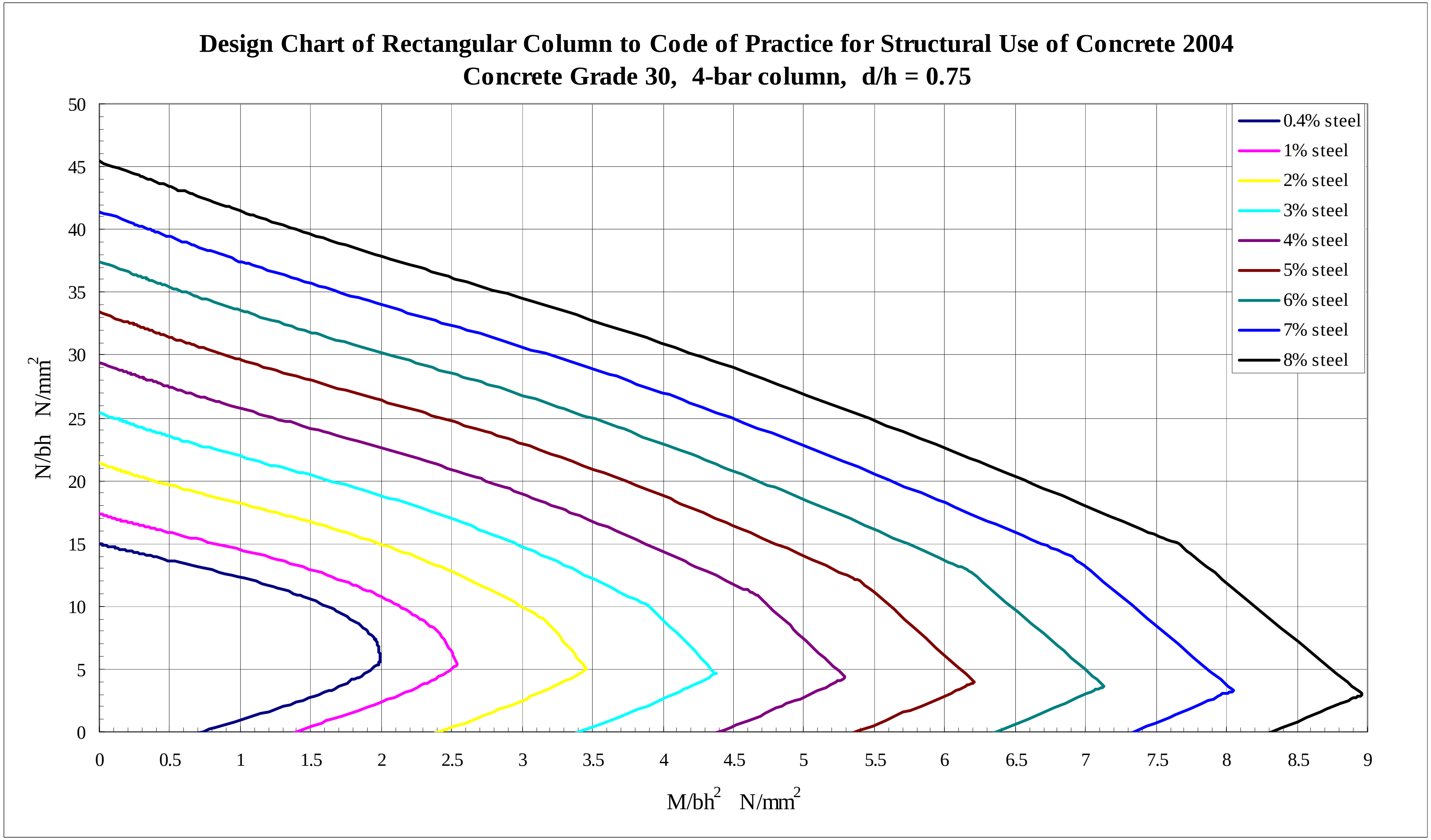
$$\frac{N_u}{bh}=\frac{0.67f_{cu}}{\gamma_m}+0.87f_y\left[\frac{11}{8}-\frac{7}{8}\frac{h}{x}\left(1-\frac{d'}{h}\right)\right]\frac{A_{sb}}{bh}$$

$$\Rightarrow\frac{A_{sb}}{bh}=\frac{\frac{N_u}{bh}-\frac{0.67f_{cu}}{\gamma_m}}{0.87f_y\left[\frac{11}{8}-\frac{7}{8}\frac{h}{x}\left(1-\frac{d'}{h}\right)\right]} \qquad \text{(Eqn F-79)}$$

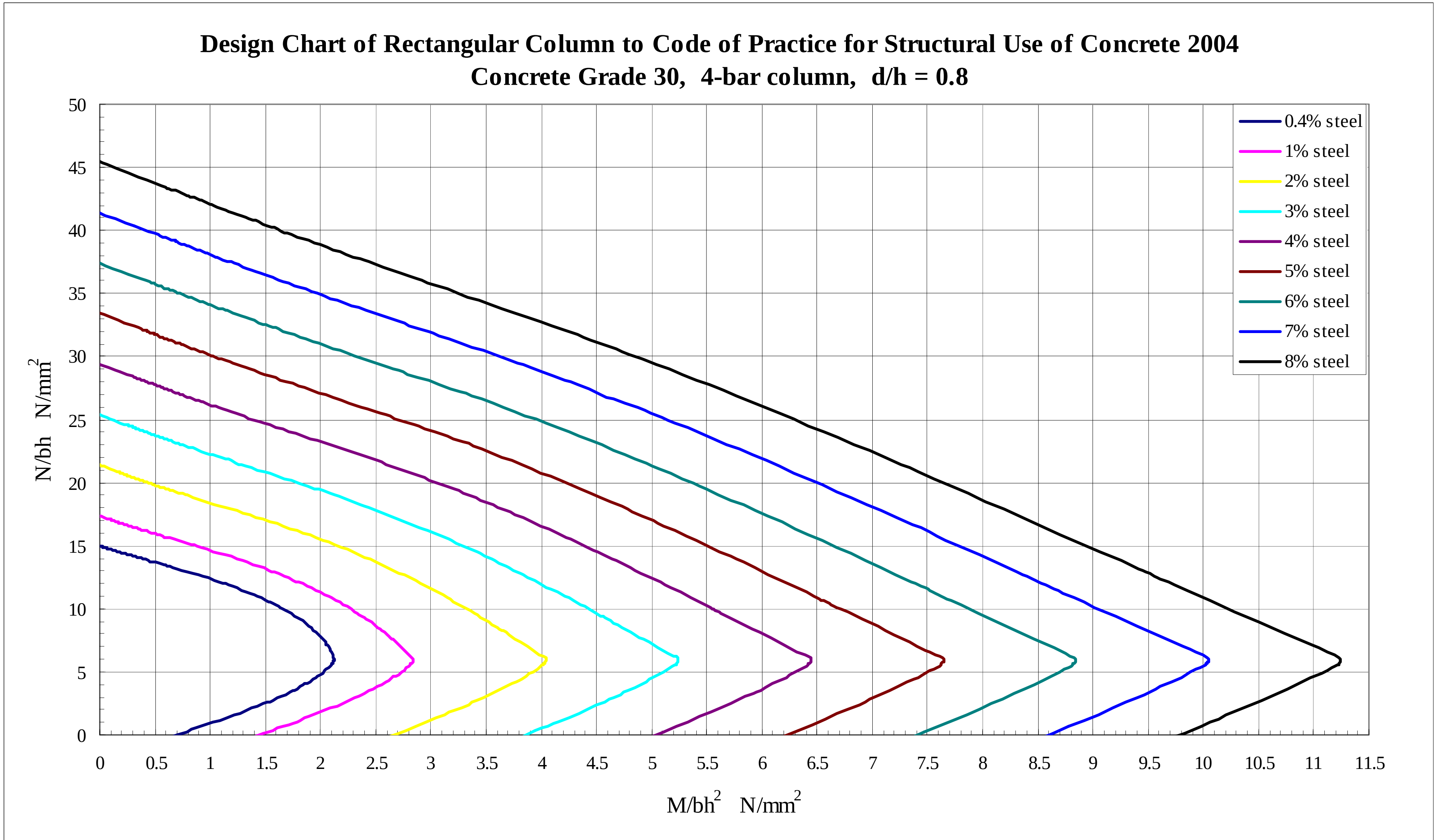
Case 7 – where $x/h \ge 7/3$

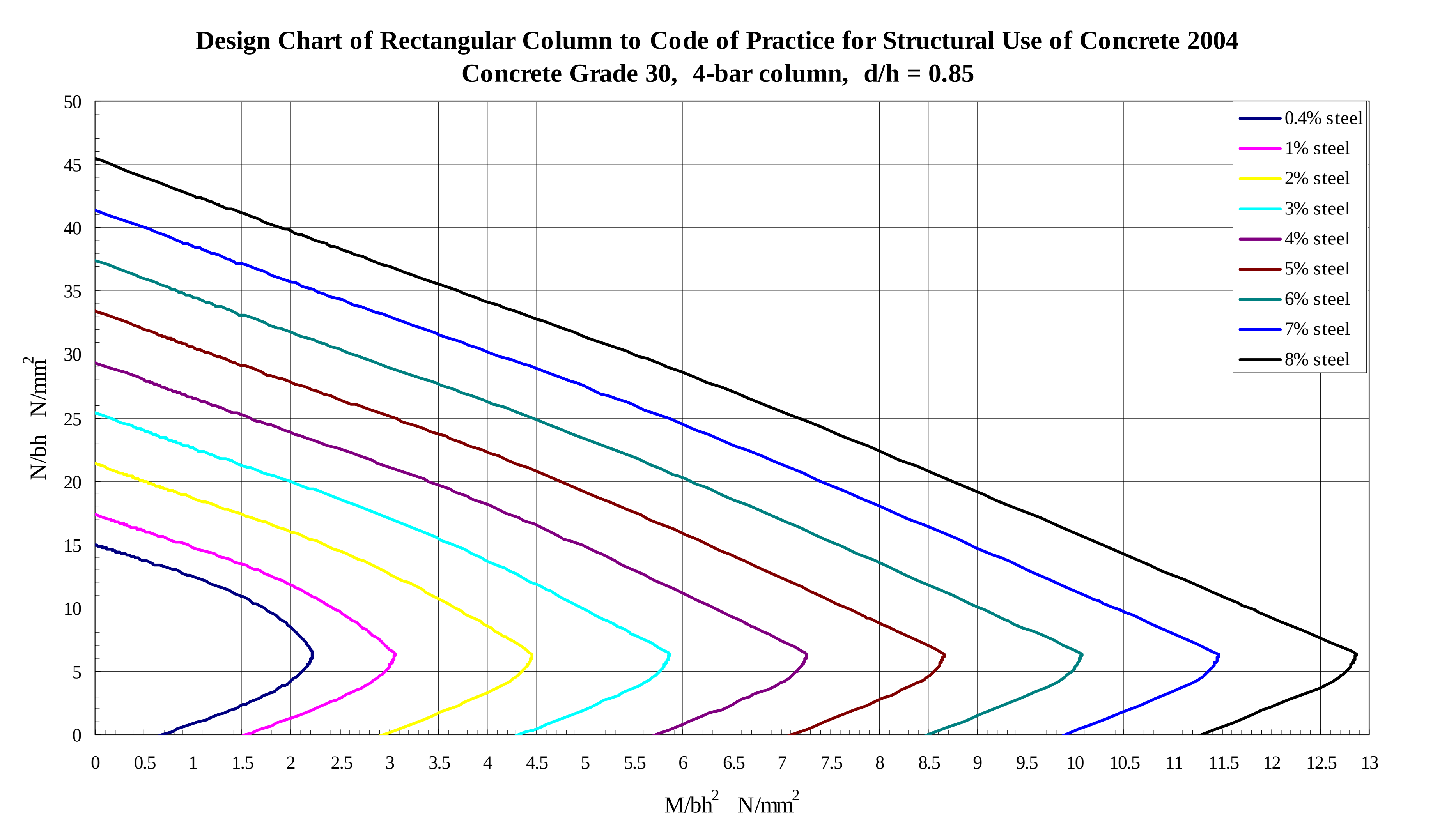
$$\frac{N_u}{bh}=\frac{0.67f_{cu}}{\gamma_m}+0.87f_y\frac{A_{sb}}{bh}\Rightarrow\frac{A_{sb}}{bh}=\left(\frac{N_u}{bh}-\frac{0.67f_{cu}}{\gamma_m}\right)\frac{1}{0.87f_y} \qquad \text{(Eqn F-80)}$$

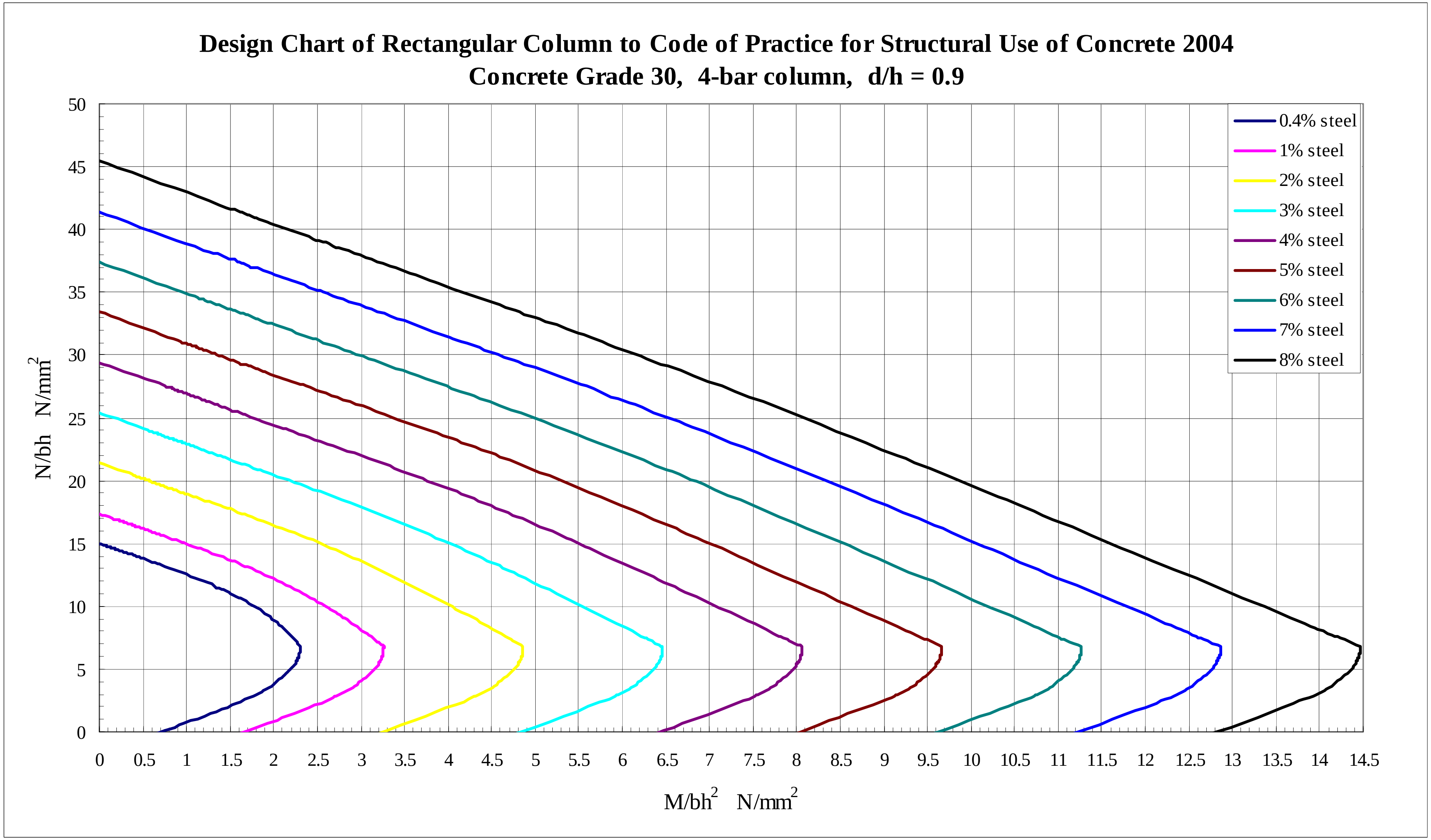
$$\frac{M_u}{bh^2}=0 \qquad \text{(Eqn F-81)}$$

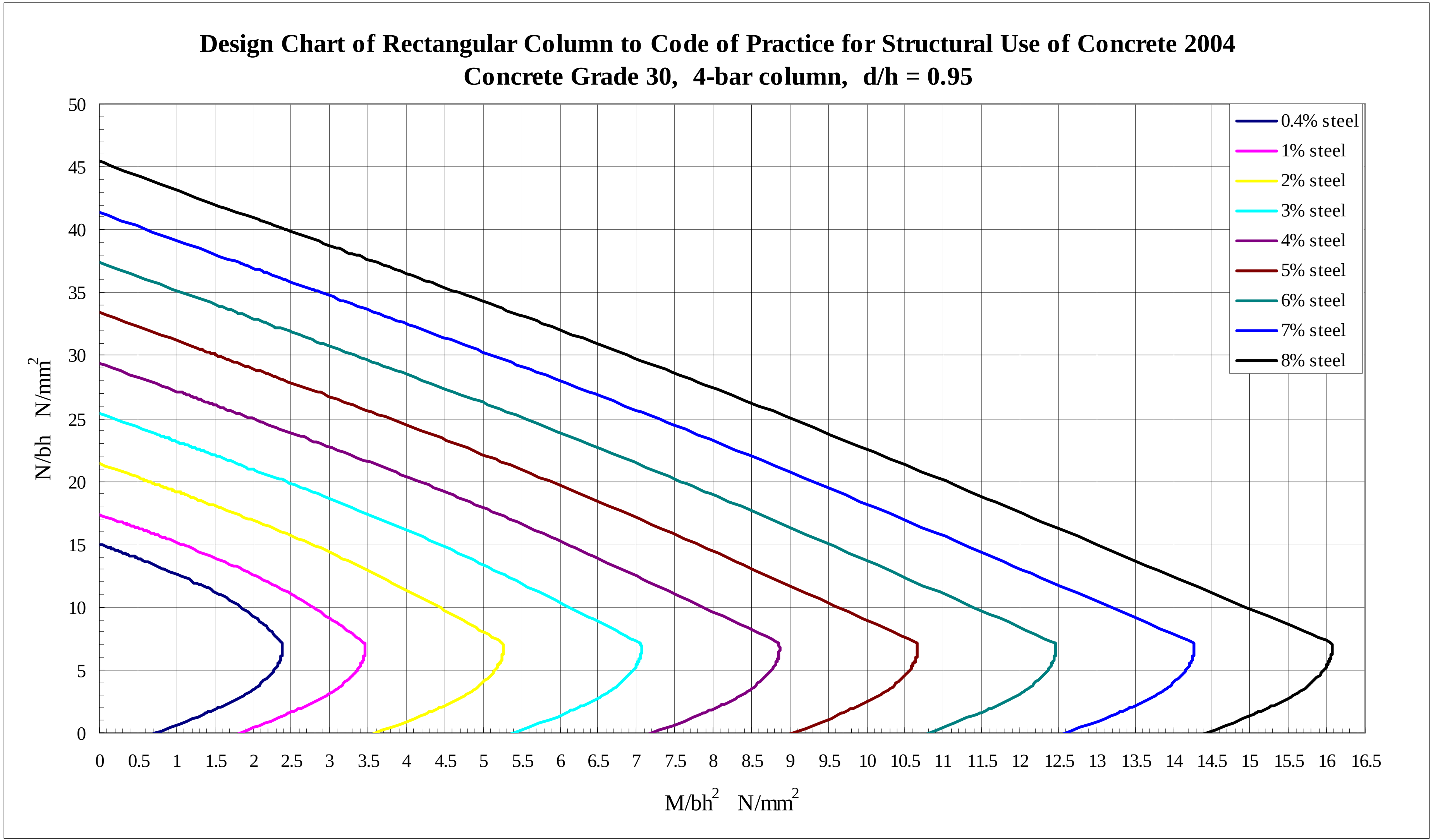


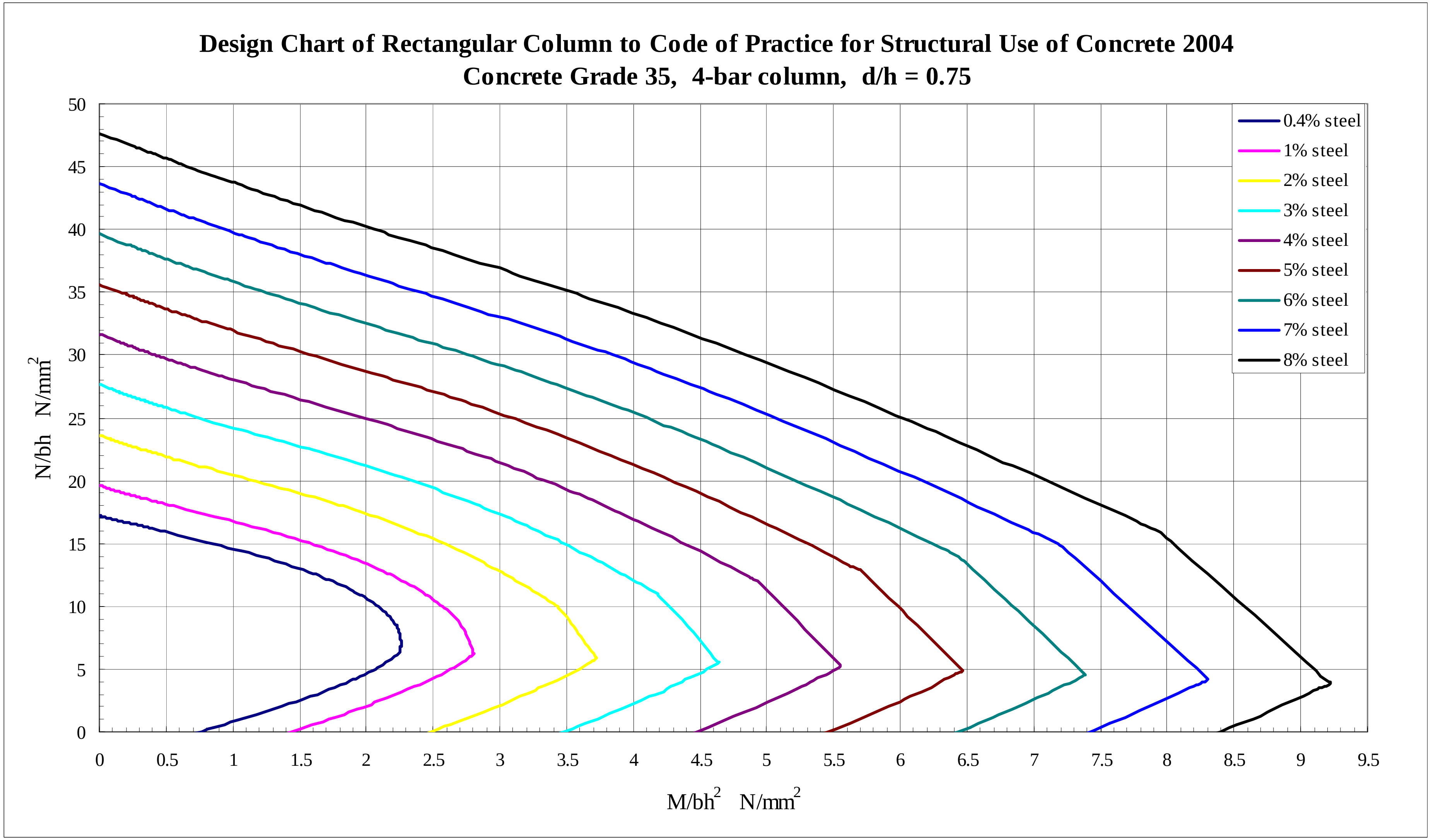
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 30, 4-bar column, d/h = 0.75
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5.5
6
6.5
7
7.5
8
8.5
9

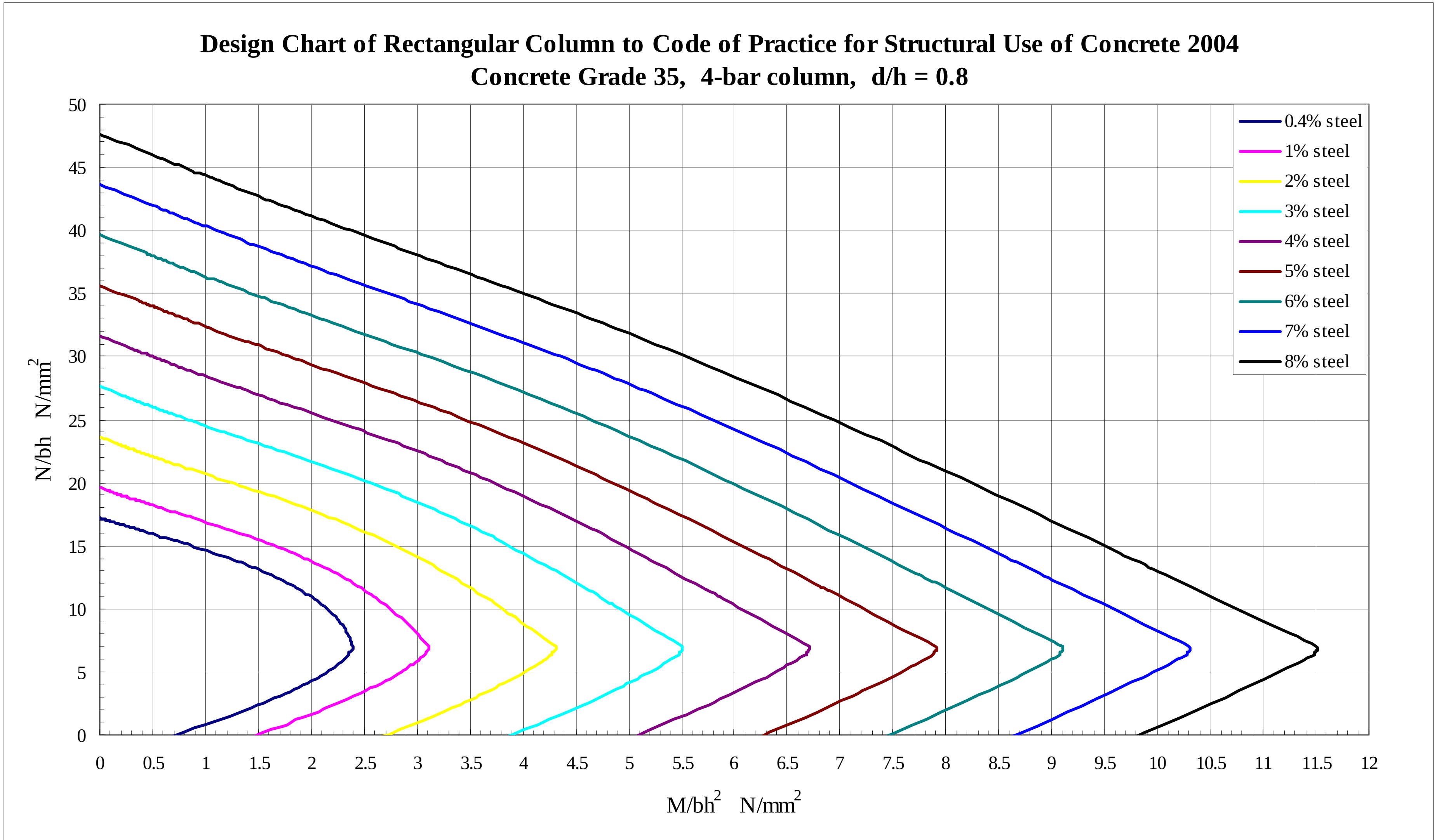

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 30, 4-bar column, d/h = 0.8
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 30, 4-bar column, d/h = 0.85
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel

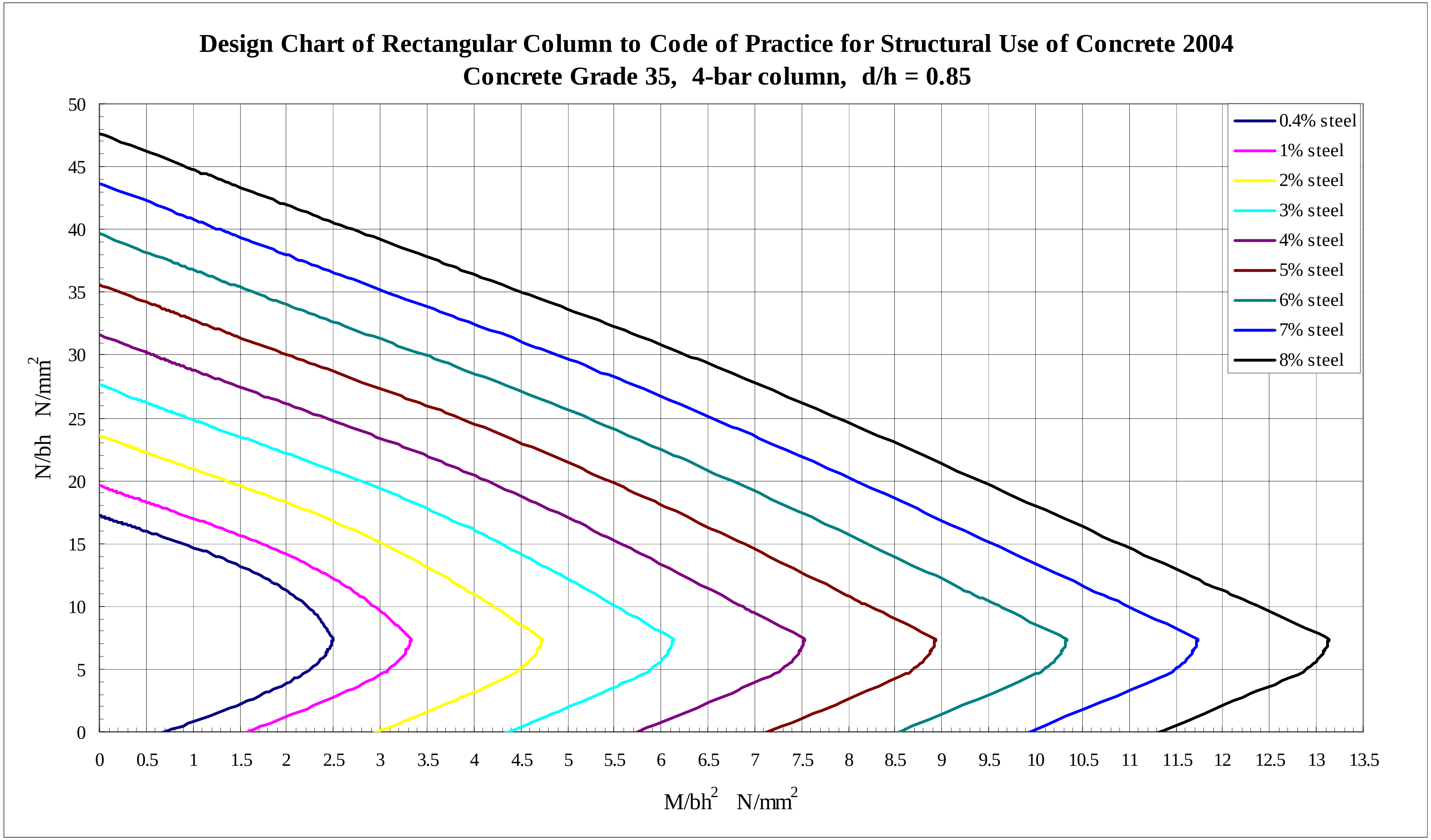

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 30, 4-bar column, d/h = 0.9
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10.5
11
11.5
12
12.5
13
13.5
14
14.5

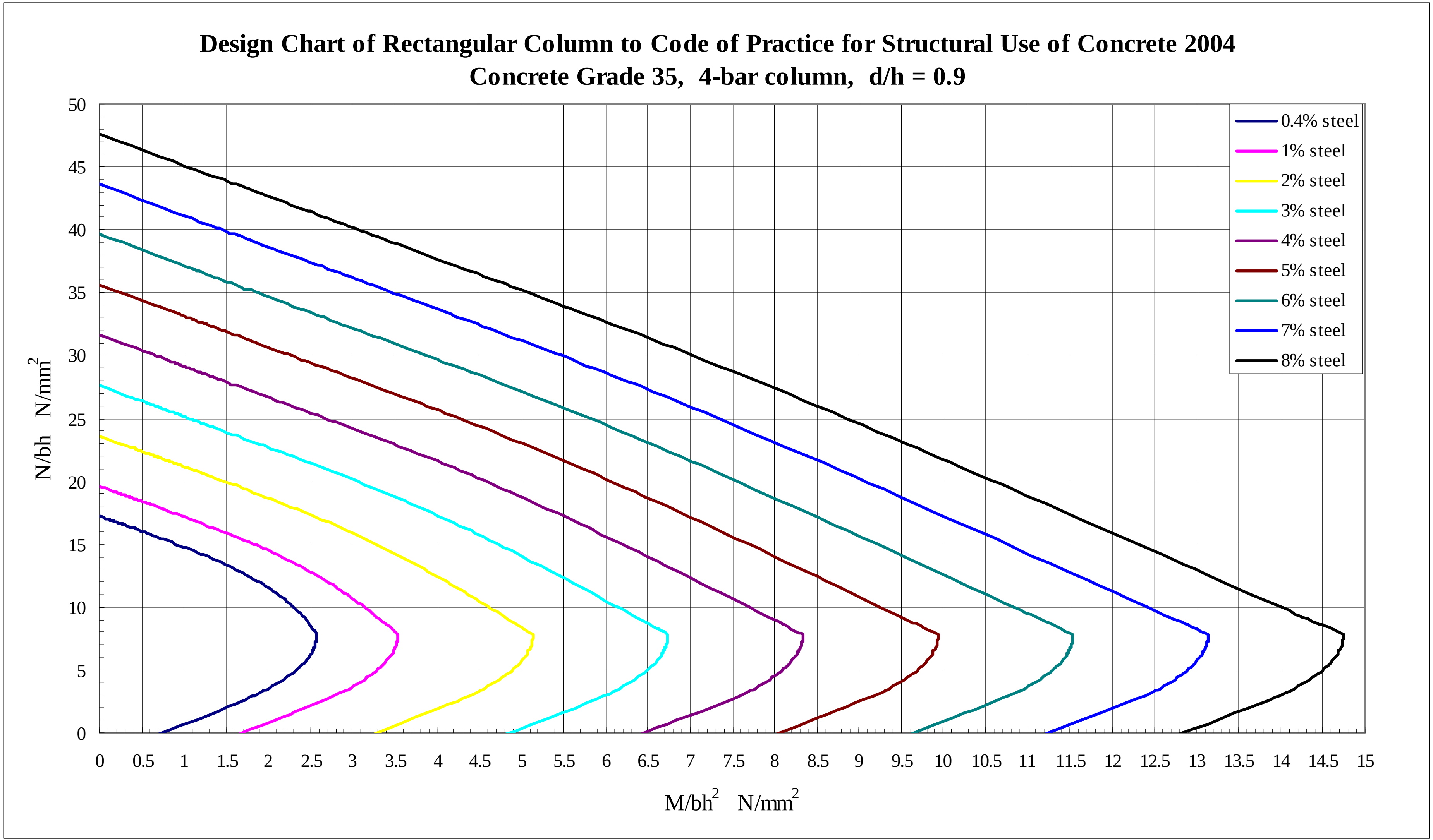

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 30, 4-bar column, d/h = 0.95
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14
14.5
15
15.5
16
16.5

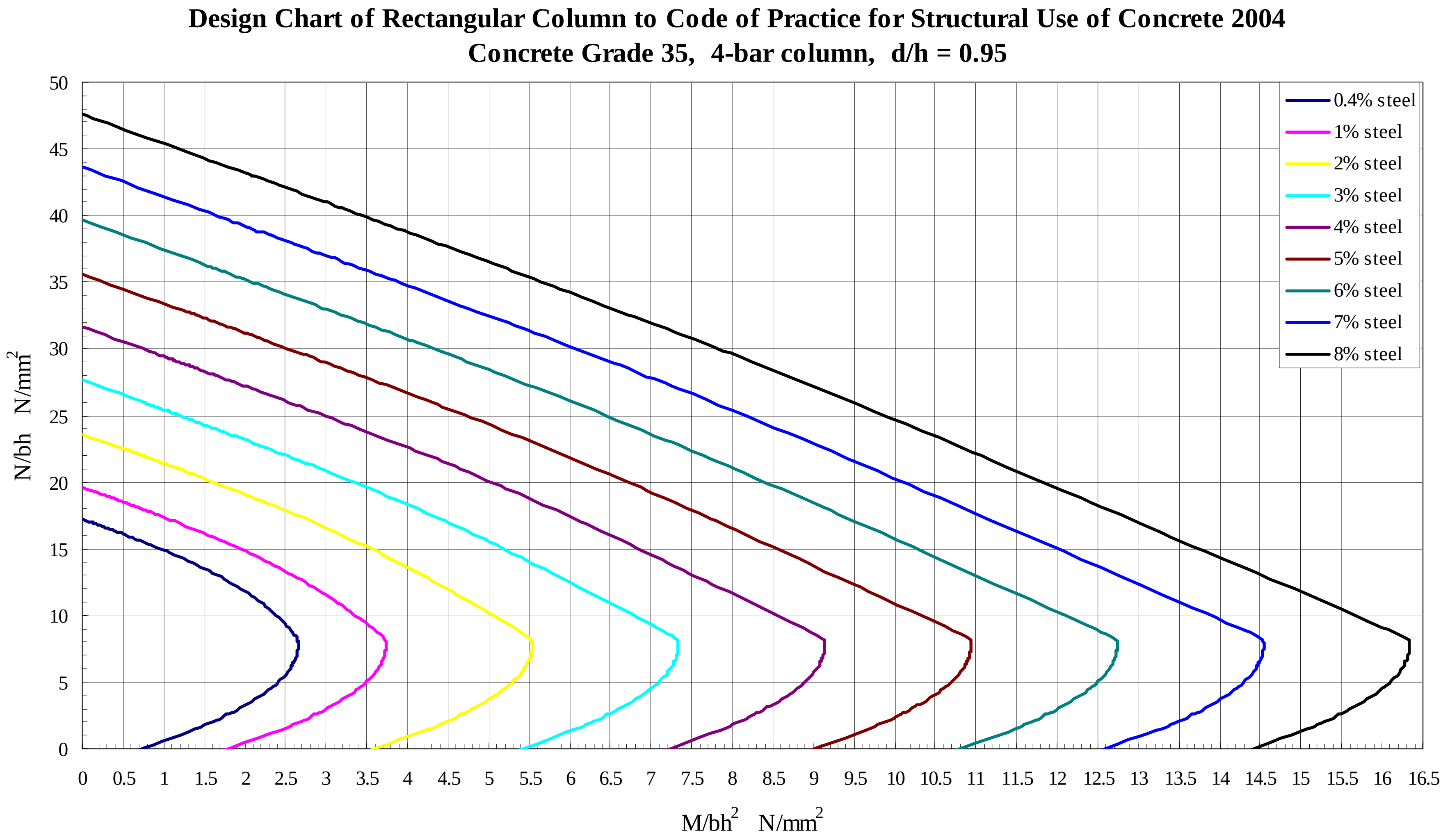

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 35, 4-bar column, d/h = 0.75
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5

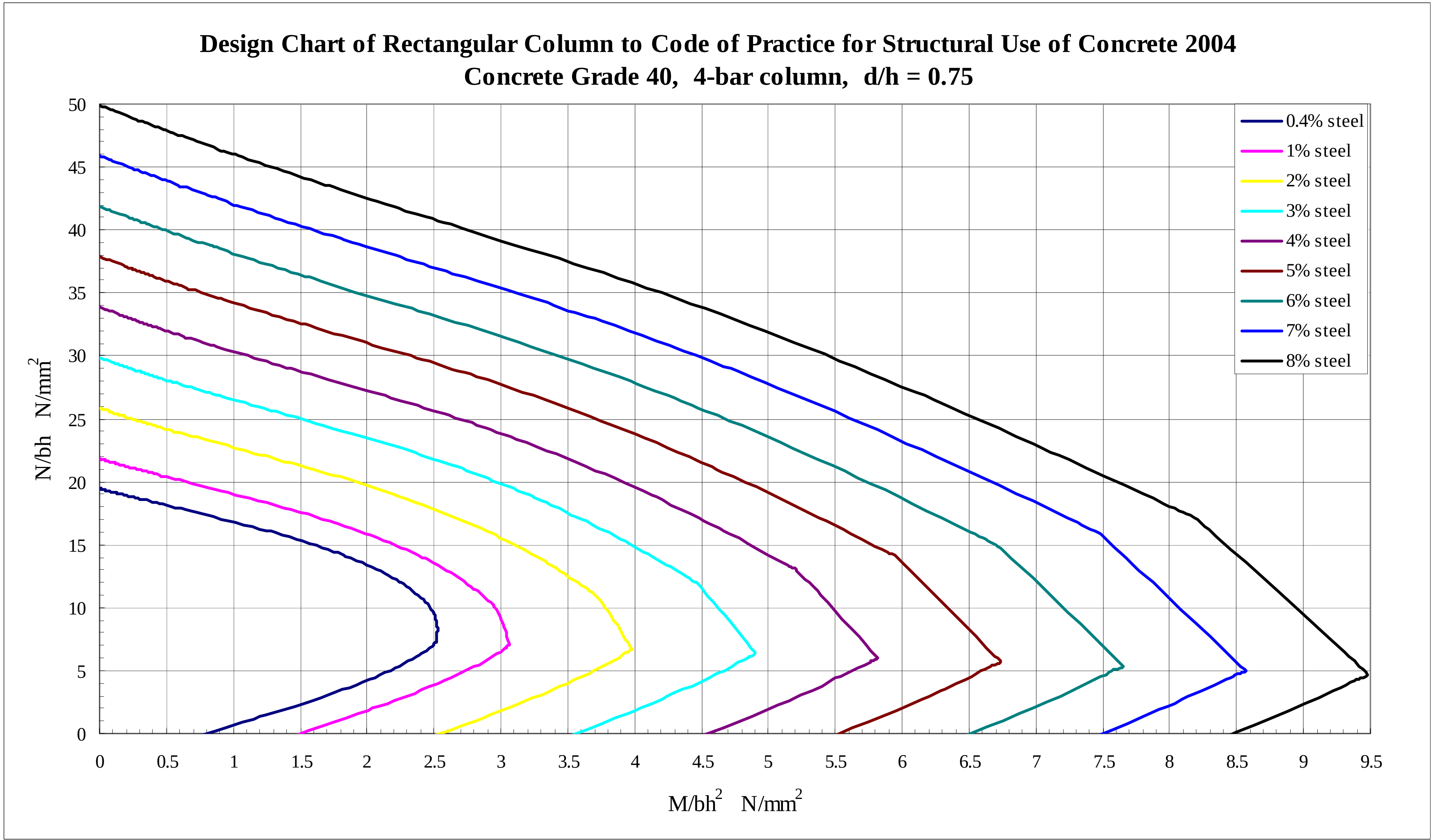


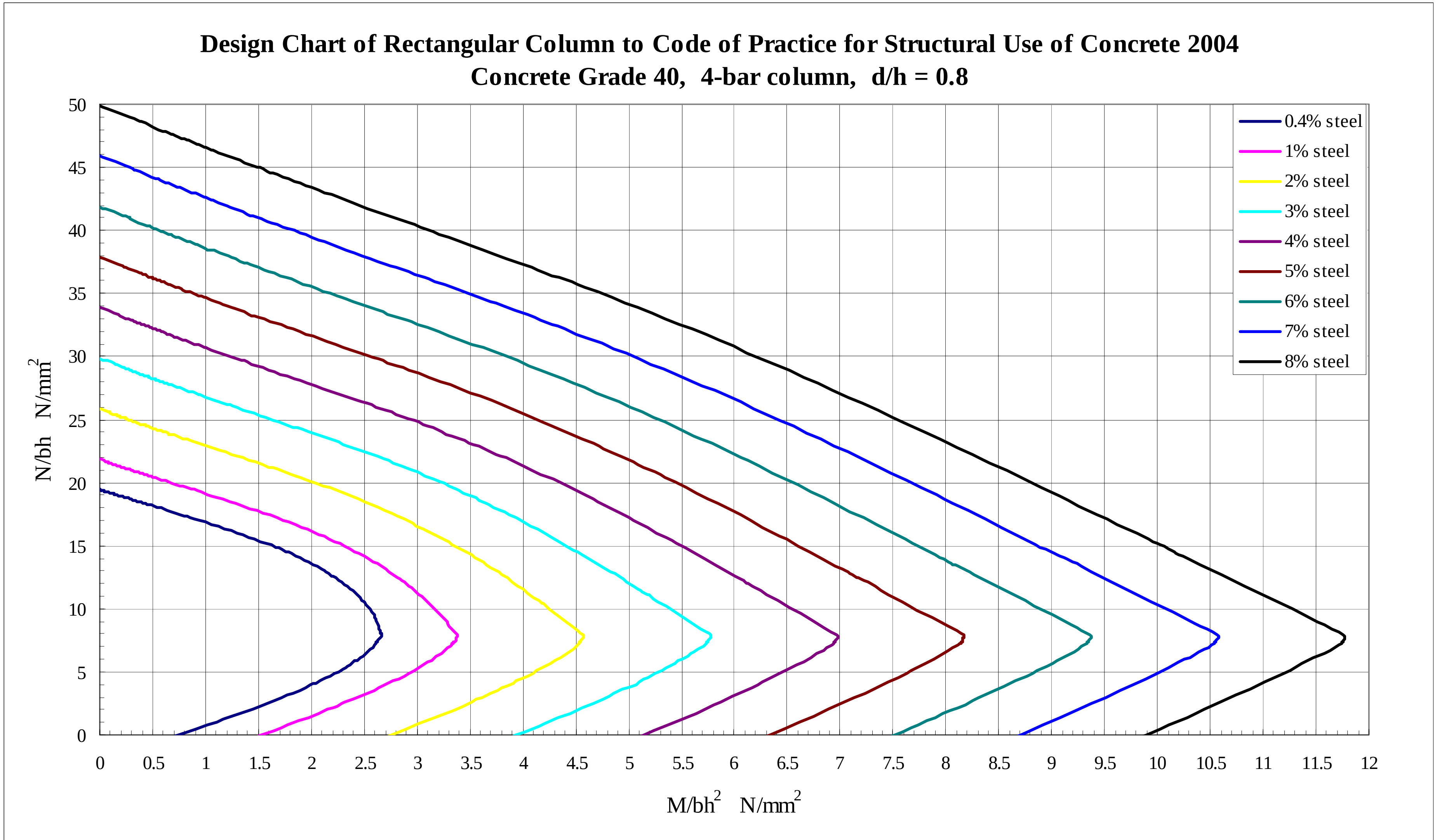
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 35, 4-bar column, d/h = 0.8
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12

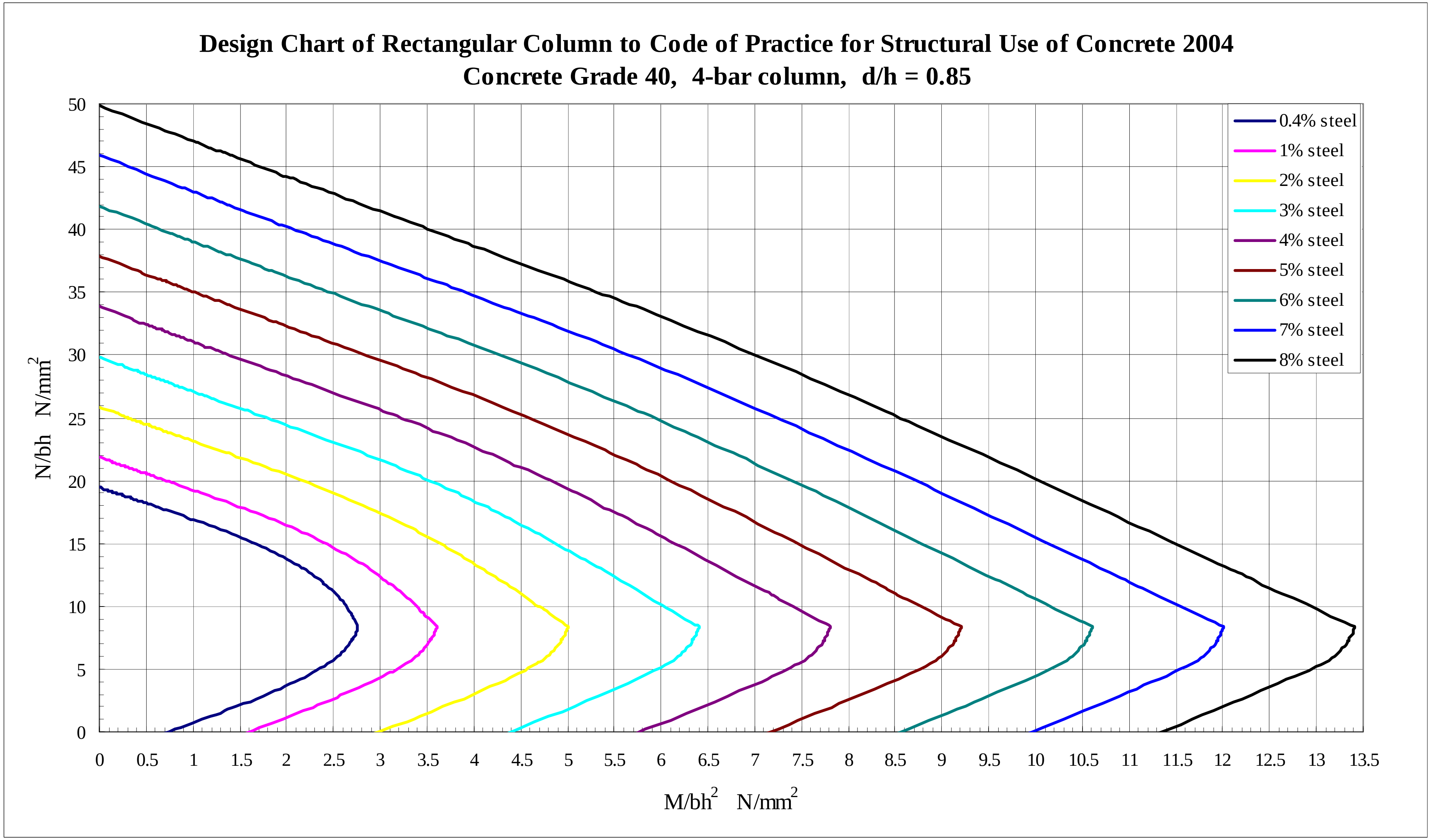

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 35, 4-bar column, d/h = 0.85
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5

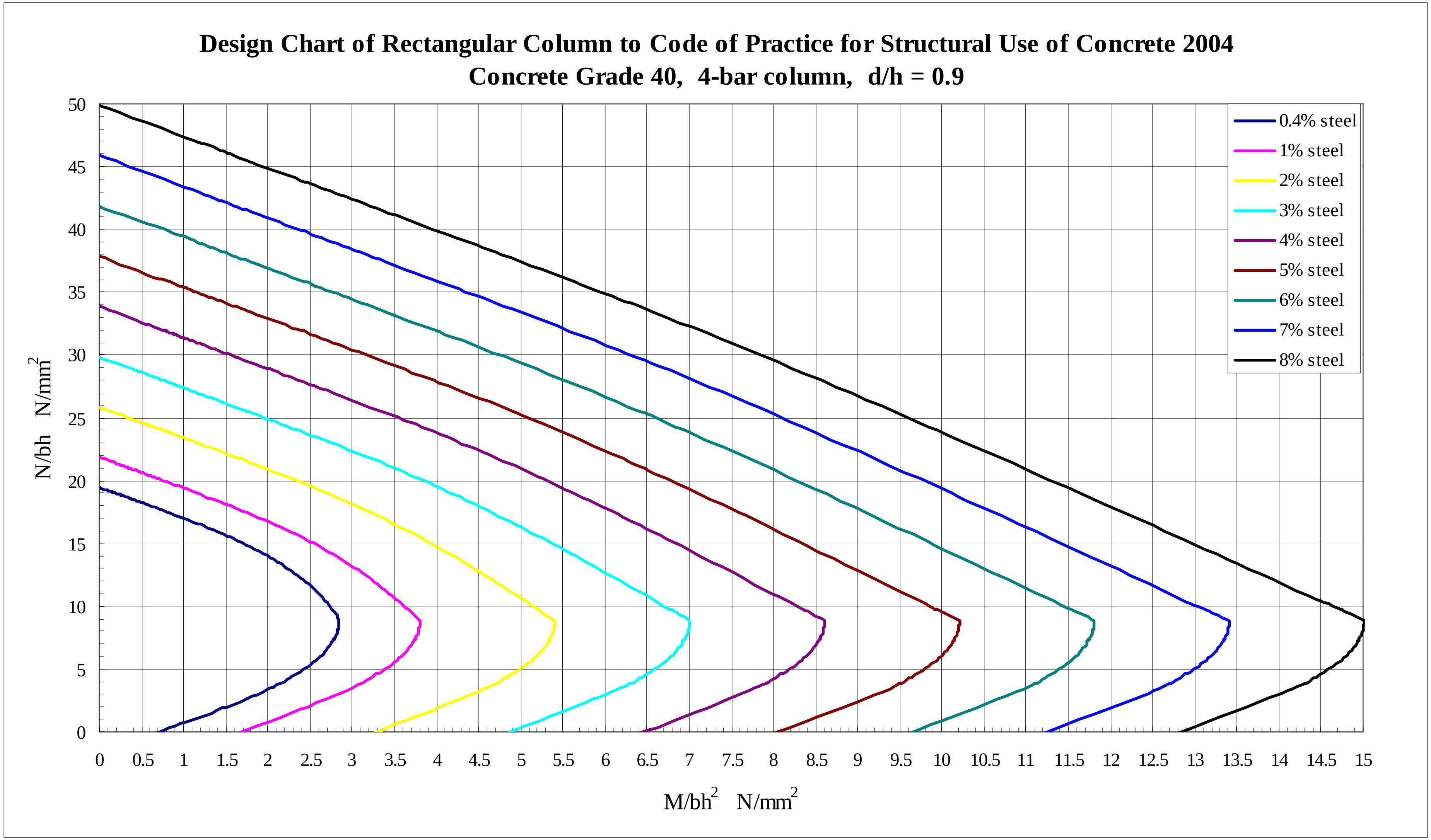

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 35, 4-bar column, d/h = 0.9
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14
14.5
15

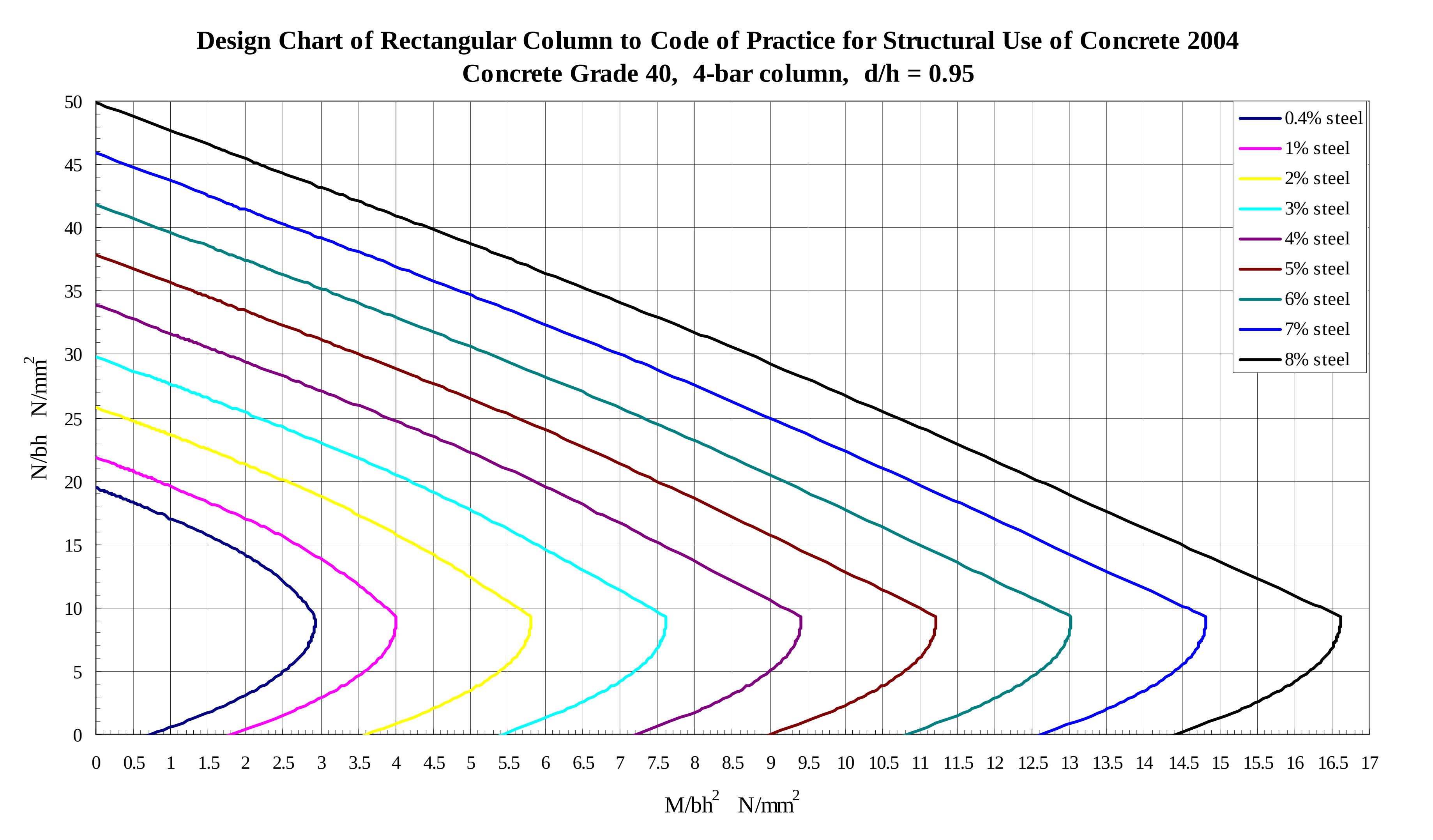

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 35, 4-bar column, d/h = 0.95
N/bh N/mm2
M/bh2 N/mm2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14
14.5
15
15.5
16
16.5
5
10
15
20
25
30
35
40
45
50
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel

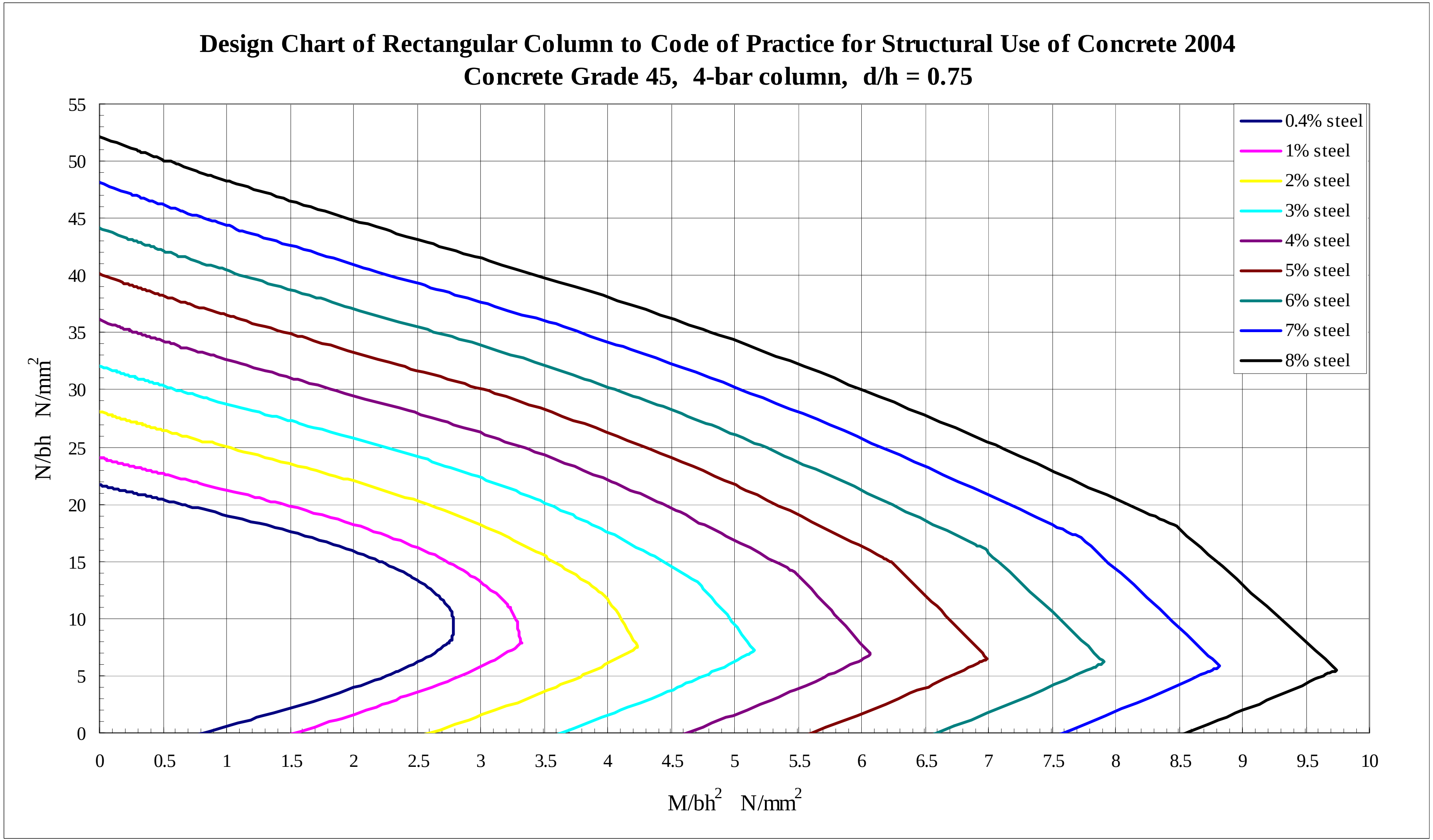

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 40, 4-bar column, d/h = 0.75
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel

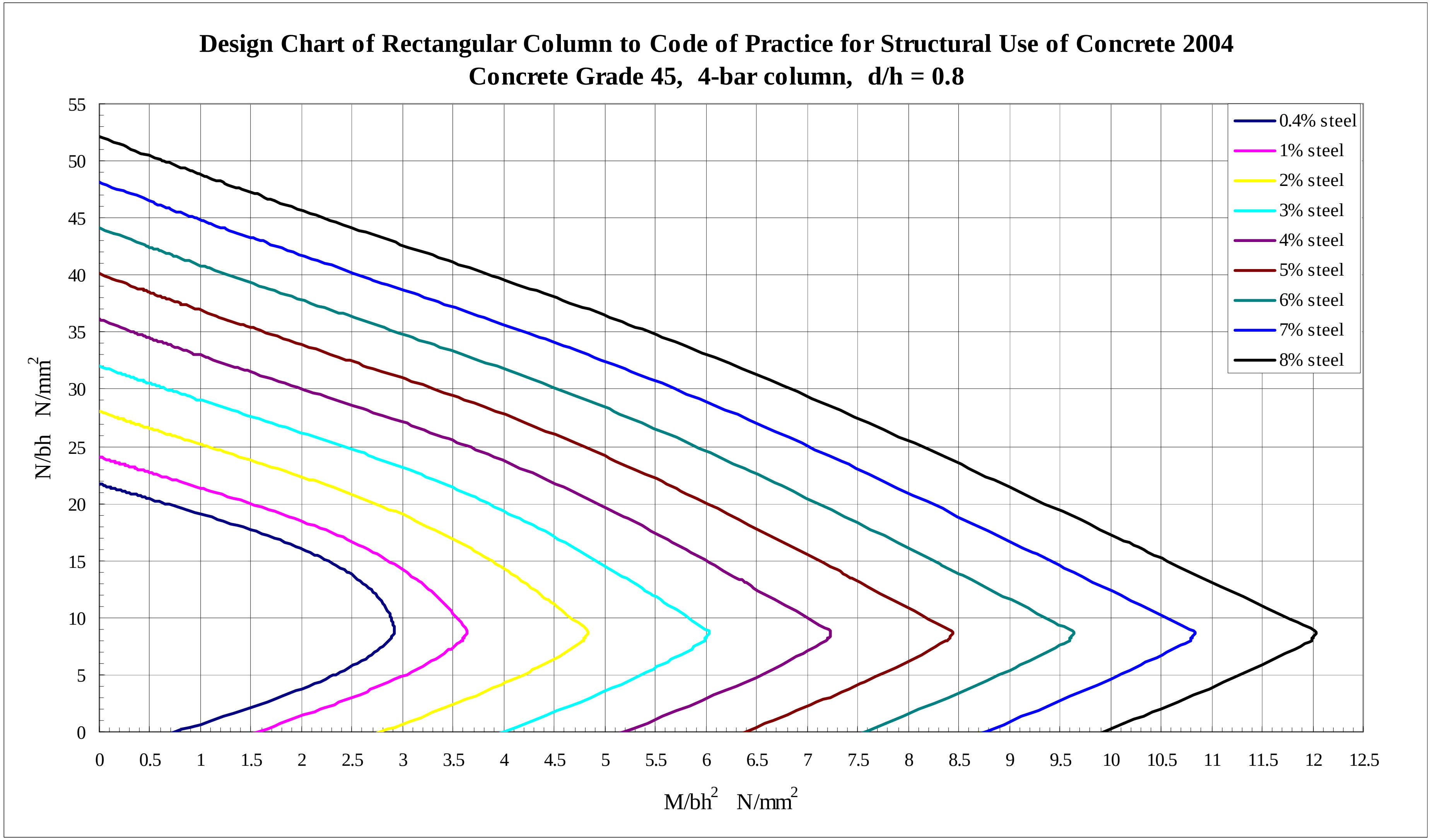

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 40, 4-bar column, d/h = 0.8
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10.5
11
11.5
12

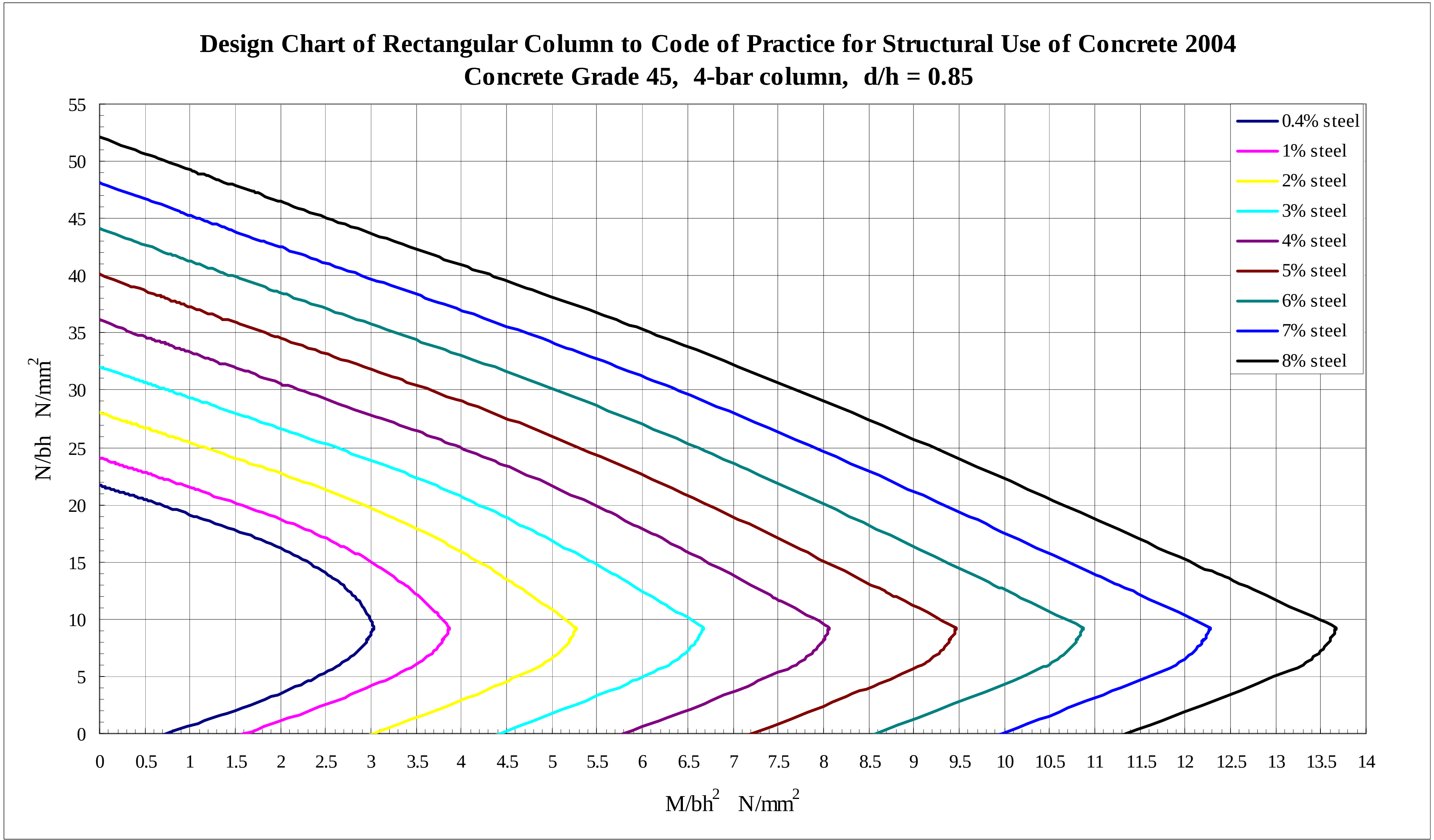

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 40, 4-bar column, d/h = 0.85
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 40, 4-bar column, d/h = 0.9
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14
14.5
15

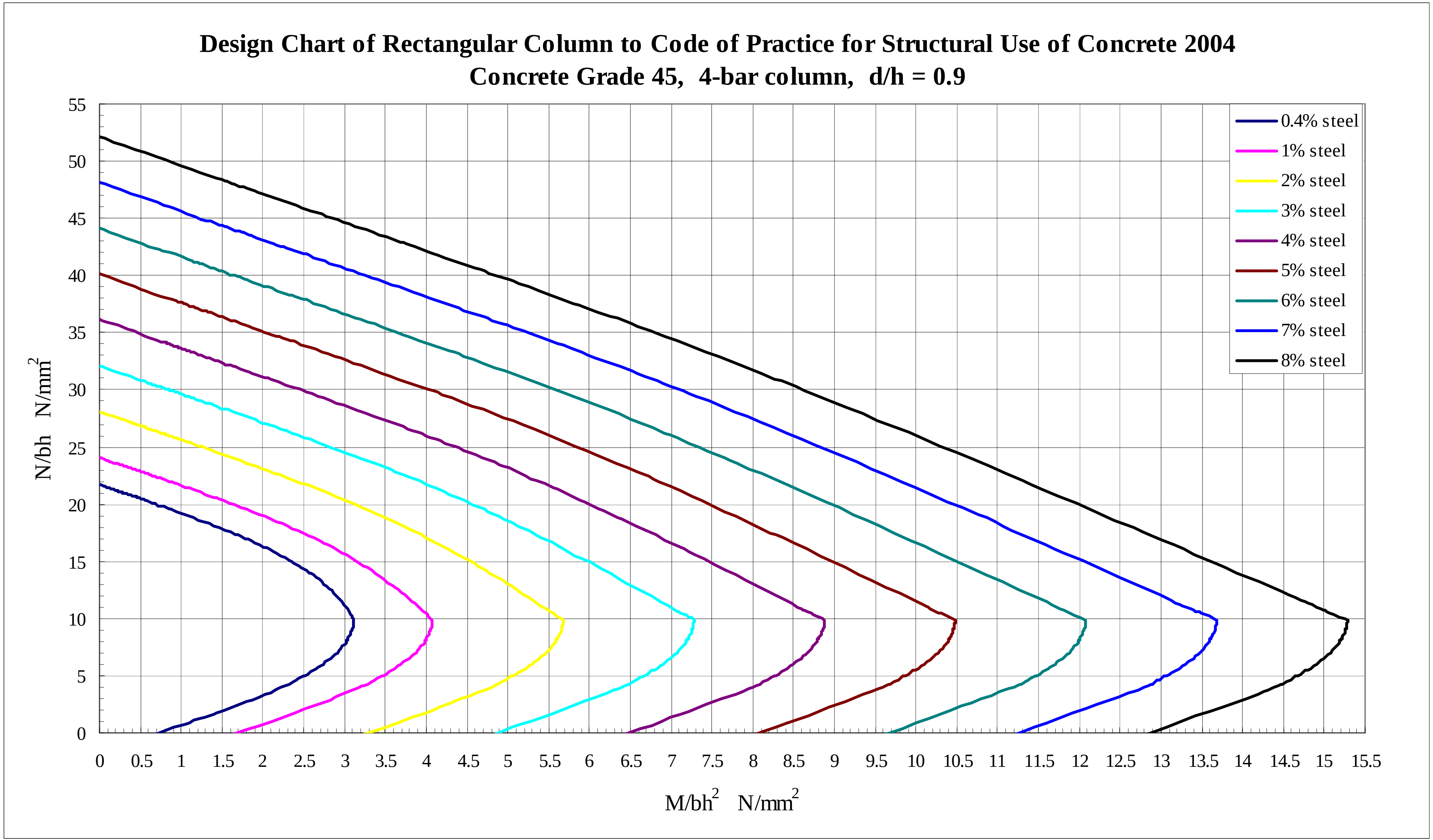

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 40, 4-bar column, d/h = 0.95
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh² N/mm²
M/bh² N/mm²
0 5 10 15 20 25 30 35 40 45 50
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10 10.5 11 11.5 12 12.5 13 13.5 14 14.5 15 15.5 16 16.5 17

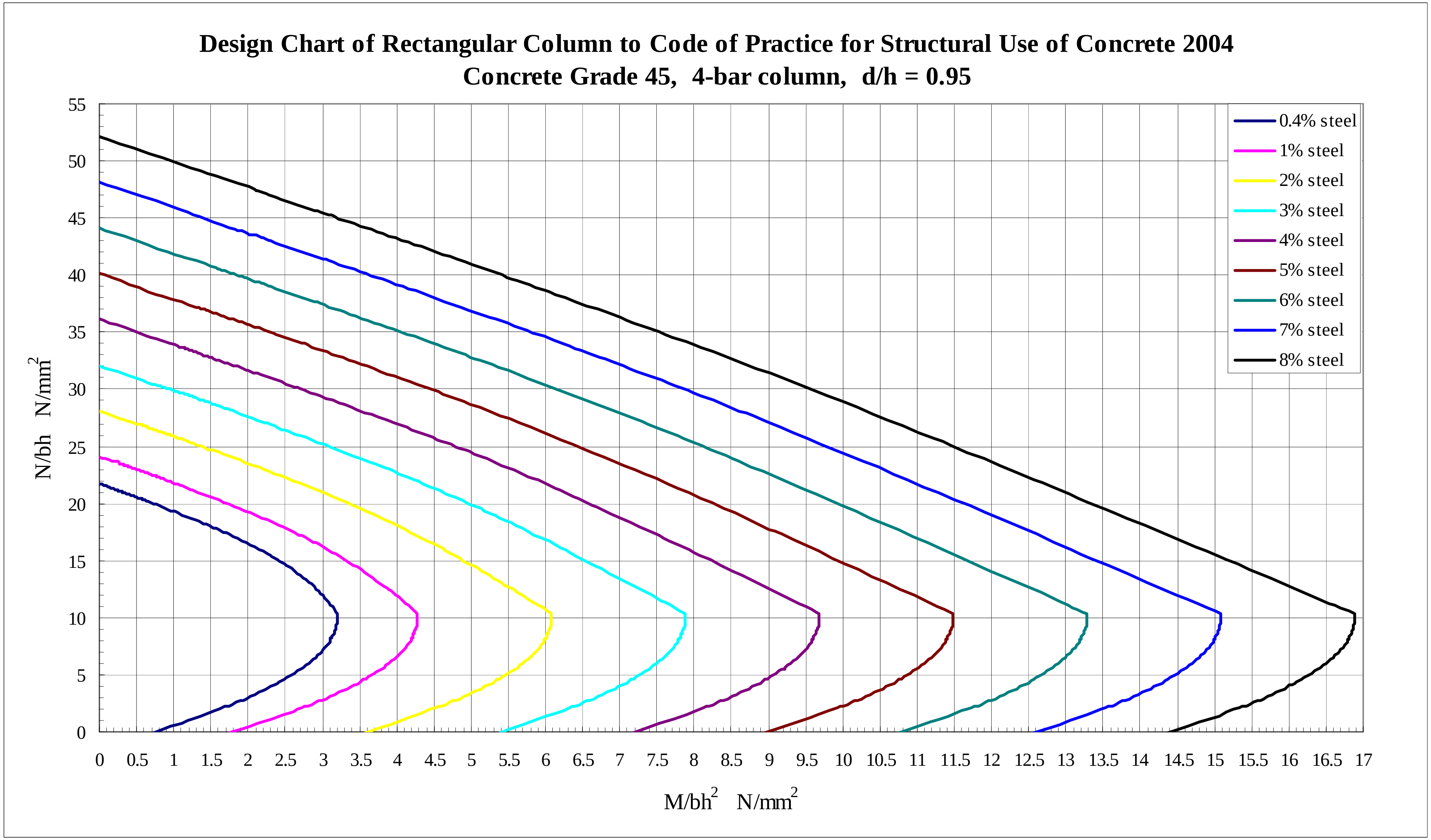

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 45, 4-bar column, d/h = 0.75
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10

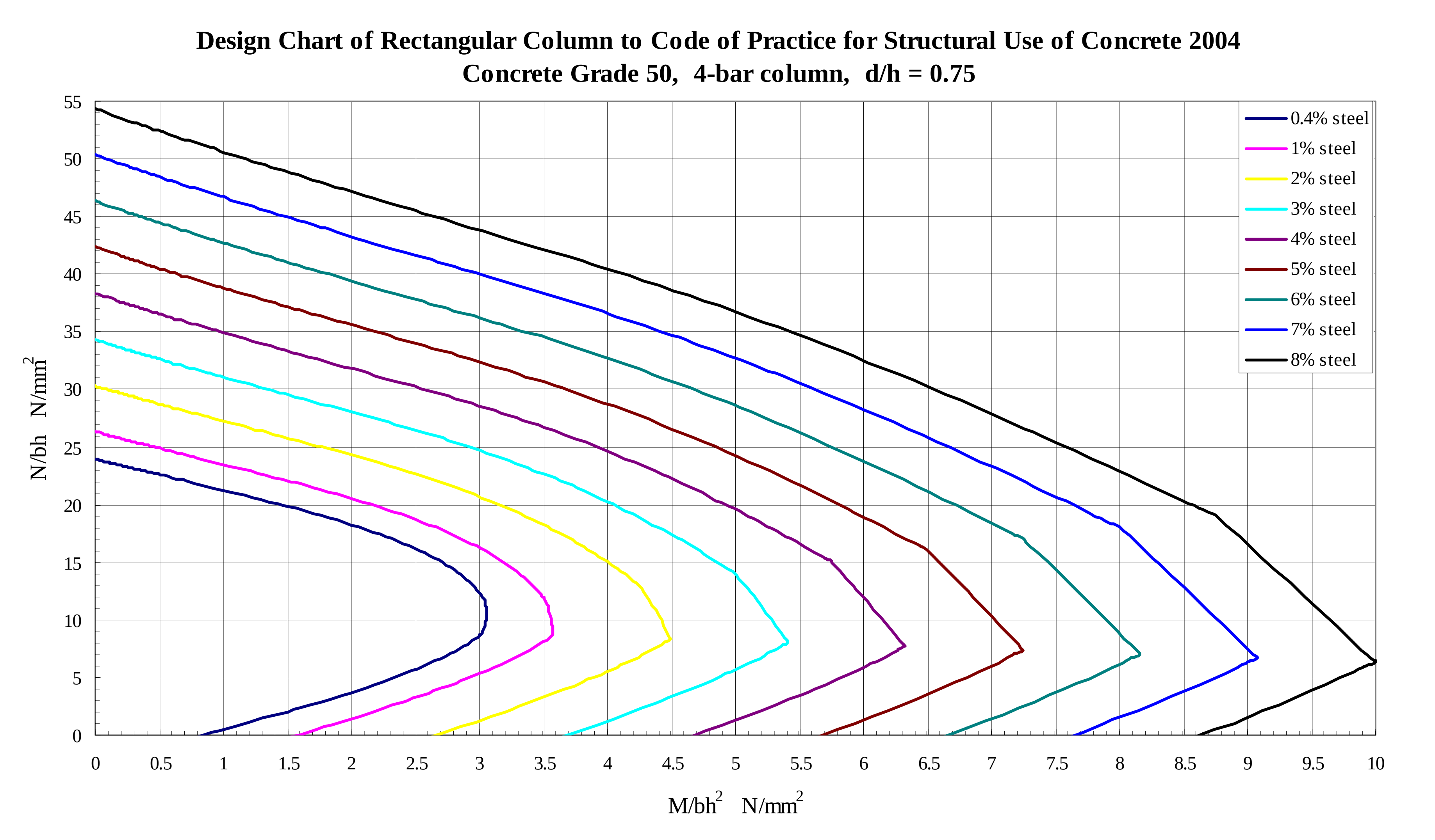

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 45, 4-bar column, d/h = 0.8
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5

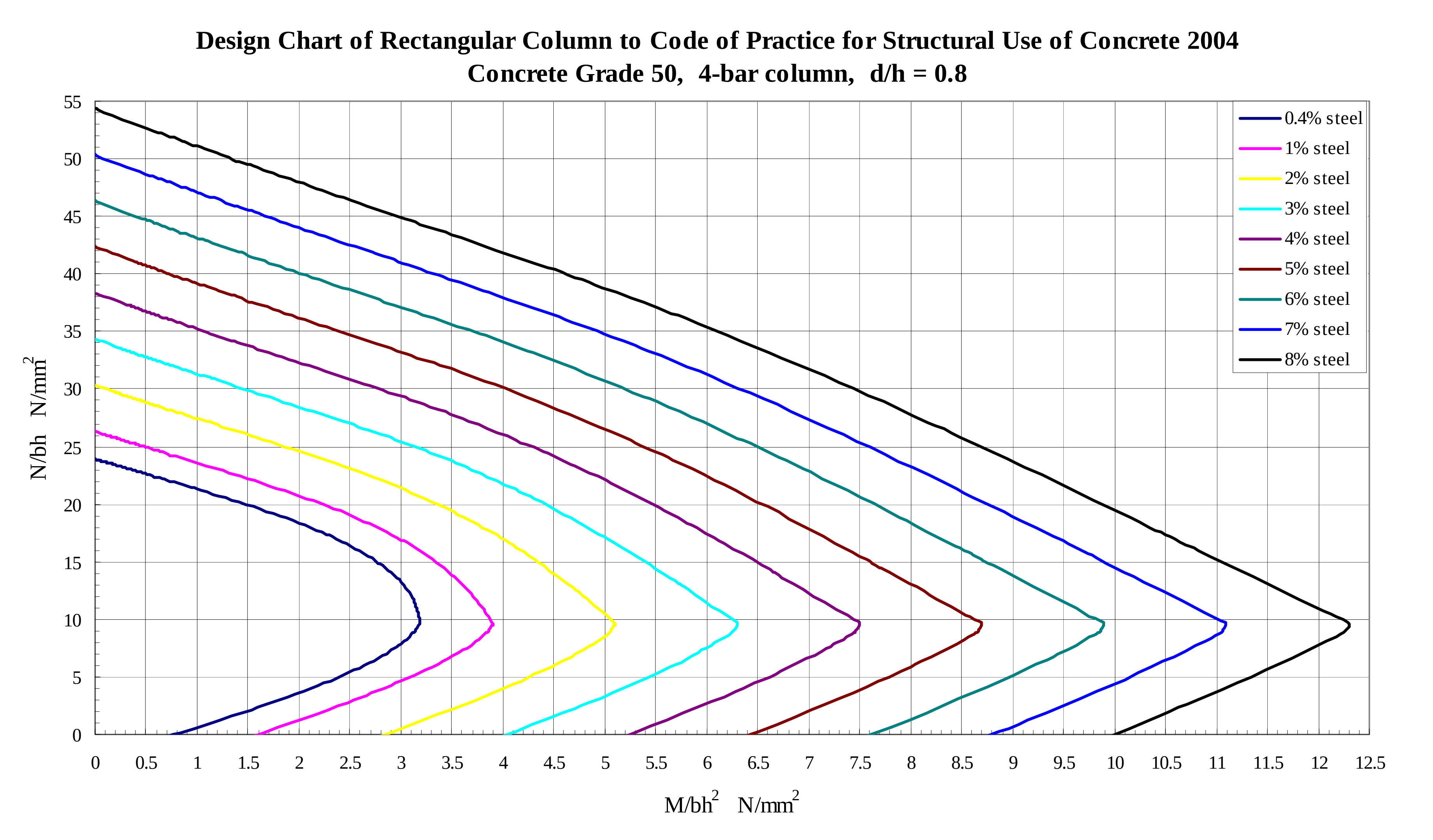


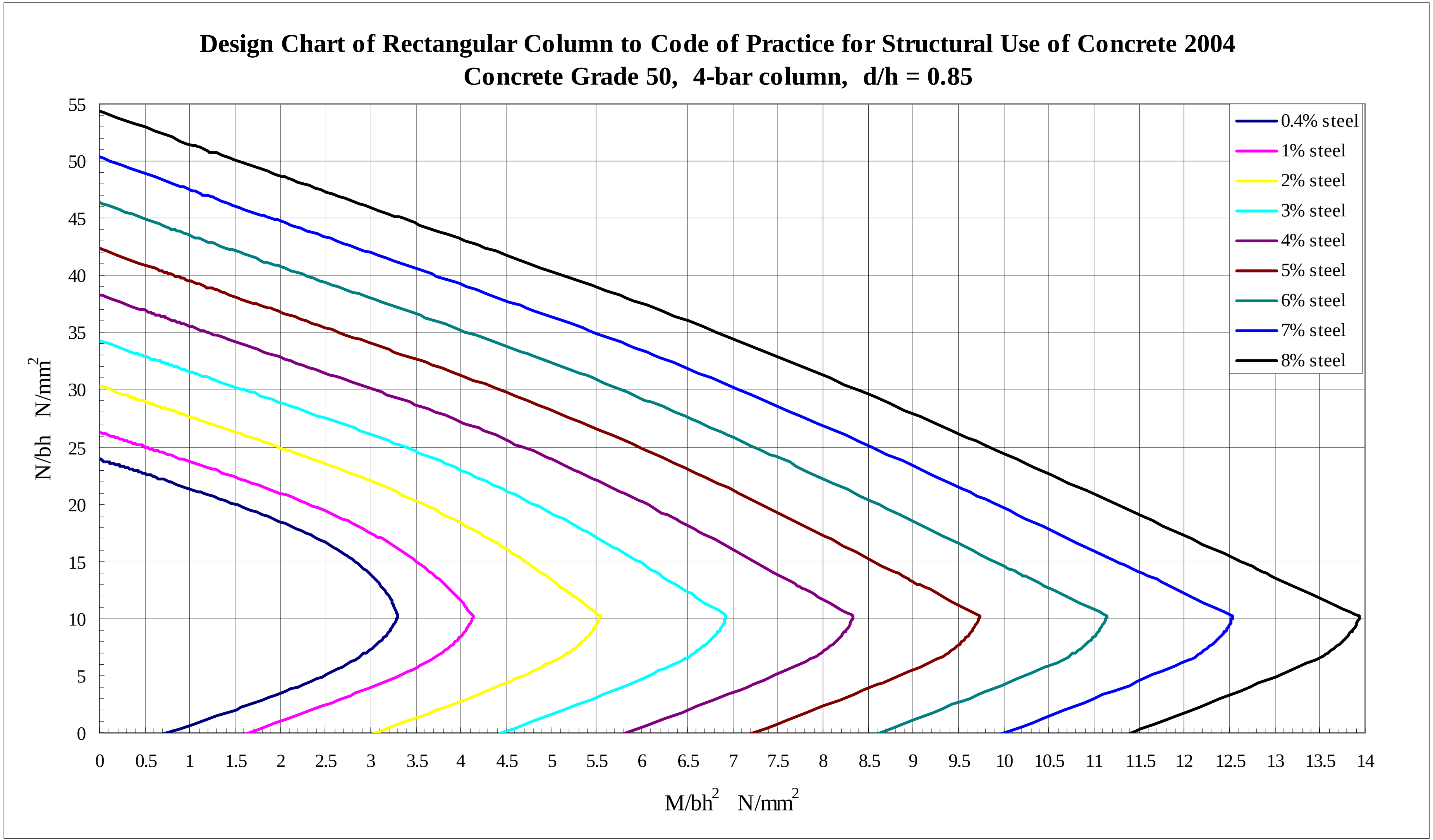
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 45, 4-bar column, d/h = 0.85
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 45, 4-bar column, d/h = 0.9
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10.5
11
11.5
12
12.5
13
13.5
14
14.5
15
15.5

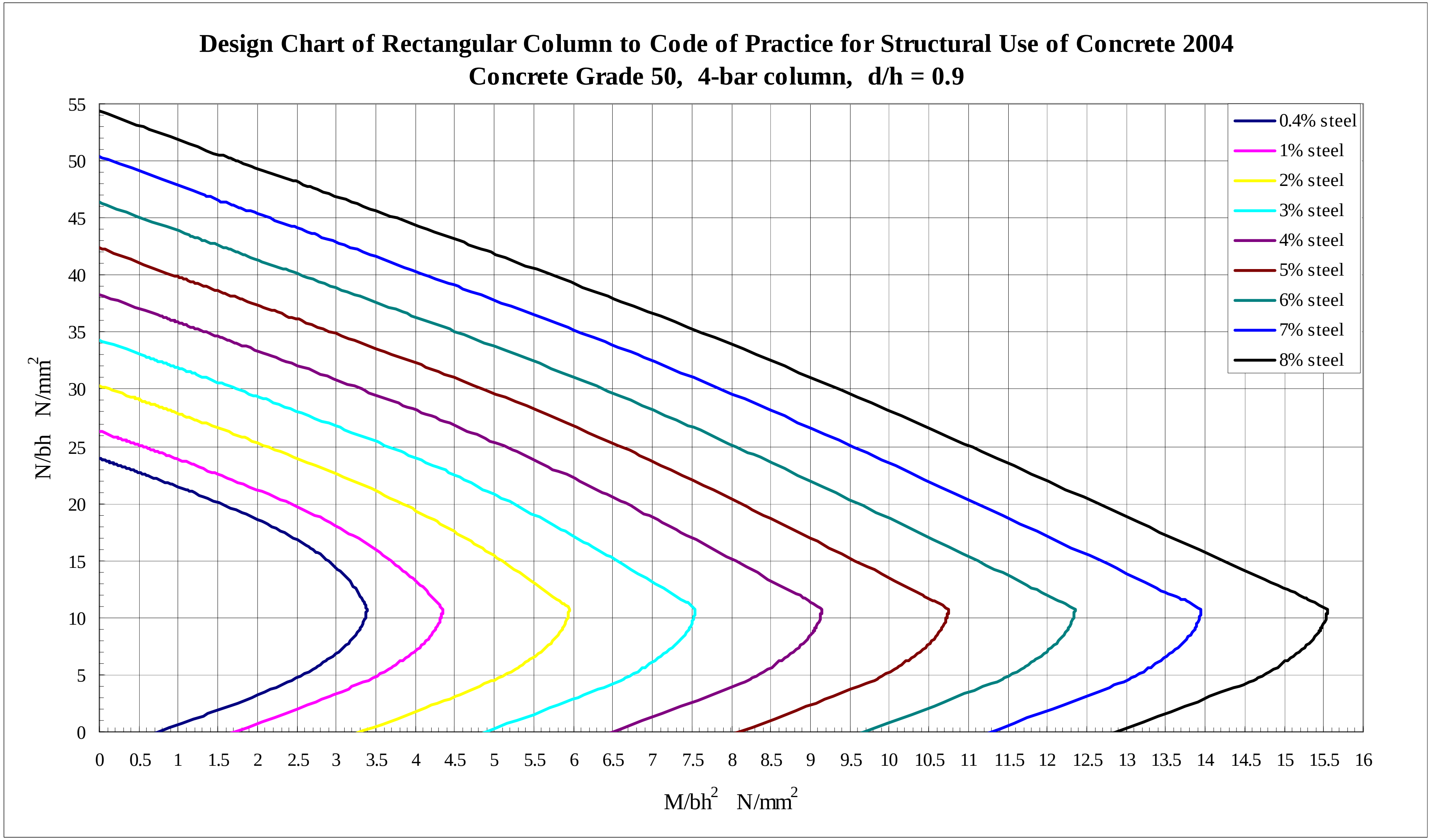

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 45, 4-bar column, d/h = 0.95
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10 10.5 11 11.5 12 12.5 13 13.5 14 14.5 15 15.5 16 16.5 17


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 50, 4-bar column, d/h = 0.75
N/bh N/mm2
M/bh2 N/mm2
0 5 10 15 20 25 30 35 40 45 50 55
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 50, 4-bar column, d/h = 0.8
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5

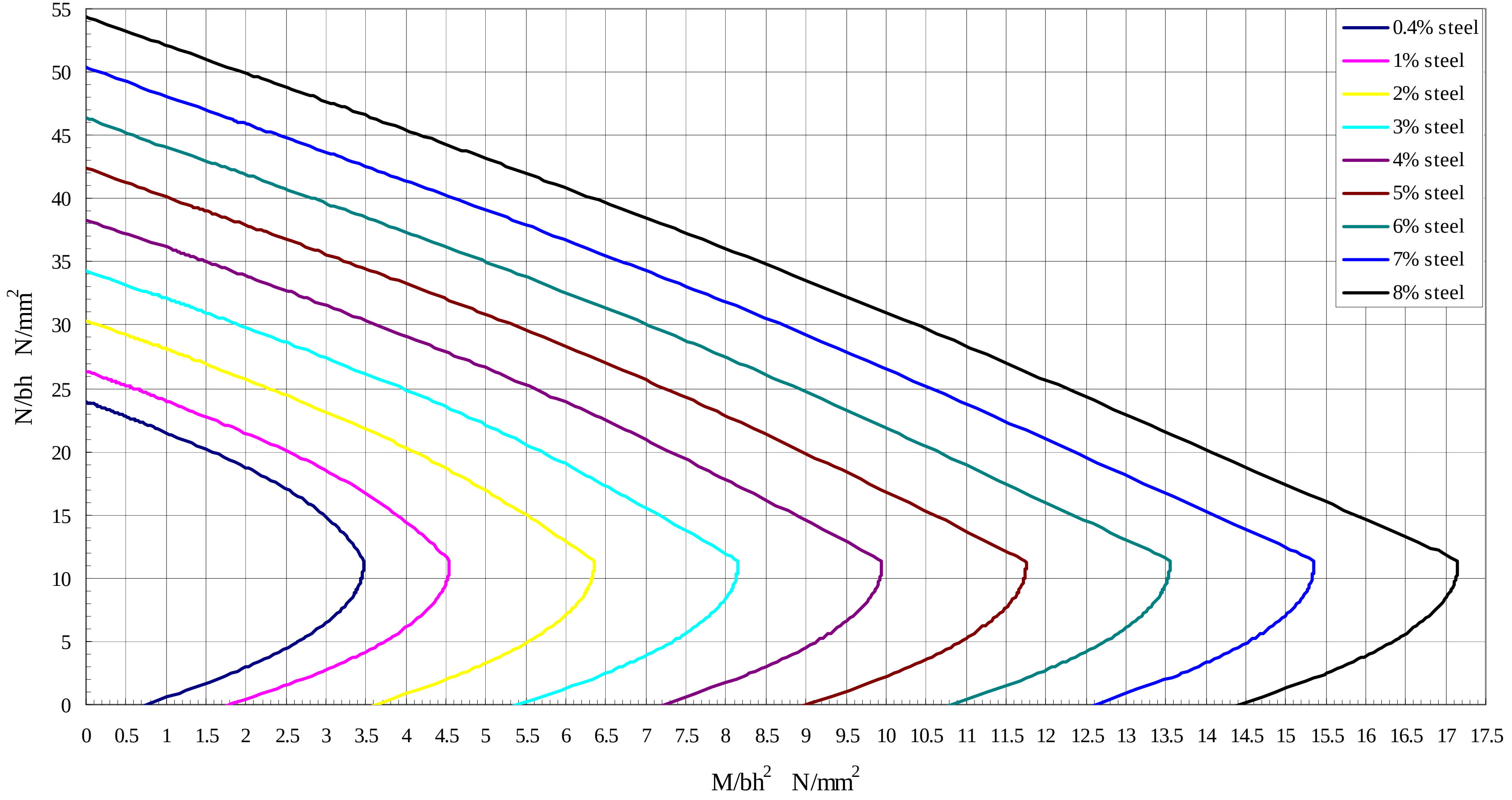


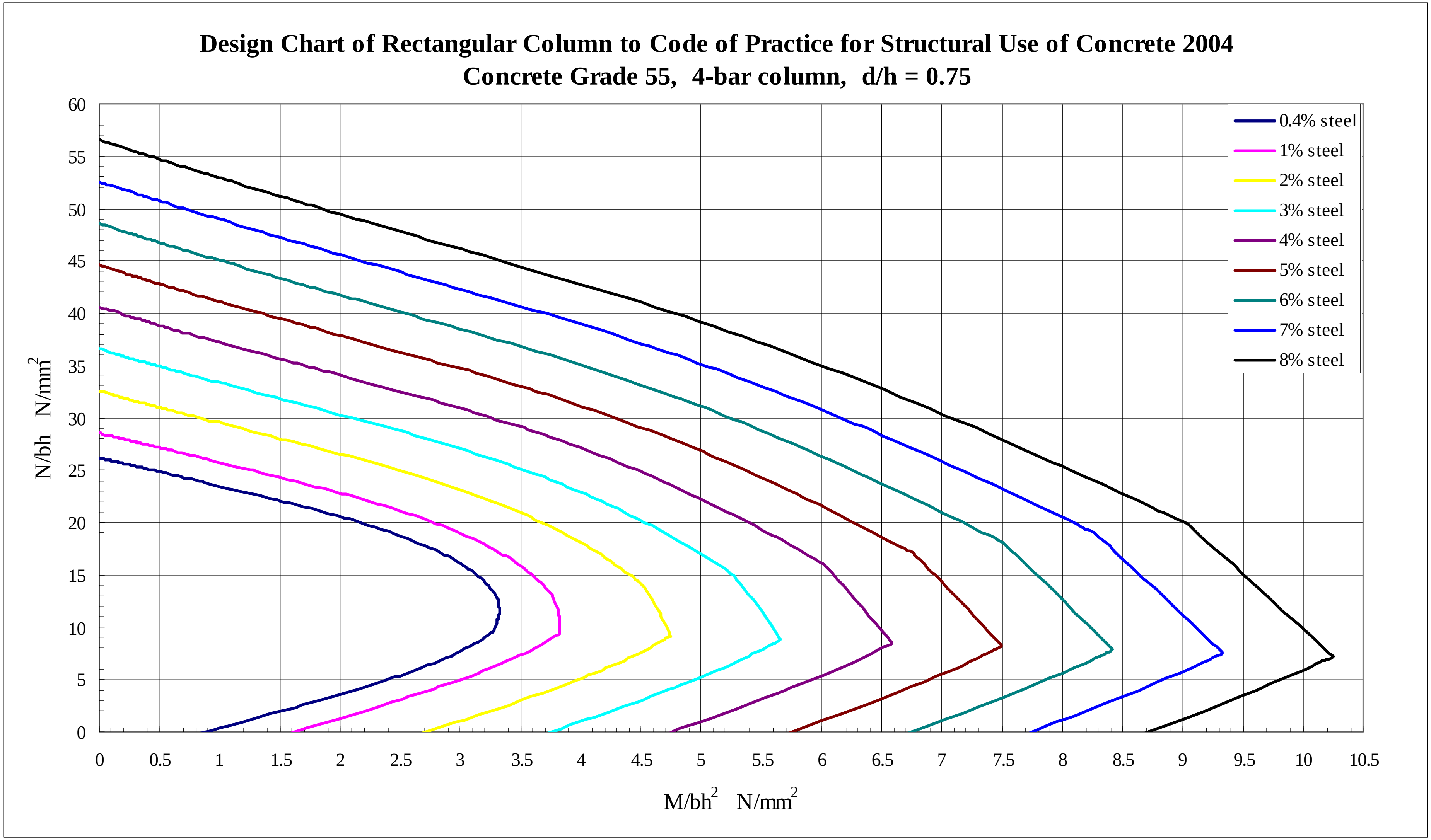
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 50, 4-bar column, d/h = 0.85
N/bh N/mm2
M/bh2 N/mm2
0 5 10 15 20 25 30 35 40 45 50 55
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10 10.5 11 11.5 12 12.5 13 13.5 14
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel

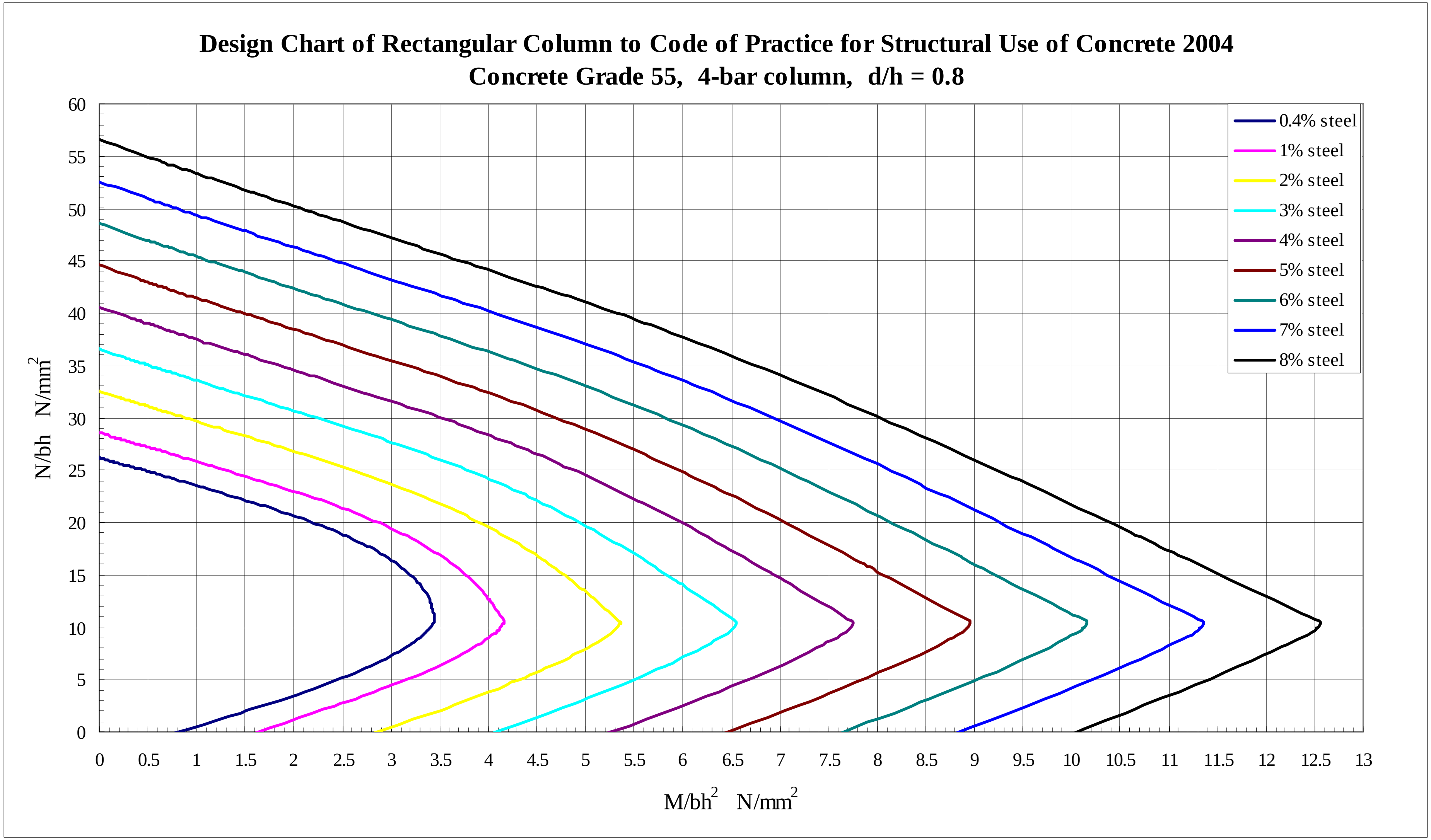


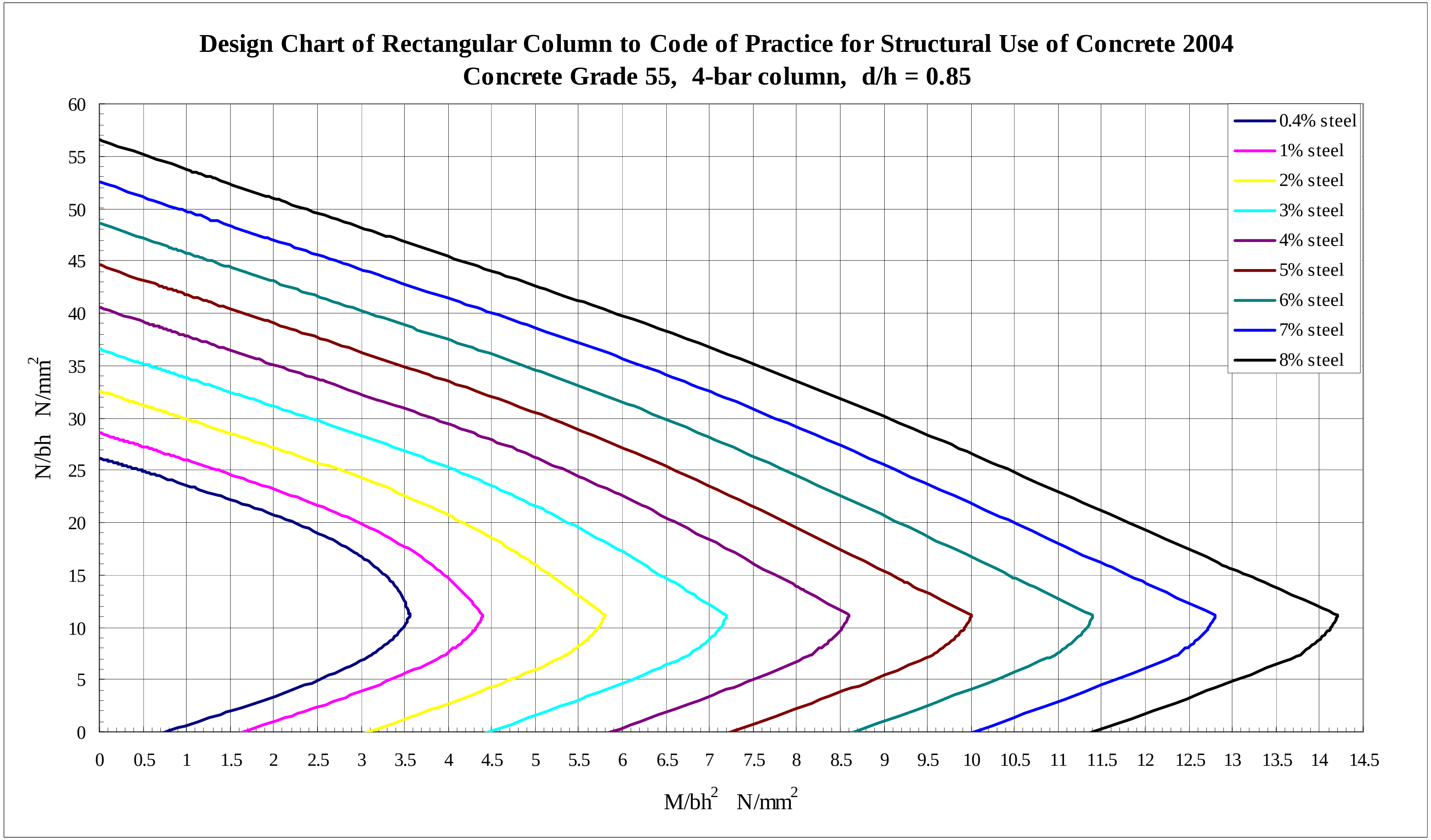
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 50, 4-bar column, d/h = 0.9
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14
14.5
15
15.5
16
55
50
45
40
35
30
25
20
15
10
5

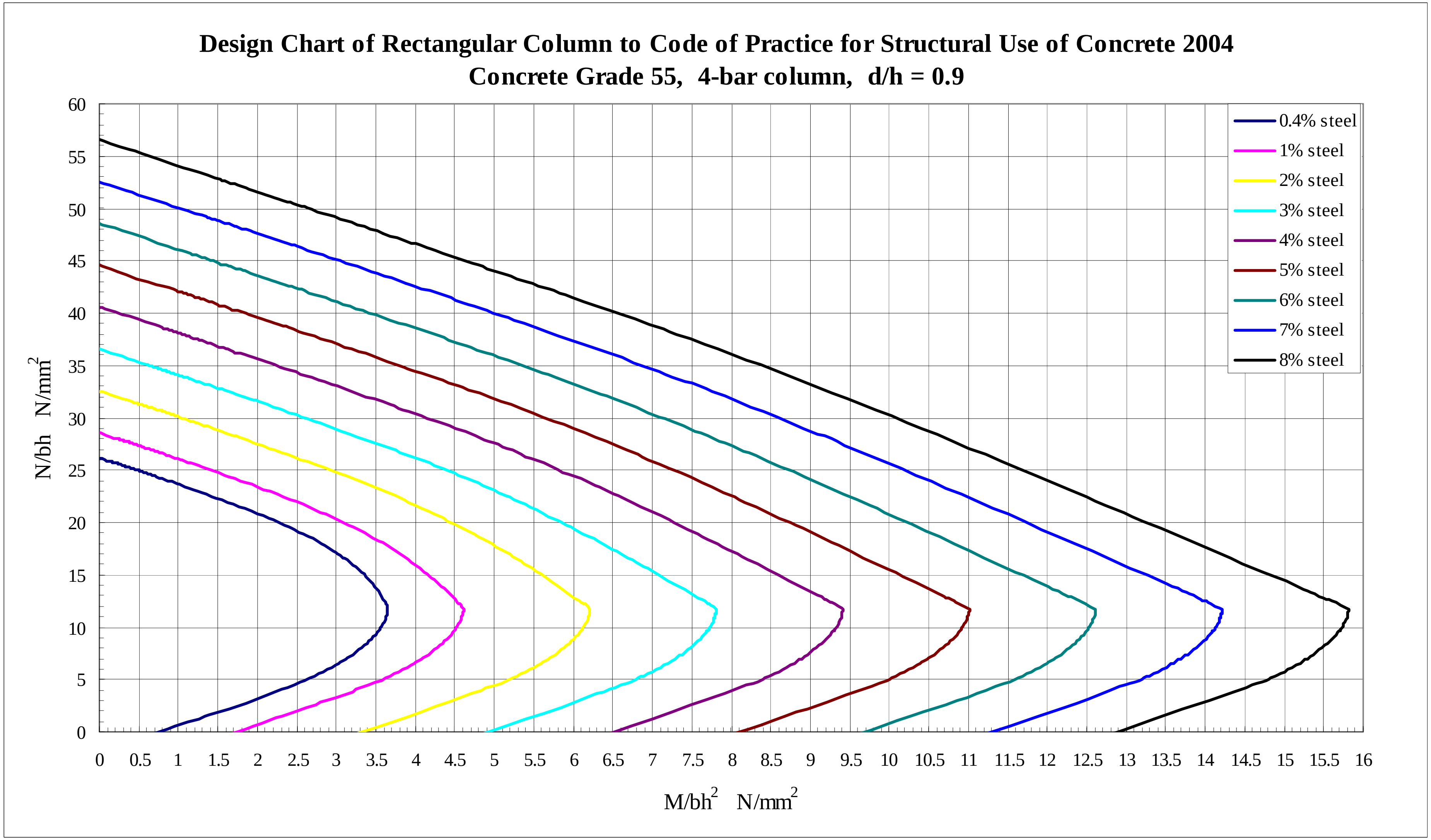
Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 50, 4-bar column, d/h = 0.95

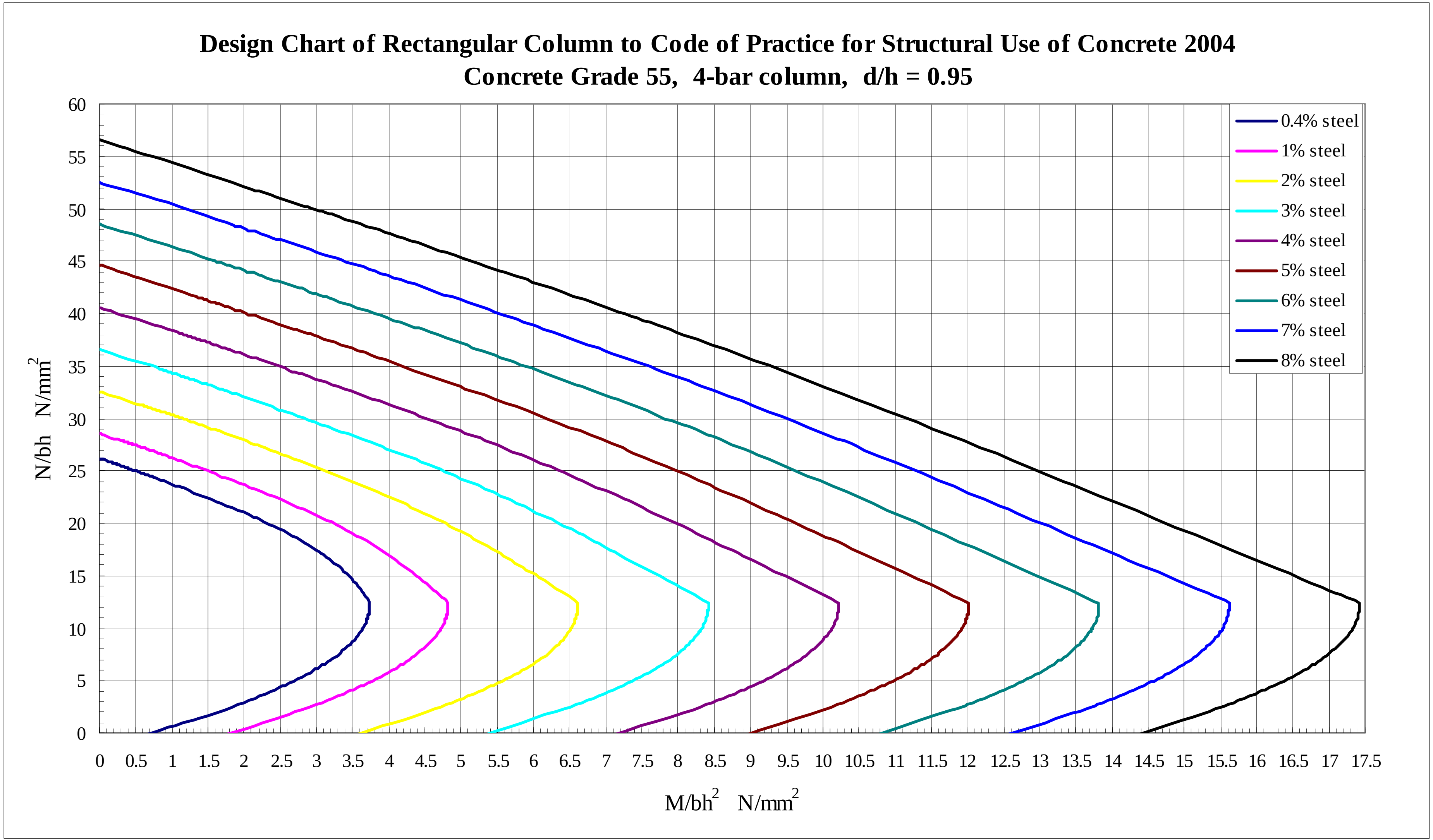


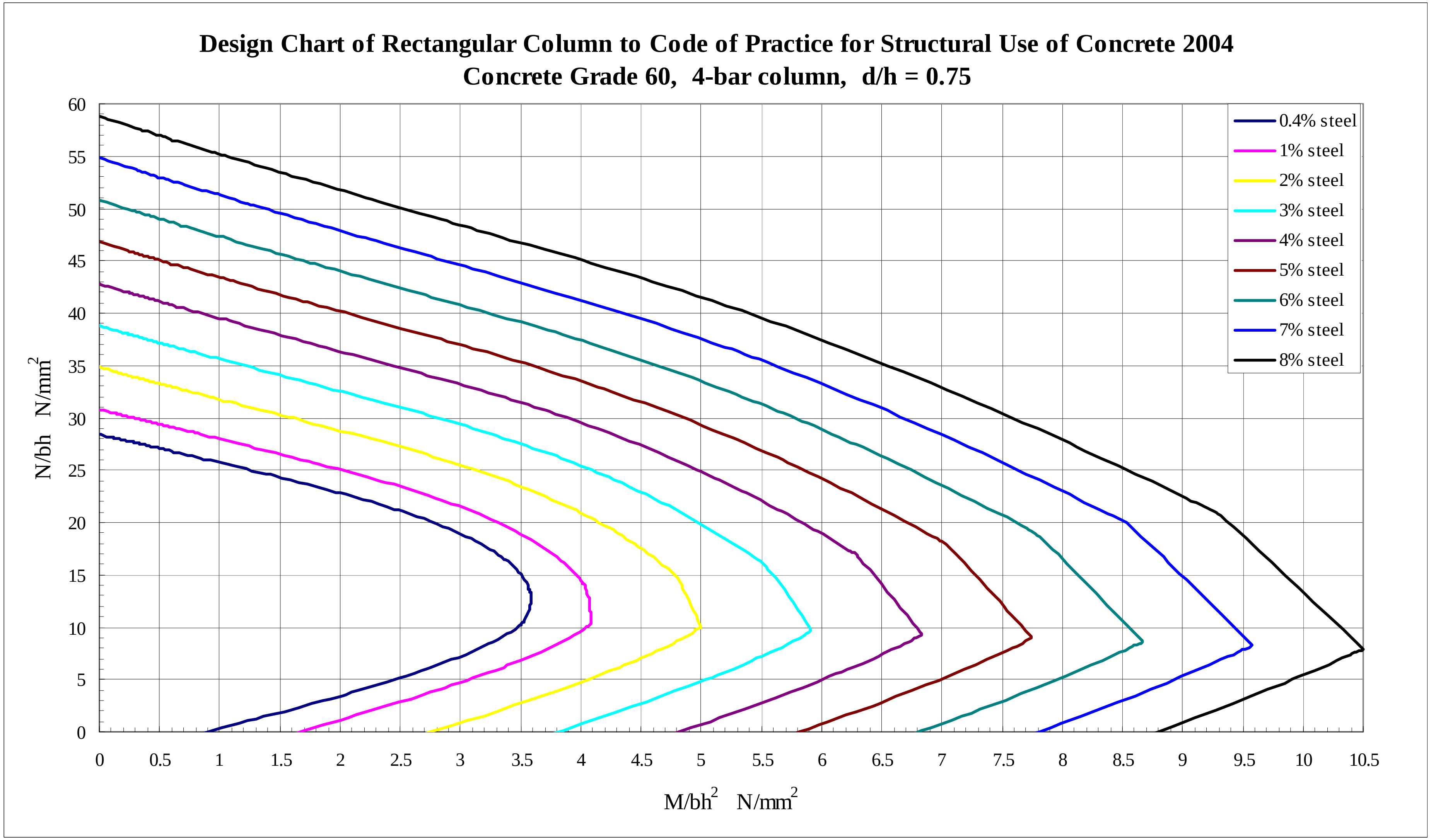

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 55, 4-bar column, d/h = 0.75
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
60
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5

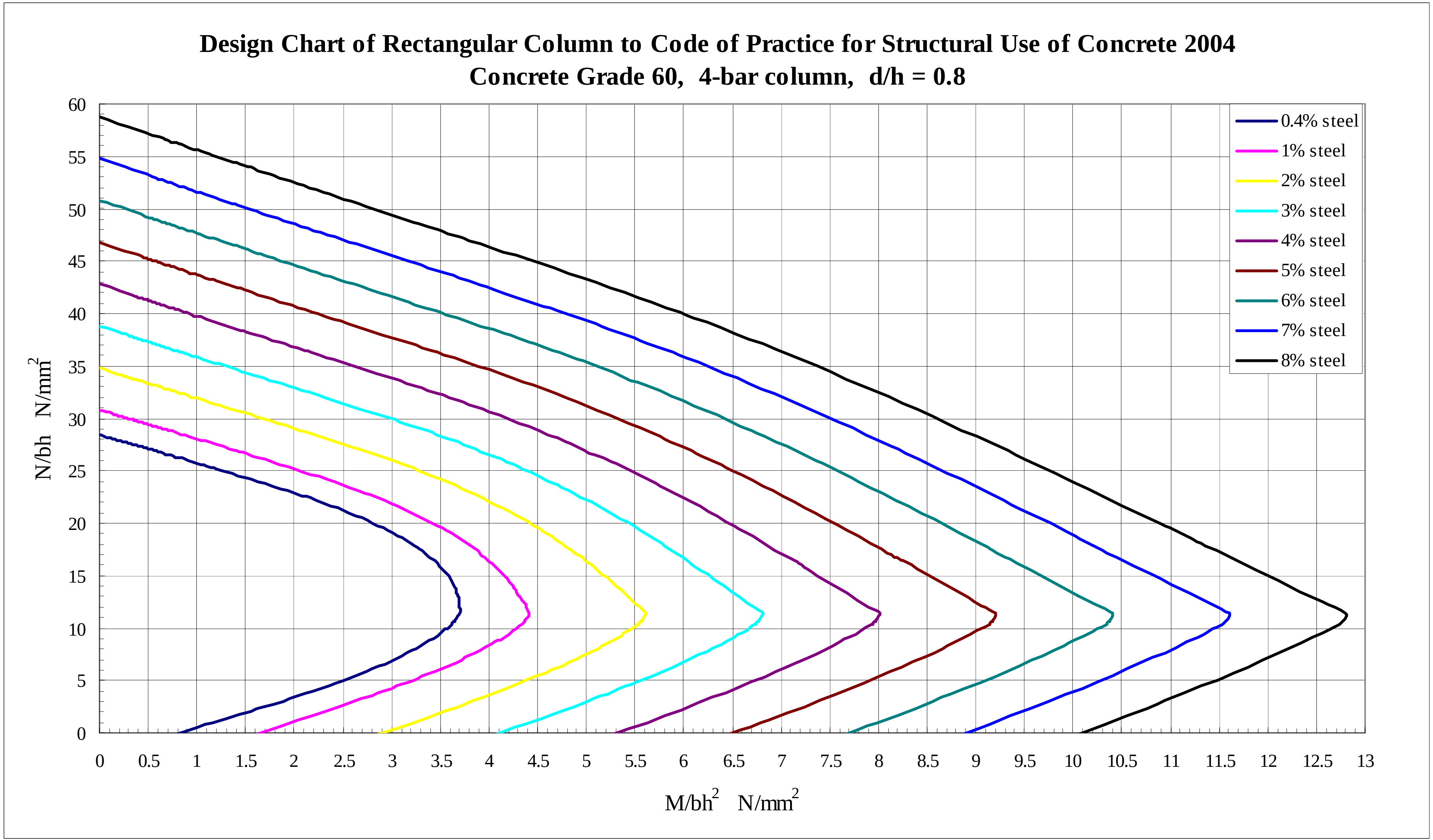

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 55, 4-bar column, d/h = 0.8
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
60
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 55, 4-bar column, d/h = 0.85
N/bh N/mm^2
M/bh^2 N/mm^2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
60
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10.5
11
11.5
12
12.5
13
13.5
14
14.5

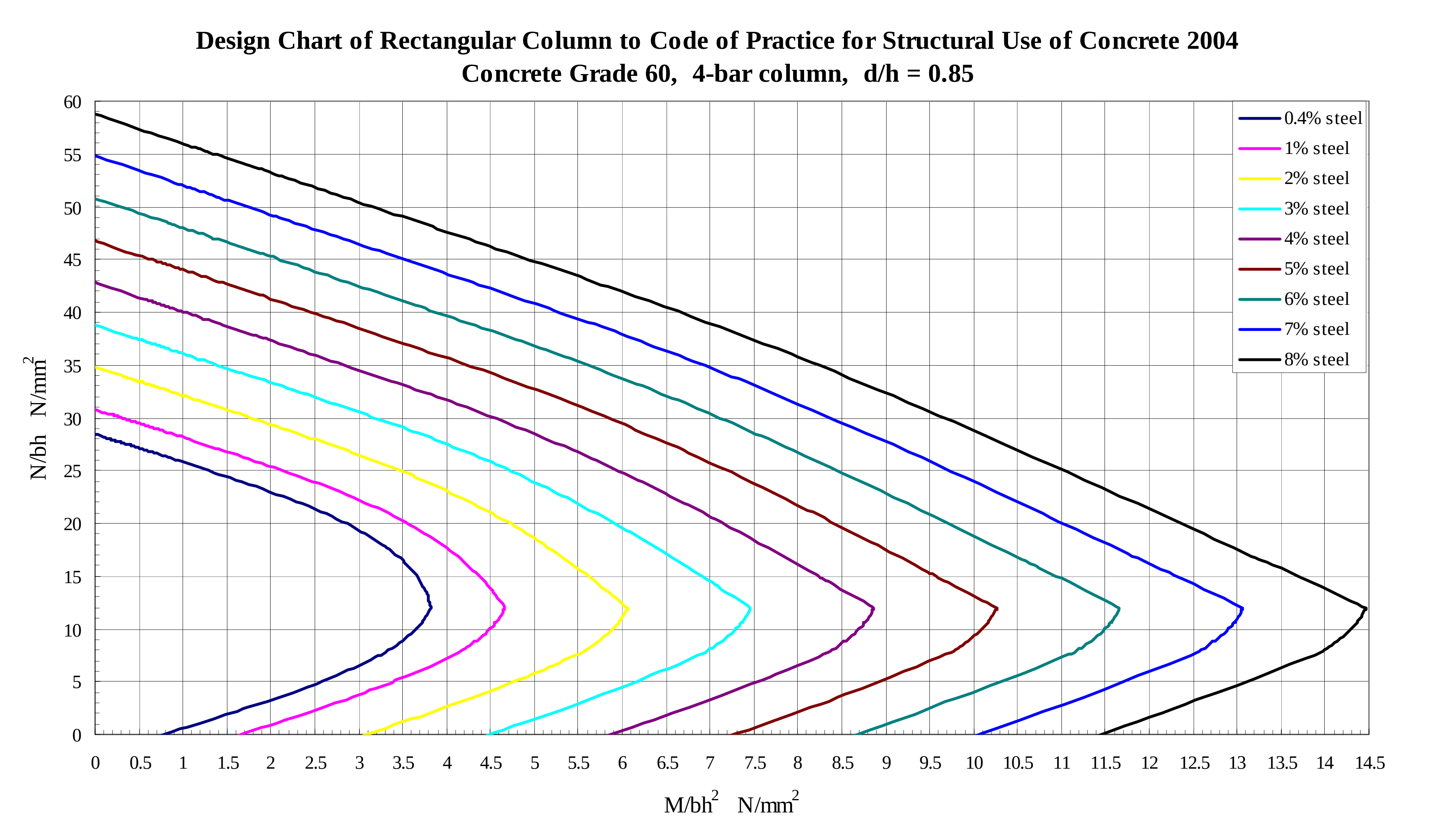

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 55, 4-bar column, d/h = 0.9
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
60
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14
14.5
15
15.5
16

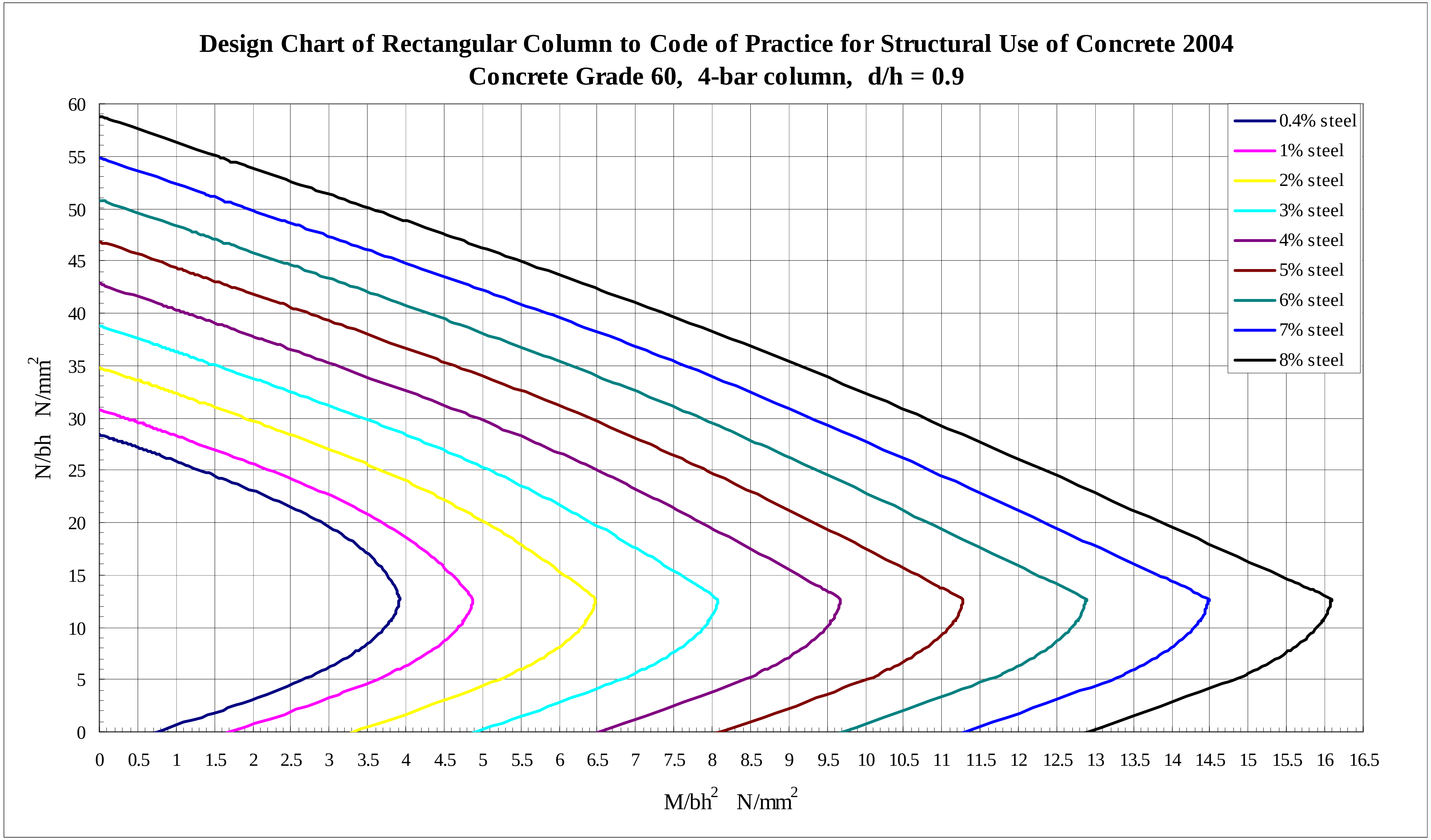

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 55, 4-bar column, d/h = 0.95
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
60
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10 10.5 11 11.5 12 12.5 13 13.5 14 14.5 15 15.5 16 16.5 17 17.5

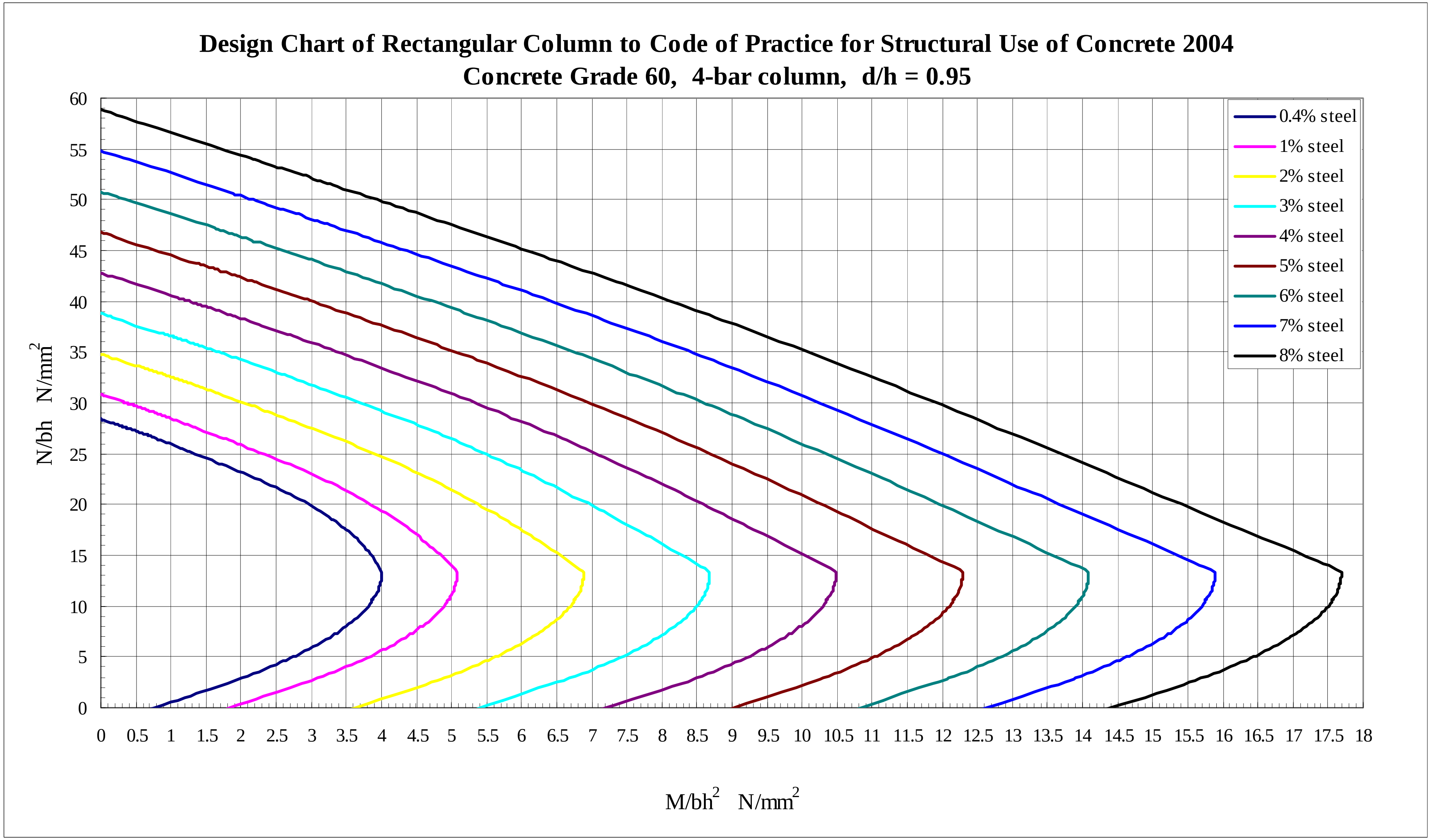

Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 60, 4-bar column, d/h = 0.75
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
60
55
50
45
40
35
30
25
20
15
10
5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5



Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 60, 4-bar column, d/h = 0.8
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh² N/mm²
M/bh² N/mm²
0
5
10
15
20
25
30
35
40
45
50
55
60
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 60, 4-bar column, d/h = 0.85
N/bh² N/mm²
M/bh² N/mm²
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
60
55
50
45
40
35
30
25
20
15
10
5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14
14.5


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 60, 4-bar column, d/h = 0.9
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0 5 10 15 20 25 30 35 40 45 50 55 60
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10 10.5 11 11.5 12 12.5 13 13.5 14 14.5 15 15.5 16 16.5


Design Chart of Rectangular Column to Code of Practice for Structural Use of Concrete 2004
Concrete Grade 60, 4-bar column, d/h = 0.95
N/bh² N/mm²
M/bh² N/mm²
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
55
60
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10 10.5 11 11.5 12 12.5 13 13.5 14 14.5 15 15.5 16 16.5 17 17.5 18

Rectangular Column R.C. Design to Code of Practice for Structural Use of Concrete 2004 - 4 bar column

Project :

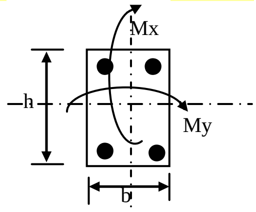
Column Mark **Floor**

f_{cu} = 35 N/mm^2 f_y = 460 N/mm^2 E_c = 23700 N/mm^2

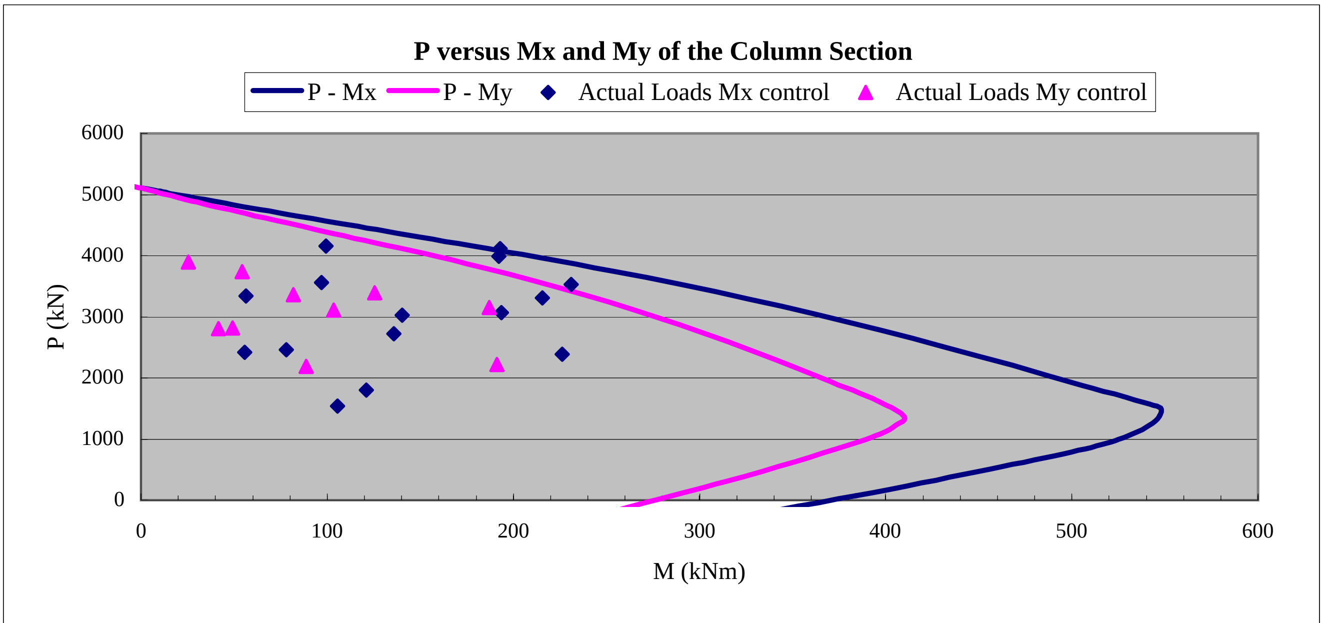
b = 400 h = 500 b' = 330.00 h' = 430.00 cover= 50 bar size = 40

Basic Load Case

Load Case No.	1	2	3	4	5	6
Load Case	D.L.	L.L.	Wx	Wy	W45	W135
Axial Load P (kN)	2304.7	582.1	-362.17	-545.1	56.92	82.09
Moment M_x (kNm)	29.13	32.11	47.1	-75.12	98.1	8.93
Moment M_y (kNm)	-31.33	16.09	2.15	44.2	76.99	35.21



		N (kN)	Mx (kNm)	My (kNm)			N/bh (N/mm^2)	M/bh^2 (N/mm^2)	d/h / d/b	x/h / y/h	Steel (%)	Steel area (mm^2)
Load Comb 1	1.4D+1.6L	4157.9	92.158	-18.118	Mx' =	99.411	20.79	0.9941	0.86	1.1701	1.9429	3885.7
Load Comb 2	1.2(D+L+Wx)	3029.6	130.01	-15.708	Mx' =	140.12	15.148	1.4012	0.86	0.9702	0.8	1600
Load Comb 3	1.2(D+L-Wx)	3898.8	16.968	-20.868	My' =	25.447	19.494	0.3181	0.825	1.3301	1.1905	2380.9
Load Comb 4	1.2(D+L+Wy)	2810	-16.656	34.752	My' =	41.507	14.05	0.5188	0.825	1.0602	0.8	1600
Load Comb 5	1.2(D+L-Wy)	4118.3	163.63	-71.328	Mx' =	192.82	20.591	1.9282	0.86	1.0405	2.4722	4944.4
Load Comb 6	1.2(D+L+W45)	3532.5	191.21	74.1	Mx' =	231.22	17.662	2.3122	0.86	0.9448	2.0502	4100.4
Load Comb 7	1.2(D+L-W45)	3395.9	-44.232	-110.68	My' =	125.49	16.979	1.5686	0.825	0.979	1.4098	2819.6
Load Comb 8	1.2(D+L+W135)	3562.7	84.204	23.964	Mx' =	96.983	17.813	0.9698	0.86	1.0901	1.1681	2336.1
Load Comb 9	1.2(D+L-W135)	3365.7	62.772	-60.54	My' =	81.79	16.828	1.0224	0.825	1.04	0.9815	1963
Load Comb 10	1.4(D+Wx)	2719.5	106.72	-40.852	Mx' =	135.67	13.598	1.3567	0.86	0.9324	0.8	1600
Load Comb 11	1.4(D-Wx)	3733.6	-25.158	-46.872	My' =	54.208	18.668	0.6776	0.825	1.15	1.226	2452.1
Load Comb 12	1.4(D+Wy)	2463.4	-64.386	18.018	Mx' =	78.184	12.317	0.7818	0.86	0.9619	0.8	1600
Load Comb 13	1.4(D-Wy)	3989.7	145.95	-105.74	Mx' =	192.25	19.949	1.9225	0.86	1.0306	2.2936	4587.2
Load Comb 14	1.4(D+W45)	3306.3	178.12	63.924	Mx' =	215.64	16.531	2.1564	0.86	0.9327	1.663	3326.1
Load Comb 15	1.4(D-W45)	3146.9	-96.558	-151.65	My' =	186.88	15.734	2.336	0.825	0.8772	1.6942	3388.3
Load Comb 16	1.4(D+W135)	3341.5	53.284	5.432	Mx' =	56.433	16.708	0.5643	0.86	1.1401	0.8	1600
Load Comb 17	1.4(D-W135)	3111.7	28.28	-93.156	My' =	103.6	15.558	1.2949	0.825	0.9803	0.8449	1689.7
Load Comb 18	1.0D+1.4Wx	1797.7	95.07	-28.32	Mx' =	120.97	8.9883	1.2097	0.86	0.7146	0.8	1600
Load Comb 19	1.0D-1.4Wx	2811.7	-36.81	-34.34	My' =	49.26	14.059	0.6158	0.825	1.0401	0.8	1600
Load Comb 20	1.0D+1.4Wy	1541.6	-76.038	30.55	Mx' =	105.72	7.7078	1.0572	0.86	0.5779	0.8	1600
Load Comb 21	1.0D-1.4Wy	3067.8	134.3	-93.21	Mx' =	193.56	15.339	1.9356	0.86	0.923	1.2156	2431.2
Load Comb 22	1.0D+1.4W45	2384.4	166.47	76.456	Mx' =	226.37	11.922	2.2637	0.86	0.7836	0.8	1600
Load Comb 23	1.0D-1.4W45	2225	-108.21	-139.12	My' =	191.32	11.125	2.3914	0.825	0.7278	0.8	1600
Load Comb 24	1.0D+1.4W135	2419.6	41.632	17.964	Mx' =	55.564	12.098	0.5556	0.86	0.9858	0.8	1600
Load Comb 25	1.0D-1.4W135	2189.8	16.628	-80.624	My' =	88.722	10.949	1.109	0.825	0.8718	0.8	1600
									Steel required =		2.4722	4944.4



Rectangular Column R.C. Design to Code of Practice for Structural Use of Concrete 2004

Project :

Column **Floor**

f_{cu} = 35 N/mm^2 f_y = 460 N/mm^2 E_c = 23700 N/mm^2

b = 1500 h = 2000 b' = 1285.17 h' = 1684.31 cover= 50

Steel provided :				
	15	Y	40	(Along each long sides h, excluding corner bars)
	12	Y	40	(Along each short sides b, excluding corner bars)
	4	Y	40	(Corner bars)

Total Steel Area = 72885 mm^2 Steel Percentage = 2.43 % Max. Ultimate Load = 76069 kN

Area of Steel per mm length for the long sides bars (including corner bars) = 21.36 mm^2/mm

Area of Steel along long sides (excluding corner bars) = 37699 mm^2

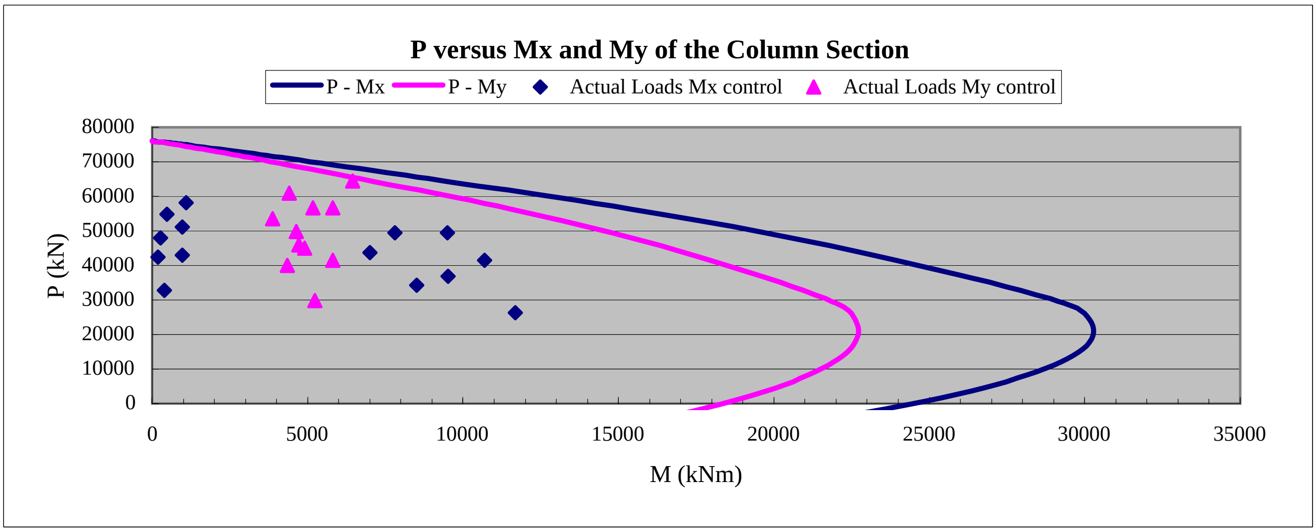
Area of Steel per mm length for the short sides bars (including corner bars) = 23.46 mm^2/mm

Area of Steel along short sides (excluding corner bars) = 30159 mm^2

Basic Load Case

Load Case No.	1	2	3	4	5	6
Load Case	D.L.	L.L.	Wx	Wy	W45	W135
Axial Load P (kN)	37872	1101	-3628.1	-2611.1	-5692.3	8209.2
Moment M_x (kNm)	-291.3	-37.11	470.81	-3700	-1750.3	4892.9
Moment M_y (kNm)	-31.33	16.09	5.17	2700	2764	-3520.2

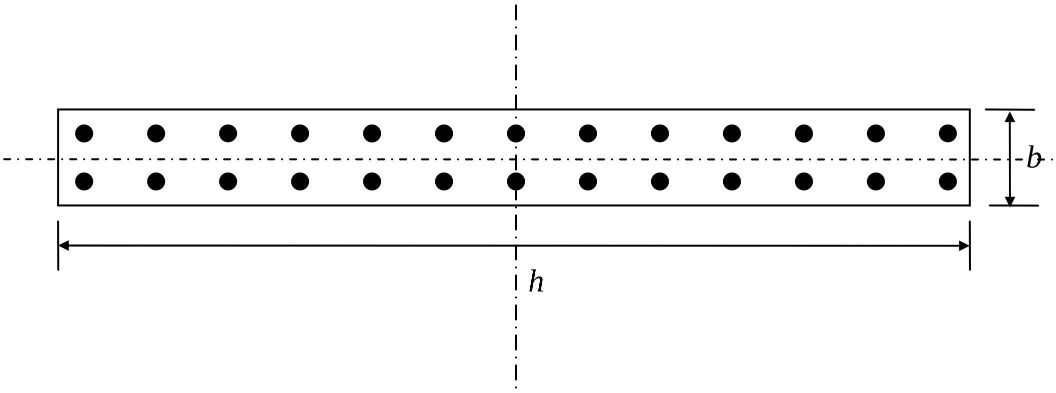
		P	M_x	M_y						
Load Comb 1	1.4D+1.6L	54782	-467.2	-18.118	Mx' =	476.55	Mux =	16452		Section OK
Load Comb 2	1.2(D+L+Wx)	42413	170.88	-12.084	Mx' =	179.2	Mux =	23474		Section OK
Load Comb 3	1.2(D+L-Wx)	51121	-959.06	-24.492	Mx' =	973.01	Mux =	18743		Section OK
Load Comb 4	1.2(D+L+Wy)	43634	-4834.1	3221.7	Mx' =	6999.6	Mux =	22861		Section OK
Load Comb 5	1.2(D+L-Wy)	49900	4045.9	-3258.3	My' =	4639	Muy =	14871		Section OK
Load Comb 6	1.2(D+L+W45)	39936	-2494.5	3298.5	My' =	4352.2	Muy =	18945		Section OK
Load Comb 7	1.2(D+L-W45)	53598	1706.3	-3335.1	My' =	3865.6	Muy =	13130		Section OK
Load Comb 8	1.2(D+L+W135)	56618	5477.4	-4242.6	My' =	5801.2	Muy =	11590		Section OK
Load Comb 9	1.2(D+L-W135)	36916	-6265.5	4206	Mx' =	9507.3	Mux =	26076		Section OK
Load Comb 10	1.4(D+Wx)	47941	251.31	-36.624	Mx' =	273.77	Mux =	20575		Section OK
Load Comb 11	1.4(D-Wx)	58099	-1067	-51.1	Mx' =	1090.8	Mux =	14185		Section OK
Load Comb 12	1.4(D+Wy)	49365	-5587.8	3736.1	Mx' =	7805.2	Mux =	19771		Section OK
Load Comb 13	1.4(D-Wy)	56676	4772.2	-3823.9	My' =	5179.4	Muy =	11560		Section OK
Load Comb 14	1.4(D+W45)	45051	-2858.3	3825.7	My' =	4911.9	Muy =	16951		Section OK
Load Comb 15	1.4(D-W45)	60989	2042.6	-3913.4	My' =	4416.8	Muy =	9164		Section OK
Load Comb 16	1.4(D+W135)	64513	6442.2	-4972.2	My' =	6446.8	Muy =	7132		Section OK
Load Comb 17	1.4(D-W135)	41527	-7257.9	4884.4	Mx' =	10685	Mux =	23910		Section OK
Load Comb 18	1.0D+1.4Wx	32792	367.83	-24.092	Mx' =	387.89	Mux =	27896		Section OK
Load Comb 19	1.0D-1.4Wx	42951	-950.43	-38.568	Mx' =	976.72	Mux =	23206		Section OK
Load Comb 20	1.0D+1.4Wy	34216	-5471.3	3748.7	Mx' =	8512.2	Mux =	27278		Section OK
Load Comb 21	1.0D-1.4Wy	41527	4888.7	-3811.3	My' =	5808.5	Muy =	18343		Section OK
Load Comb 22	1.0D+1.4W45	29902	-2741.8	3838.3	My' =	5236.3	Muy =	22460		Section OK
Load Comb 23	1.0D-1.4W45	45841	2159.2	-3900.9	My' =	4707.8	Muy =	16626		Section OK
Load Comb 24	1.0D+1.4W135	49364	6558.7	-4959.6	Mx' =	9502.2	Mux =	19771		Section OK
Load Comb 25	1.0D-1.4W135	26379	-7141.3	4897	Mx' =	11689	Mux =	29921		Section OK



Appendix G

Derivation of Design Formulae for Walls to Rigorous Stress Strain Curve of Concrete

Derivation of Design Formulae for Shear Walls to Rigorous Stress Strain Curve of Concrete



As similar to the exercise in Appendix F for columns, the exercise in this Appendix is repeated for walls by which $\frac{A_{sb}}{bh}$ are set to zero in the various cases 1 to 7, using the equations summarized in Appendix F.

Cases 1 to 3 – where $x/h \le 7/11$

By (Eqn F-18) or (Eqn F-32) or (Eqn F-36) of Appendix F

$$\frac{N_u}{bh} = \frac{0.67 f_{cu}}{3\gamma_m}\left(3 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h} + 0.87 f_y\left(2\frac{x}{h} - 1\right)\frac{A_{sh}}{bh}$$

$$\frac{A_{sh}}{bh} = \frac{\frac{N_u}{bh} - \frac{0.67 f_{cu}}{3\gamma_m}\left(3 - \frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}}{0.87 f_y\left(2\frac{x}{h} - 1\right)} \qquad \text{(Eqn G-1)}$$

Substituting into (Eqn F-31) or (Eqn F-34) or (Eqn F-38) and putting $\frac{A_{sb}}{bh} = 0$

$$\frac{M_u}{bh^2} = \frac{0.67 f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left\{\frac{1}{2}\left(1 - \frac{x}{h}\right) - \frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}} + \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\left(\frac{x}{h}\right) - \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\left(\frac{x}{h}\right)\right\} + 0.87 f_y \frac{A_{sh}}{bh}\left[\left(\frac{x}{h}\right) - \frac{163}{147}\left(\frac{x}{h}\right)^2\right]$$

$$\Rightarrow \frac{0.67 f_{cu}}{\gamma_m}\left\{\frac{16}{147} + \frac{131}{441}\frac{\varepsilon_0}{\varepsilon_{ult}} - \frac{1}{6}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right\}\left(\frac{x}{h}\right)^3 + \left\{\frac{0.67 f_{cu}}{\gamma_m}\left[\frac{1}{2} - \frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}} + \frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right] - \frac{N_u}{bh}\frac{163}{147}\right\}\left(\frac{x}{h}\right)^2$$

$$+\left[\frac{N_u}{bh} - \frac{2M_u}{bh^2} - \frac{0.67 f_{cu}}{\gamma_m}\left(\frac{1}{2} - \frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\right]\frac{x}{h} + \frac{M_u}{bh^2} = 0 \qquad \text{(Eqn G-2)}$$

Upon solving $\frac{x}{h}$, back substituting into (Eqn G-1) to calculate $\frac{A_{sh}}{bh}$

Case 4 – where $7/11 < x/h \le 1$

Referring to (Eqn F-39) of Appendix F and setting $\frac{A_{sb}}{bh}=0$

$$\frac{0.67 f_{cu}}{3\gamma_m}\left(3-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}+0.87 f_y \frac{A_{sh}}{bh}\left(-\frac{9}{56}\frac{x}{h}-\frac{7}{8}\frac{h}{x}+\frac{7}{4}\right)=\frac{N_u}{bh}$$

$$\Rightarrow \frac{A_{sh}}{bh}=\frac{\frac{N_u}{bh}-\frac{0.67 f_{cu}}{3\gamma_m}\left(3-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}}{0.87 f_y\left(-\frac{9}{56}\frac{x}{h}-\frac{7}{8}\frac{h}{x}+\frac{7}{4}\right)} \qquad \text{(Eqn G-3)}$$

Substituting into (Eqn F-41) with $\frac{A_{sb}}{bh}=0$

$$\frac{M_u}{bh^2}=\frac{0.67 f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left\{\frac{1}{2}\left(1-\frac{x}{h}\right)-\frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}}+\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}\left(\frac{x}{h}\right)-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\left(\frac{x}{h}\right)\right\}$$

$$+0.87 f_y\left[\frac{7}{48}\frac{h}{x}-\frac{9}{112}\frac{x}{h}+\frac{9}{392}\left(\frac{x}{h}\right)^2\right]\frac{A_{sh}}{bh}$$

$$\Rightarrow \frac{M_u}{bh^2}\left(\frac{7}{4}-\frac{9}{56}\frac{x}{h}-\frac{7}{8}\frac{h}{x}\right)=\frac{0.67 f_{cu}}{\gamma_m}\left(\frac{x}{h}\right)\left\{\left(\frac{1}{2}-\frac{1}{6}\frac{\varepsilon_0}{\varepsilon_{ult}}\right)+\left[\frac{1}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{1}{2}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{h}\right)\right\}\left(\frac{7}{4}-\frac{9}{56}\frac{x}{h}-\frac{7}{8}\frac{h}{x}\right)$$

$$+\left[\frac{7}{48}\frac{h}{x}-\frac{9}{112}\frac{x}{h}+\frac{9}{392}\left(\frac{x}{h}\right)^2\right]\left[\frac{N_u}{bh}-\frac{0.67 f_{cu}}{3\gamma_m}\left(3-\frac{\varepsilon_0}{\varepsilon_{ult}}\right)\frac{x}{h}\right]$$

$$\Rightarrow \frac{0.67 f_{cu}}{\gamma_m}\left[-\frac{45}{784}+\frac{9}{196}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{3}{224}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]\left(\frac{x}{h}\right)^4$$

$$+\left[\frac{0.67 f_{cu}}{\gamma_m}\left[\frac{7}{8}-\frac{7}{12}\frac{\varepsilon_0}{\varepsilon_{ult}}+\frac{7}{48}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]-\frac{9}{392}\frac{N_u}{bh}\right]\left(\frac{x}{h}\right)^3$$

$$+\left[-\frac{9}{56}\frac{M_u}{bh^2}+\frac{0.67 f_{cu}}{\gamma_m}\frac{7}{8}\left[-1.5+\frac{2}{3}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{1}{12}\left(\frac{\varepsilon_0}{\varepsilon_{ult}}\right)^2\right]+\frac{9}{112}\frac{N_u}{bh}\right]\left(\frac{x}{h}\right)^2$$

$$+\frac{7}{4}\frac{M_u}{bh^2}+\frac{0.67 f_{cu}}{3\gamma_m}\frac{7}{12}\left[3-\frac{\varepsilon_0}{\varepsilon_{ult}}\right]\frac{x}{h}+\left[-\frac{7}{8}\frac{M_u}{bh^2}-\frac{7}{48}\frac{N_u}{bh}\right]=0 \qquad \text{(Eqn G-4)}$$

Upon solving $\frac{x}{h}$, back-substitute into (Eqn G-3) to solve for $\frac{A_{sh}}{bh}$

<u>Case 5 – where $1 < x/h \le 1/(1-\varepsilon_0/\varepsilon_{ult})$</u>

Referring to (Eqn F-52) and setting $\frac{A_{sb}}{bd}=0$

$$\frac{N_u}{bh}=E_c\varepsilon_{ult}\left[-\frac{1}{2}-\frac{1}{6}\frac{\varepsilon_0^{\,2}}{\varepsilon_{ult}^{\,2}}+\frac{1}{6}\frac{\varepsilon_{ult}}{\varepsilon_0}+\frac{1}{2}\frac{\varepsilon_0}{\varepsilon_{ult}}\right]\frac{x}{h}+E_c\varepsilon_{ult}\left(1-\frac{1}{2}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)$$

$$+E_c\varepsilon_{ult}\left(\frac{1}{2}\frac{\varepsilon_{ult}}{\varepsilon_0}-\frac{1}{2}\right)\frac{h}{x}-\frac{E_c\varepsilon_{ult}^{\ 2}}{6\varepsilon_0}\left(\frac{h}{x}\right)^2+0.87f_y\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{9}{56}\frac{x}{h}\right)\frac{A_{sh}}{bh} \qquad \text{(Eqn G-5)}$$

Substituting for $0.87f_y\dfrac{A_{sh}}{bh}$ into (Eqn F-57), again setting $\dfrac{A_{sb}}{bd}=0$

$$\frac{M}{bh^2}=E_c\varepsilon_{ult}\left(-\frac{1}{24}\frac{\varepsilon_0^{\ 3}}{\varepsilon_{ult}^{\ 3}}+\frac{1}{6}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}-\frac{1}{4}\frac{\varepsilon_0}{\varepsilon_{ult}}+\frac{1}{6}-\frac{1}{24}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)\left(\frac{x}{h}\right)^2$$

$$+E_c\varepsilon_{ult}\left[-\frac{1}{12}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}+\frac{1}{4}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{1}{4}+\frac{1}{12}\frac{\varepsilon_{ult}}{\varepsilon_0}\right]\frac{x}{h}+E_c\varepsilon_{ult}\left(\frac{1}{12}-\frac{1}{12}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)\frac{h}{x}+E_c\varepsilon_{ult}\frac{1}{24}\frac{\varepsilon_{ult}}{\varepsilon_0}\left(\frac{h}{x}\right)^2$$

$$+0.87f_y\left[\frac{7}{48}\frac{h}{x}-\frac{9}{112}\frac{x}{h}+\frac{9}{392}\left(\frac{x}{h}\right)^2\right]\frac{A_{sh}}{bh}$$

$$\Rightarrow\frac{M}{bh^2}\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{9}{56}\frac{x}{h}\right)=E_c\varepsilon_{ult}\left(-\frac{1}{24}\frac{\varepsilon_0^{\ 3}}{\varepsilon_{ult}^{\ 3}}+\frac{1}{6}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}-\frac{1}{4}\frac{\varepsilon_0}{\varepsilon_{ult}}+\frac{1}{6}-\frac{1}{24}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{9}{56}\frac{x}{h}\right)\left(\frac{x}{h}\right)^2$$

$$+E_c\varepsilon_{ult}\left[-\frac{1}{12}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}+\frac{1}{4}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{1}{4}+\frac{1}{12}\frac{\varepsilon_{ult}}{\varepsilon_0}\right]\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{9}{56}\frac{x}{h}\right)\frac{x}{h}+E_c\varepsilon_{ult}\left(\frac{1}{12}-\frac{1}{12}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{9}{56}\frac{x}{h}\right)\frac{h}{x}$$

$$+E_c\varepsilon_{ult}\frac{1}{24}\frac{\varepsilon_{ult}}{\varepsilon_0}\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{9}{56}\frac{x}{h}\right)\left(\frac{h}{x}\right)^2+\frac{N_u}{bh}\left[\frac{7}{48}\frac{h}{x}-\frac{9}{112}\frac{x}{h}+\frac{9}{392}\left(\frac{x}{h}\right)^2\right]$$

$$-E_c\varepsilon_{ult}\left[-\frac{1}{2}-\frac{1}{6}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}+\frac{1}{6}\frac{\varepsilon_{ult}}{\varepsilon_0}+\frac{1}{2}\frac{\varepsilon_0}{\varepsilon_{ult}}\right]\frac{x}{h}\left[\frac{7}{48}\frac{h}{x}-\frac{9}{112}\frac{x}{h}+\frac{9}{392}\left(\frac{x}{h}\right)^2\right]$$

$$-E_c\varepsilon_{ult}\left(1-\frac{1}{2}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)\left[\frac{7}{48}\frac{h}{x}-\frac{9}{112}\frac{x}{h}+\frac{9}{392}\left(\frac{x}{h}\right)^2\right]-E_c\varepsilon_{ult}\left(\frac{1}{2}\frac{\varepsilon_{ult}}{\varepsilon_0}-\frac{1}{2}\right)\frac{h}{x}\left[\frac{7}{48}\frac{h}{x}-\frac{9}{112}\frac{x}{h}+\frac{9}{392}\left(\frac{x}{h}\right)^2\right]$$

$$+\frac{E_c\varepsilon_{ult}^{\ 2}}{6\varepsilon_0}\left(\frac{h}{x}\right)^2\left[\frac{7}{48}\frac{h}{x}-\frac{9}{112}\frac{x}{h}+\frac{9}{392}\left(\frac{x}{h}\right)^2\right]$$

$$\Rightarrow E_c\varepsilon_{ult}\left(\frac{3}{448}\frac{\varepsilon_0^{\ 3}}{\varepsilon_{ult}^{\ 3}}-\frac{9}{392}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}+\frac{45}{1568}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{3}{196}+\frac{9}{3136}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)\left(\frac{x}{h}\right)^3$$

$$+\left[E_c\varepsilon_{ult}\left(\frac{-7}{96}\frac{\varepsilon_0^{\ 3}}{\varepsilon_{ult}^{\ 3}}+\frac{7}{24}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}-\frac{7}{16}\frac{\varepsilon_0}{\varepsilon_{ult}}+\frac{79}{294}-\frac{289}{4704}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)+\frac{9}{392}\frac{N_u}{bh}\right]\left(\frac{x}{h}\right)^2$$

$$+\left[E_c\varepsilon_{ult}\left(\frac{7}{193}\frac{\varepsilon_0^{\ 3}}{\varepsilon_{ult}^{\ 3}}-\frac{7}{24}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}+\frac{21}{32}\frac{\varepsilon_0}{\varepsilon_{ult}}-\frac{289}{588}+\frac{1229}{9408}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)+\frac{9}{56}\frac{M}{bh^2}-\frac{9}{112}\frac{N_u}{bh}\right]\left(\frac{x}{h}\right)$$

$$+\left[E_c\varepsilon_{ult}\left(\frac{7}{72}\frac{\varepsilon_0^{\ 2}}{\varepsilon_{ult}^{\ 2}}-\frac{7}{24}\frac{\varepsilon_0}{\varepsilon_{ult}}+\frac{5}{21}-\frac{281}{7056}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)-\frac{7}{4}\frac{M}{bh^2}\right]+\left(-\frac{E_c\varepsilon_{ult}^{\ 2}}{\varepsilon_0}\frac{125}{1344}+\frac{7}{8}\frac{M}{bh^2}+\frac{7}{48}\frac{N_u}{bh}\right)\frac{h}{x}$$

$$+\frac{E_c\varepsilon_{ult}^{\ 2}}{\varepsilon_0}\frac{7}{96}\left(\frac{h}{x}\right)^2-\frac{E_c\varepsilon_{ult}^{\ 2}}{\varepsilon_0}\frac{7}{576}\left(\frac{h}{x}\right)^3=0 \qquad \text{(Eqn G-6)}$$

which is in fact an equation of 6th power in $\dfrac{x}{h}$. $\dfrac{x}{h}$ is to be solved by numerical

method. By back-substituting $\frac{x}{h}$ into (Eqn G-5)

$$\frac{N_u}{bh} = E_c\varepsilon_{ult}\left[-\frac{1}{2}-\frac{1}{6}\frac{\varepsilon_0{}^2}{\varepsilon_{ult}{}^2}+\frac{1}{6}\frac{\varepsilon_{ult}}{\varepsilon_0}+\frac{1}{2}\frac{\varepsilon_0}{\varepsilon_{ult}}\right]\frac{x}{h}+E_c\varepsilon_{ult}\left(1-\frac{1}{2}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)$$

$$+E_c\varepsilon_{ult}\left(\frac{1}{2}\frac{\varepsilon_{ult}}{\varepsilon_0}-\frac{1}{2}\right)\frac{h}{x}-\frac{E_c\varepsilon_{ult}{}^2}{6\varepsilon_0}\left(\frac{h}{x}\right)^2+0.87f_y\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{9}{56}\frac{x}{h}\right)\frac{A_{sh}}{bh}$$

$$\frac{A_{sh}}{bh} = \frac{\frac{N_u}{bh}-E_c\varepsilon_{ult}\left[-\frac{1}{2}-\frac{1}{6}\frac{\varepsilon_0{}^2}{\varepsilon_{ult}{}^2}+\frac{1}{6}\frac{\varepsilon_{ult}}{\varepsilon_0}+\frac{1}{2}\frac{\varepsilon_0}{\varepsilon_{ult}}\right]\frac{x}{h}-E_c\varepsilon_{ult}\left(1-\frac{1}{2}\frac{\varepsilon_{ult}}{\varepsilon_0}\right)-E_c\varepsilon_{ult}\left(\frac{1}{2}\frac{\varepsilon_{ult}}{\varepsilon_0}-\frac{1}{2}\right)\frac{h}{x}+\frac{E_c\varepsilon_{ult}{}^2}{6\varepsilon_0}\left(\frac{h}{x}\right)^2}{0.87f_y\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{9}{56}\frac{x}{h}\right)}$$

Case 6 – where $1/(1-\varepsilon_0/\varepsilon_{ult}) < x/h \le 7/3$

Consider (Eqn F-58) and substituting $\frac{A_{sb}}{bh}=0$

$$\frac{N_u}{bh}=\frac{0.67f_{cu}}{\gamma_m}+0.87f_y\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{9}{56}\frac{x}{h}\right)\frac{A_{sh}}{bh}\Rightarrow\frac{A_{sh}}{bh}=\frac{\frac{N_u}{bh}-\frac{0.67f_{cu}}{\gamma_m}}{0.87f_y\left(\frac{7}{4}-\frac{7}{8}\frac{h}{x}-\frac{9}{56}\frac{x}{h}\right)} \quad \text{(Eqn G-7)}$$

Substituting into (Eqn F-60)

$$\frac{M_u}{bh^2}=0.87f_y\frac{A_{sh}}{bh}\left[\frac{7}{48}\frac{h}{x}-\frac{9}{112}\frac{x}{h}+\frac{9}{392}\left(\frac{x}{h}\right)^2\right]$$

$$\Rightarrow\frac{9}{392}\left(\frac{N_u}{bh}-\frac{0.67f_{cu}}{\gamma_m}\right)\left(\frac{x}{h}\right)^3+\left[\frac{9}{56}\frac{M_u}{bh^2}-\frac{9}{112}\left(\frac{N_u}{bh}-\frac{0.67f_{cu}}{\gamma_m}\right)\right]\left(\frac{x}{h}\right)^2$$

$$-\frac{7}{4}\frac{M_u}{bh^2}\frac{x}{h}+\left[\frac{7}{48}\left(\frac{N_u}{bh}-\frac{0.67f_{cu}}{\gamma_m}\right)+\frac{7}{8}\frac{M_u}{bh^2}\right]=0 \quad \text{(Eqn G-8)}$$

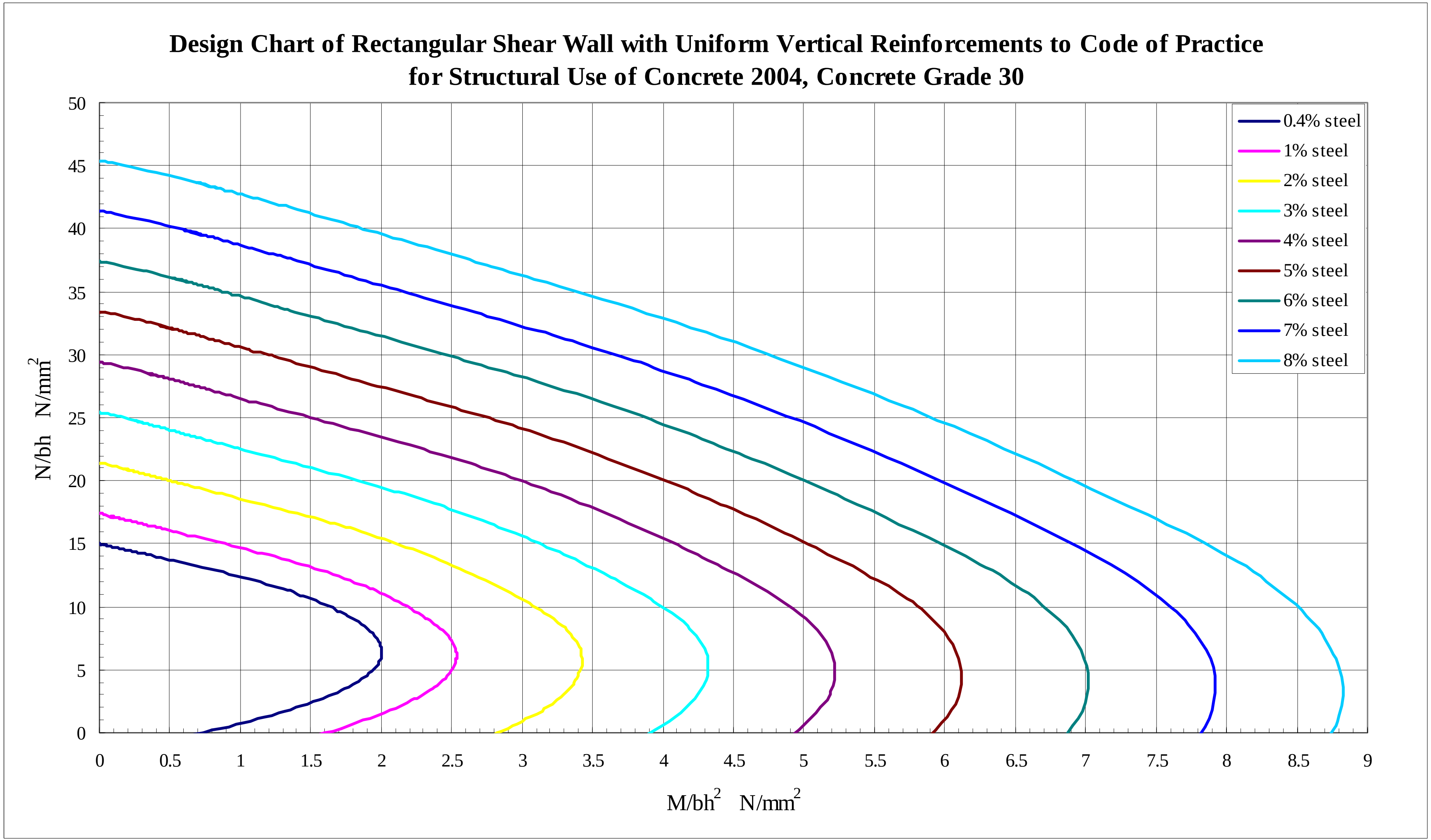
Solving the cubic equation for $\frac{x}{h}$ and substituting into (Eqn G-7) to solve for $\frac{A_{sh}}{bh}$

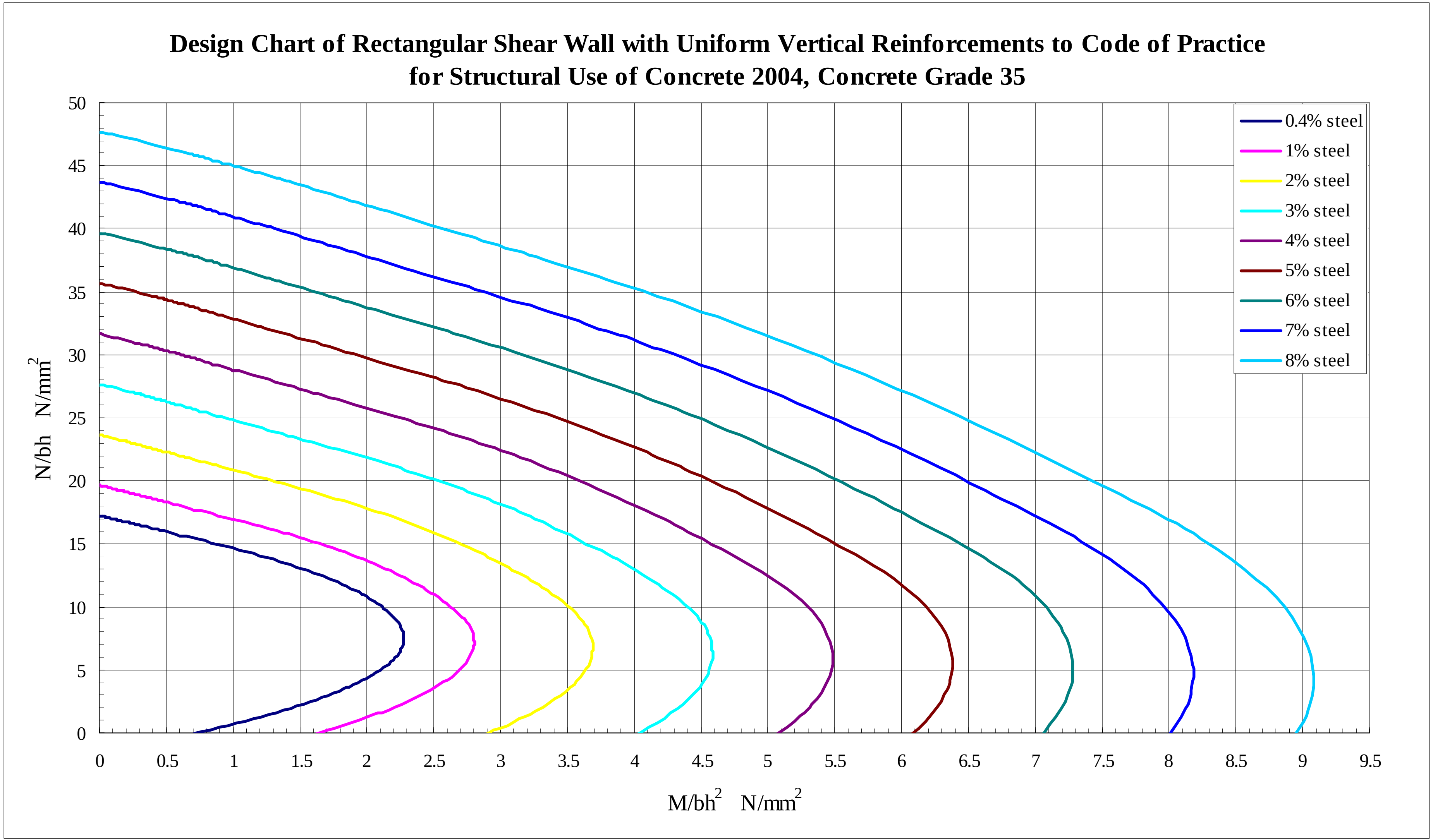
Case 7 – where $x/h > 7/3$

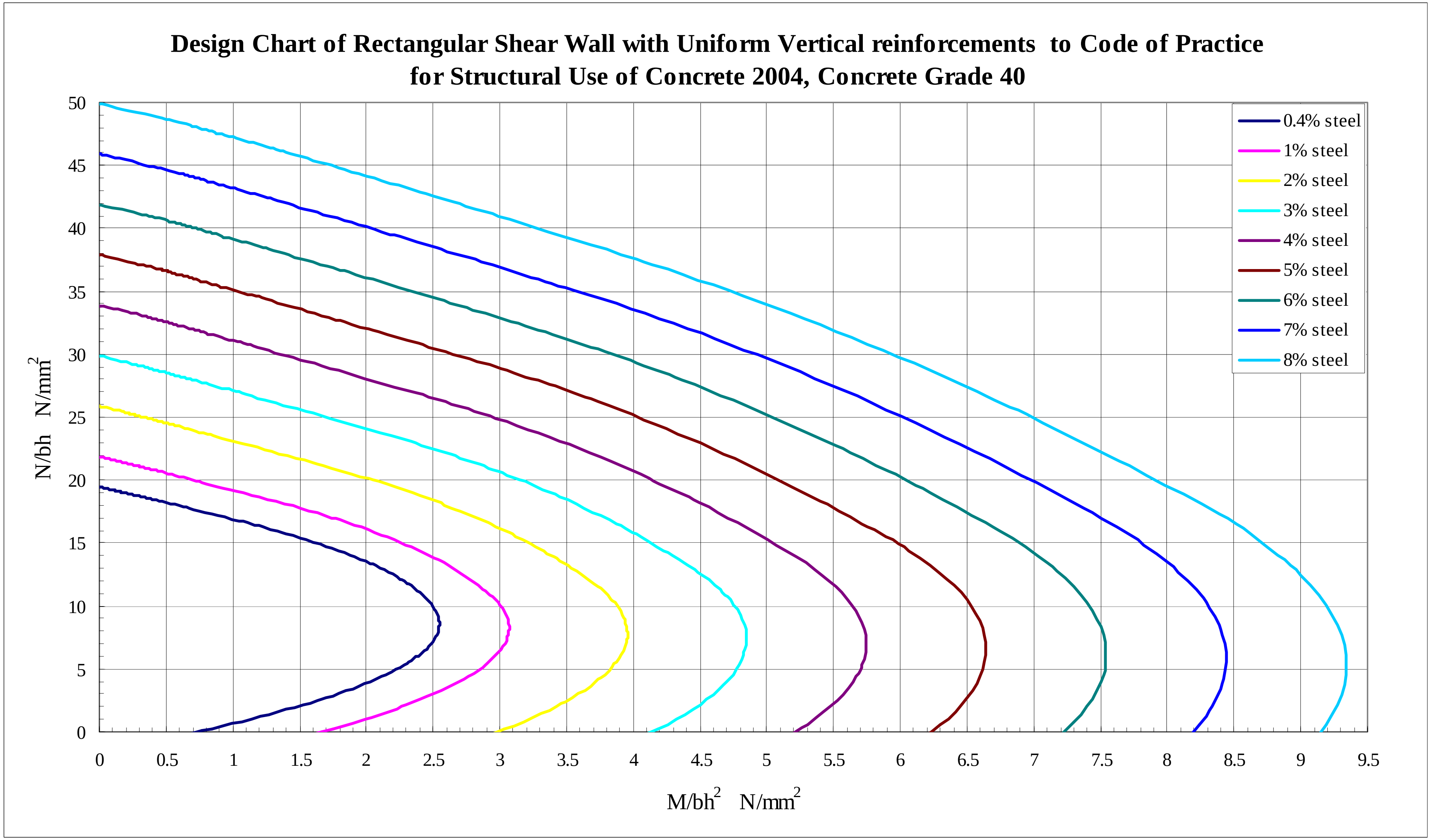
By (Eqn F-61) and setting $\frac{A_{sb}}{bh}=0$

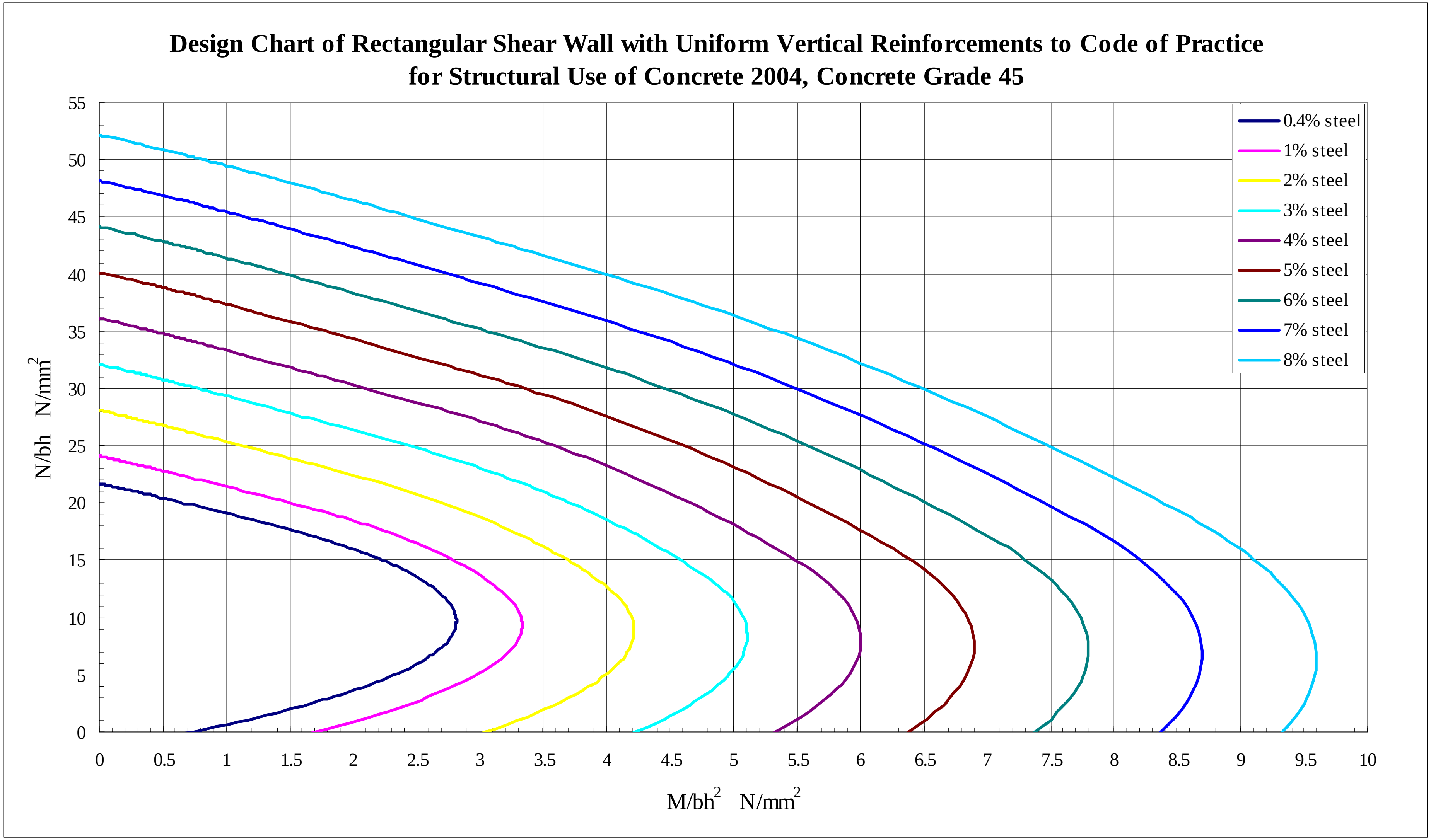
$$\frac{N_u}{bh}=\frac{0.67f_{cu}}{\gamma_m}+0.87f_y\frac{A_{sh}}{bh}\Rightarrow\frac{A_{sh}}{bh}=\left(\frac{N_u}{bh}-\frac{0.67f_{cu}}{\gamma_m}\right)\frac{1}{0.87f_y} \quad \text{(Eqn G-9)}$$

$$\frac{M_u}{bh^2}=0$$

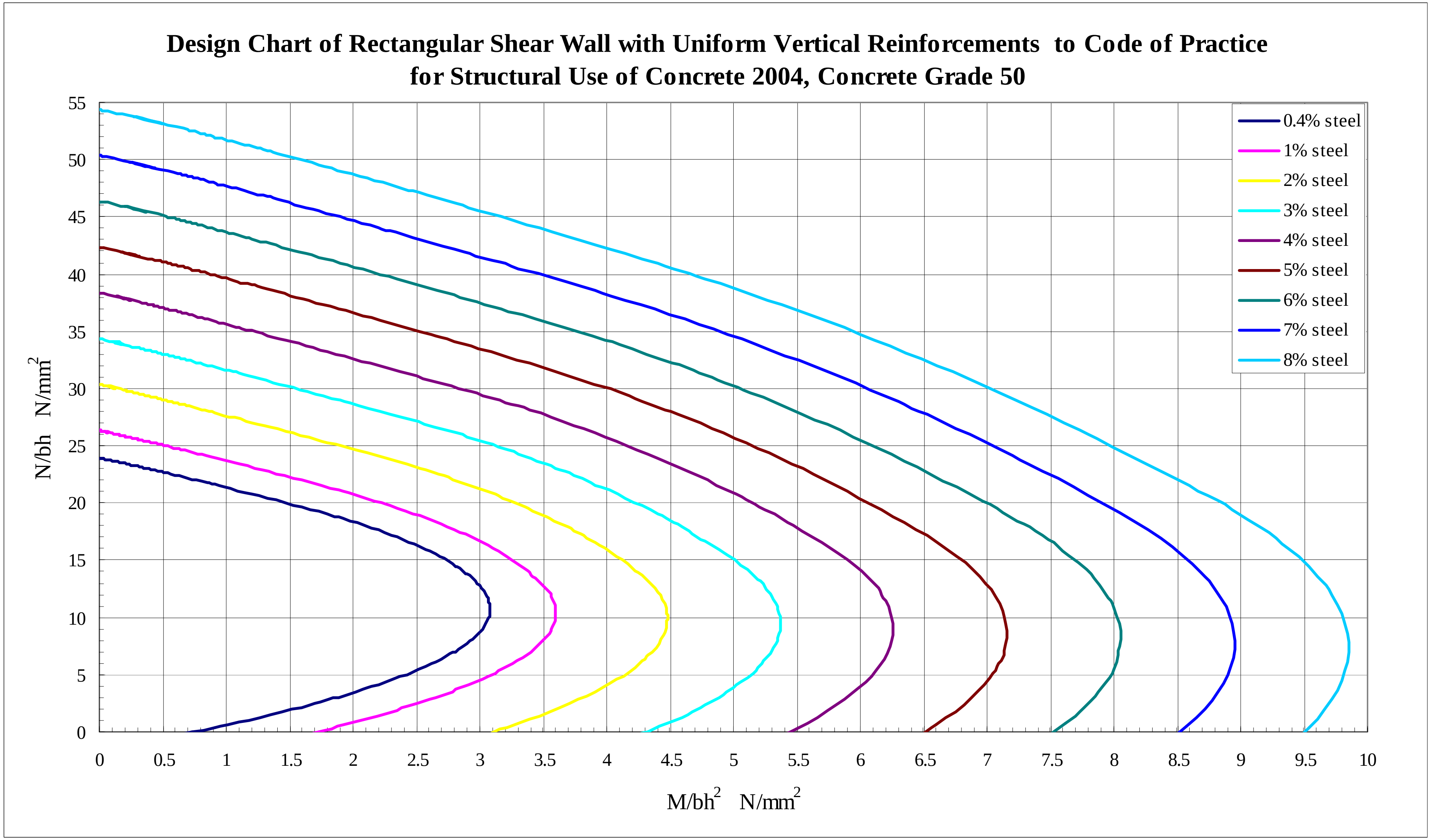

Design Chart of Rectangular Shear Wall with Uniform Vertical Reinforcements to Code of Practice for Structural Use of Concrete 2004, Concrete Grade 30
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9

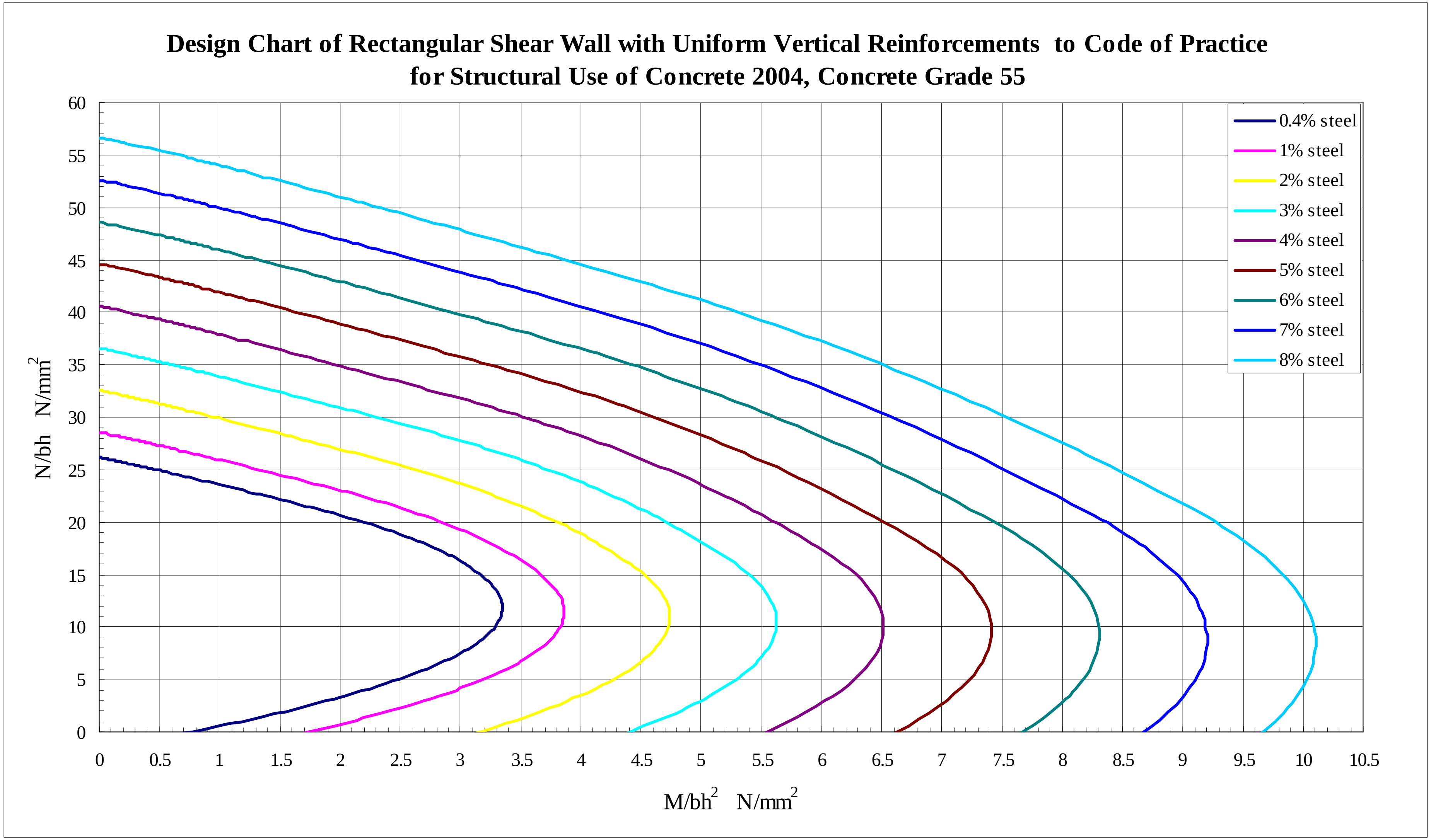

Design Chart of Rectangular Shear Wall with Uniform Vertical Reinforcements to Code of Practice
for Structural Use of Concrete 2004, Concrete Grade 35
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5

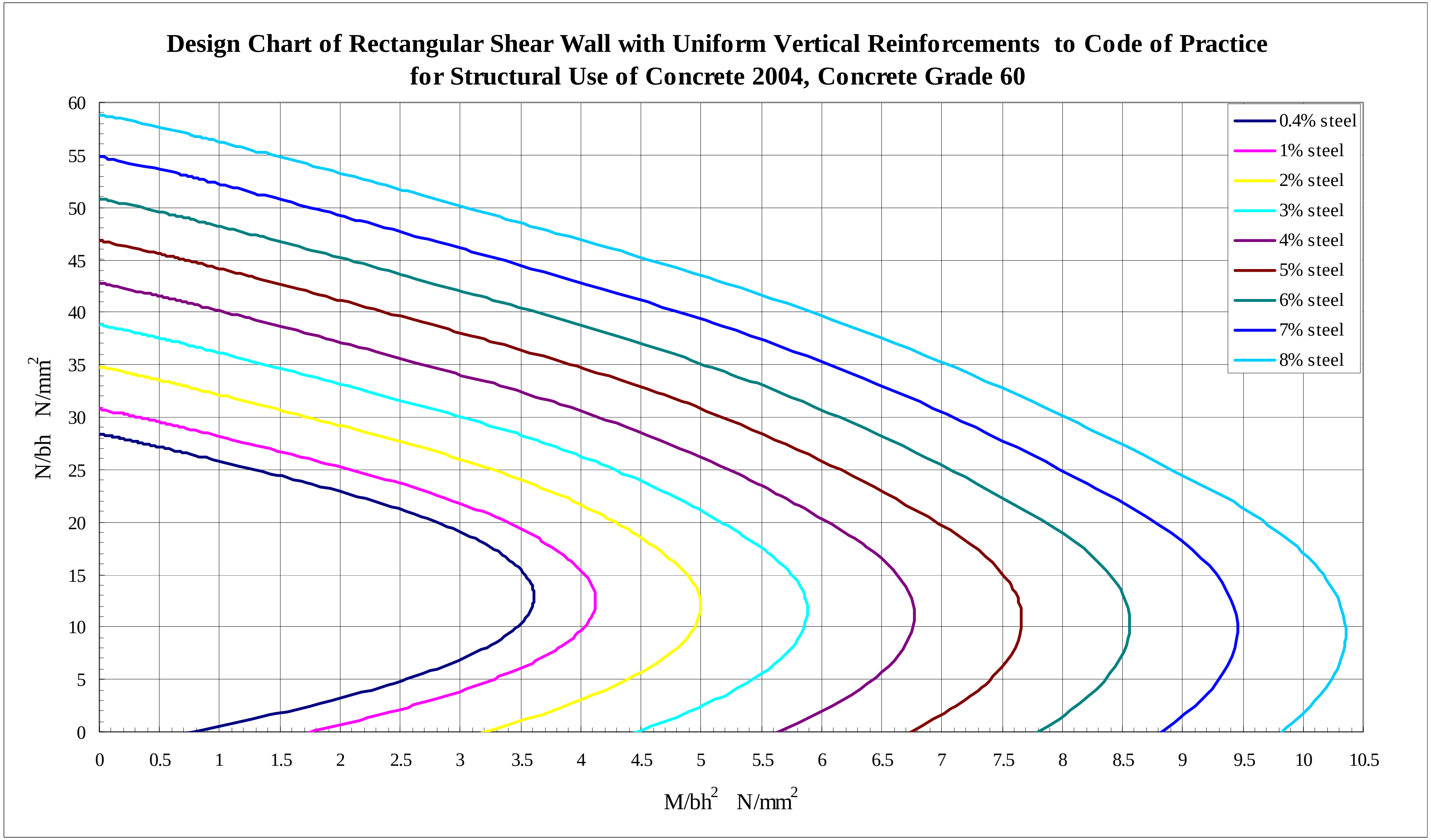

Design Chart of Rectangular Shear Wall with Uniform Vertical reinforcements to Code of Practice for Structural Use of Concrete 2004, Concrete Grade 40
N/bh N/mm2
M/bh2 N/mm2
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
0
5
10
15
20
25
30
35
40
45
50
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5



Design Chart of Rectangular Shear Wall with Uniform Vertical Reinforcements to Code of Practice for Structural Use of Concrete 2004, Concrete Grade 45
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
5
15
20
25
30
35
40
45
50
55


Design Chart of Rectangular Shear Wall with Uniform Vertical Reinforcements to Code of Practice for Structural Use of Concrete 2004, Concrete Grade 50
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10


Design Chart of Rectangular Shear Wall with Uniform Vertical Reinforcements to Code of Practice for Structural Use of Concrete 2004, Concrete Grade 55
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
5
10
15
20
25
30
35
40
45
50
55
60
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5


Design Chart of Rectangular Shear Wall with Uniform Vertical Reinforcements to Code of Practice for Structural Use of Concrete 2004, Concrete Grade 60
0.4% steel
1% steel
2% steel
3% steel
4% steel
5% steel
6% steel
7% steel
8% steel
N/bh N/mm2
M/bh2 N/mm2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
0
5
10
15
20
25
30
35
40
45
50
55
60

Shear Wall R.C. Design to Code of Practice for Structural Use of Concrete 2004 - Uniform Reinforcements

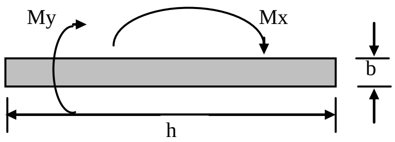
Project :

Wall Mark **Floor**

f_{cu} = 35 N/mm^2 f_y = 460 N/mm^2 E_c = 23700 N/mm^2

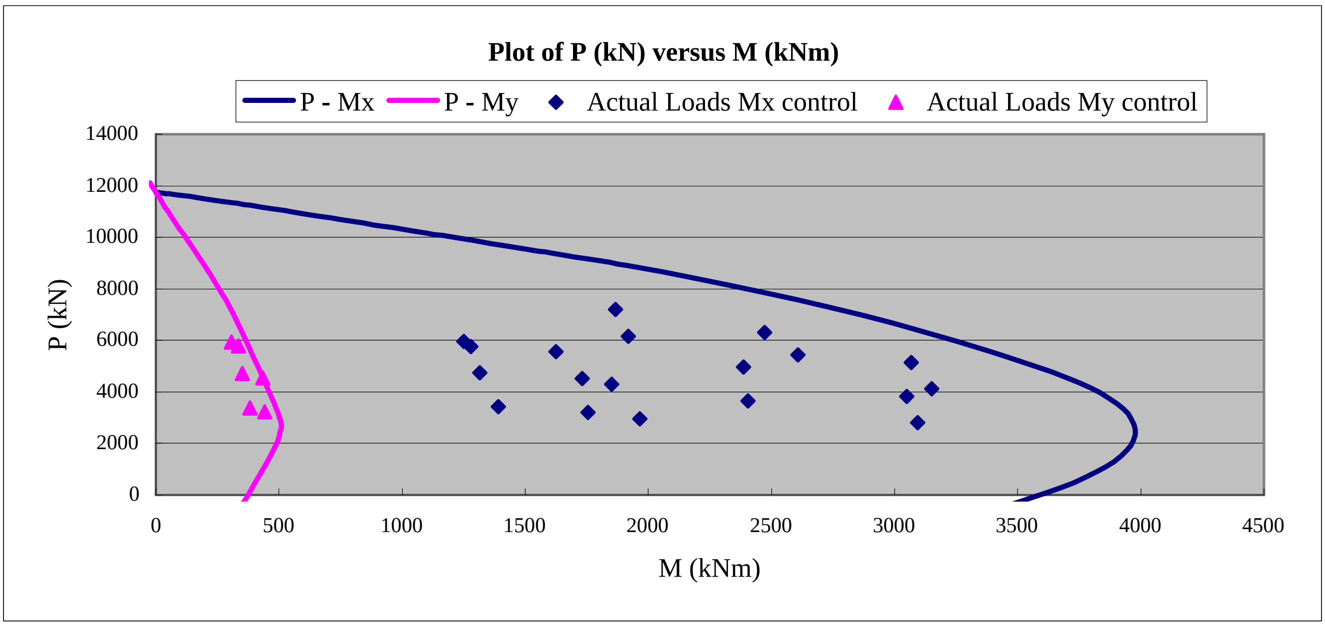
b = 200 h = 2000 b' = 165.00 h' = 1500.00 cover= 25 bar size = 20

Basic Load Case



Load Case No.	1	2	3	4	5	6
Load Case	D.L.	L.L.	Wx	Wy	W45	W135
Axial Load P (kN)	3304.7	1582.1	-362.17	-245.1	56.92	82.09
Moment M_x (kNm)	29.13	32.11	2047.1	-1275.1	1098.1	888.93
Moment M_y (kNm)	-31.33	16.09	2.15	44.2	206.5	35.21

		N (kN)	Mx (kNm)	My (kNm)			N/bh (N/mm^2)	M/bh^2 (N/mm^2)	d/h / d/b	x/h / y/b	Steel (%)	Steel area (mm^2)
Load Comb 1	1.4D+1.6L	7200	1500	100	Mx' =	1866.2	18	2.3328	-	0.8901	2.3359	9343.5
Load Comb 2	1.2(D+L+Wx)	5429.6	2530	-15.708	Mx' =	2607.8	13.574	3.2597	-	0.7152	2.3059	9223.4
Load Comb 3	1.2(D+L-Wx)	6298.8	-2383	-20.868	Mx' =	2473.2	15.747	3.0915	-	0.7772	2.5637	10255
Load Comb 4	1.2(D+L+Wy)	5570	-1456.7	34.752	Mx' =	1624.9	13.925	2.0311	-	0.8365	1.0928	4371.1
Load Comb 5	1.2(D+L-Wy)	6158.3	1603.6	-71.328	Mx' =	1918.9	15.396	2.3986	-	0.8314	1.7861	7144.5
Load Comb 6	1.2(D+L+W45)	5932.5	1391.2	229.51	My' =	306.62	14.831	3.8328	0.825	0.7508	2.7599	11040
Load Comb 7	1.2(D+L-W45)	5795.9	-1244.2	-266.09	My' =	336.52	14.49	4.2065	0.825	0.7255	3.0079	12032
Load Comb 8	1.2(D+L+W135)	5962.7	1140.2	23.964	Mx' =	1249.5	14.907	1.5618	-	0.9185	0.9046	3618.4
Load Comb 9	1.2(D+L-W135)	5765.7	-993.23	-60.54	Mx' =	1277.8	14.414	1.5972	-	0.9032	0.8122	3248.8
Load Comb 10	1.4(D+Wx)	4119.5	2906.7	-40.852	Mx' =	3150.7	10.299	3.9384	-	0.6025	2.5143	10057
Load Comb 11	1.4(D-Wx)	5133.6	-2825.2	-46.872	Mx' =	3068	12.834	3.835	-	0.6673	2.7996	11199
Load Comb 12	1.4(D+Wy)	4283.4	-1744.4	18.018	Mx' =	1849.7	10.709	2.3121	-	0.6995	0.7417	2966.7
Load Comb 13	1.4(D-Wy)	4969.7	1826	-105.74	Mx' =	2387.4	12.424	2.9842	-	0.7031	1.7913	7165.1
Load Comb 14	1.4(D+W45)	4706.3	1578.1	245.24	My' =	350.54	11.766	4.3818	0.825	0.6532	2.626	10504
Load Comb 15	1.4(D-W45)	4546.9	-1496.6	-332.96	My' =	435.07	11.367	5.4384	0.825	0.62	3.4305	13722
Load Comb 16	1.4(D+W135)	4741.5	1285.3	5.432	Mx' =	1315.1	11.854	1.6439	-	0.8274	0.4	1600
Load Comb 17	1.4(D-W135)	4511.7	-1203.7	-93.156	Mx' =	1731.6	11.279	2.1645	-	0.7373	0.6743	2697.1
Load Comb 18	1.0D+1.4Wx	2797.7	2895.1	-28.32	Mx' =	3093.4	6.9942	3.8667	-	0.5051	2.1902	8760.9
Load Comb 19	1.0D-1.4Wx	3811.7	-2836.8	-34.34	Mx' =	3050.1	9.5293	3.8126	-	0.5843	2.2839	9135.8
Load Comb 20	1.0D+1.4Wy	2961.6	-1756	30.55	Mx' =	1966	7.4039	2.4576	-	0.5307	0.607	2428.1
Load Comb 21	1.0D-1.4Wy	3647.8	1814.3	-93.21	Mx' =	2405.2	9.1196	3.0065	-	0.5941	1.3272	5308.9
Load Comb 22	1.0D+1.4W45	3384.4	1566.5	257.77	My' =	381.82	8.461	4.7727	0.825	0.5596	2.3848	9539.2
Load Comb 23	1.0D-1.4W45	3225	-1508.2	-320.43	My' =	442.13	8.0625	5.5266	0.825	0.5456	2.9149	11660
Load Comb 24	1.0D+1.4W135	3419.6	1273.6	17.964	Mx' =	1390.7	8.5491	1.7384	-	0.6436	0.4	1600
Load Comb 25	1.0D-1.4W135	3189.8	-1215.4	-80.624	Mx' =	1755.3	7.9744	2.1941	-	0.5706	0.4	1600
										Steel required =	3.4305	13722



Appendix H

Estimation of Support Stiffnesses of vertical supports to Transfer Structures

Appendix H

Estimation of Support Stiffnesses of vertical supports to Transfer Structures

Simulation of Support Stiffness in Plate Bending Structure

For support stiffness, we are referring to the force or moment required to produce unit vertical movement or unit rotation at the top of the support which are denoted by K_Z, $K_{\theta X}$, $K_{\theta Y}$ for settlement stiffness along the Z direction, and rotational stiffnesses about X and Y directions. These stiffnesses are independent parameters which can interact only through the plate structure. Most softwares allow the user either to input numerical values or structural sizes and heights of the support (which are usually walls or columns) by which the softwares can calculate numerical values for the support stiffnesses as follows :

(i) For the settlement stiffness K_Z, the value is mostly simply AE/L where A is the cross sectional of the support which is either a column or a wall, E is the Young's Modulus of the construction material and L is the free length of the column / wall. The AE/L simply measures the 'elastic shortening' of the column / wall.

Strictly speaking, the expression AE/L is only correct if the column / wall is one storey high and restrained completely from settlement at the bottom. However, if the column / wall is more than one storey high, the settlement stiffness estimation can be very complicated. It will not even be a constant value. The settlement of the support is, in fact, 'interacting' with that of others through the structural frame linking them together by transferring the axial loads in the column / wall to others through shears in the linking beams. Nevertheless, if the linking beams (usually floor beams) in the structural frame are 'flexible', the transfer of loads from one column / wall through the linking beams to the rest of the frame will generally be negligible. By ignoring such transfer, the settlement stiffness of a column / wall can be obtained by 'compounding' the settlement stiffness of the individual settlement stiffness at each floor as

$$K_Z = \frac{1}{\dfrac{L_1}{A_1E_1} + \dfrac{L_2}{A_2E_2} + \dfrac{L_3}{A_3E_3} + \ldots\ldots \dfrac{L_n}{A_nE_n}} = \frac{1}{\sum \dfrac{L_i}{A_iE_i}}$$

(ii) For the rotational stiffness, most of the existing softwares calculate the numerical values either by $\frac{4EI}{L}$ or $\frac{3EI}{L}$, depending on whether the far end of the supporting column / wall is assumed fixed or pinned (where I is the second moment of area of the column / wall section). However, apart from the assumed fixity at the far end, the formulae $\frac{4EI}{L}$ or $\frac{3EI}{L}$ are also based on the assumption that both ends of the column / wall are restrained from lateral movement (sidesway). It is obvious that the assumption will not be valid if the out-of-plane load or the structural layout is unsymmetrical where the plate will have lateral movements. The errors may be significant if the structure is to simulate a transfer plate under wind load which is in the form of an out-of-plane moment tending to overturn the structure. Nevertheless, the errors can be minimized by finding the force that will be required to restrain the slab structure from sideswaying and applying a force of the same magnitude but of opposite direction to nullify this force. This magnitude of this restraining force or nullifying force is the sum of the total shears produced in the supporting walls / columns due to the moments induced on the walls / columns from the plate analysis. However, the analysis of finding the effects on the plate by the "nullifying force" has to be done on a plane frame or a space frame structure as the 2-D plate bending model cannot cater for lateral in-plane loads. This approach is adopted by some local engineers and the procedure for analysis is illustrated in Figure H-1.

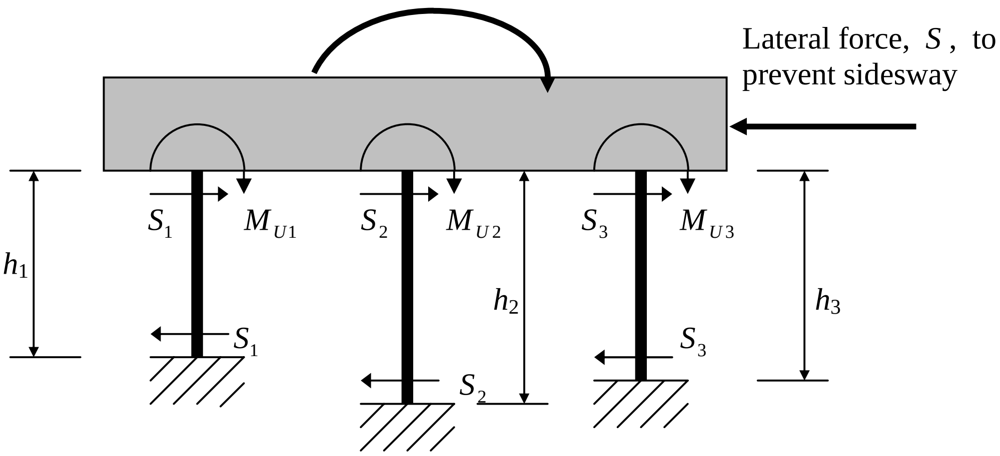


Figure H-1 Diagrammatic illustration of the restraining shear or nullifying shear

In addition, the followings should be noted :

Note : 1. If the wall / column is prismatic and the lower end is restrained from rotation, the moment at the lower end will be $M_{Li} = 0.5M_{Ui}$ (carry-over from the top); if the lower end is assumed pinned, the moment at it will be zero;

2. The shear on the wall / column will be $S_i = \dfrac{M_{Ui} + M_{Li}}{h_i}$ where M_{Ui} is obtained from plate bending analysis and the total restraining shear is $S = \sum S_i$

Appendix I

Derivation of Formulae for Rigid Cap Analysis

Derivation of Formulae for Rigid Cap Analysis

Underlying Principles of the Rigid Cap Analysis

The "Rigid Cap Analysis" method utilizes the assumption of "Rigid Cap" in the solution of pile loads beneath a single cap against out-of-plane loads, i.e. the cap is a perfectly rigid body which does not deform upon the application of loads. The cap itself may settle, translate or rotate, but as a rigid body. The deflections of a connecting pile will therefore be well defined by the movement of the cap and the location of the pile beneath the cap, taking into consideration of the connection conditions of the piles. Consider a Pile i situated from a point O on the pile cap as shown in Figure I-1 with settlement stiffness K_{iZ}

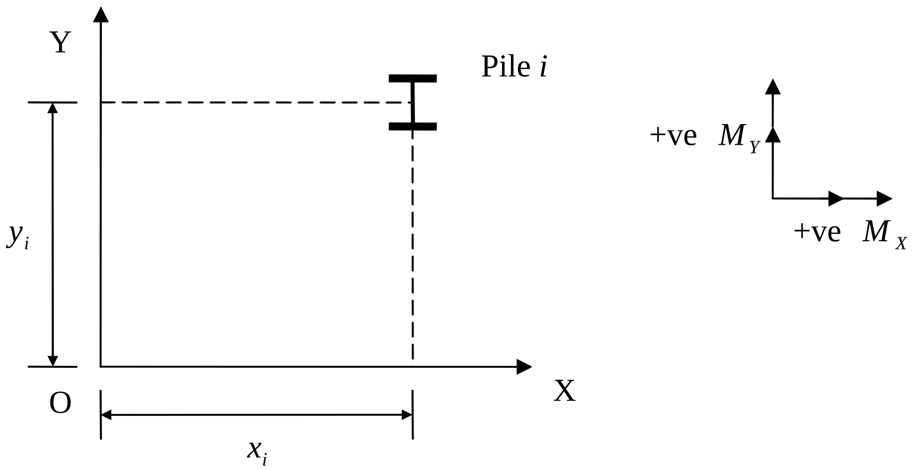


Figure I-1 – Derivation of Pile Loads under Rigid Cap

As the settlement of all piles beneath the Cap will lie in the same plane after the application of the out-of-plane load, the settlement of Pile i denoted by Δ_{iZ} can be defined by $b_O + b_1 x_i + b_2 y_i$ which is the equation for a plane in 'co-ordinate geometry' where b_O, b_1 and b_2 are constants.

The upward reaction by Pile i is $K_{iZ}(b_O + b_1 x_i + b_2 y_i)$

Summing all pile loads :

Balancing the applied load $P = \sum K_{iZ}(b_O + b_1 x_i + b_2 y_i)$

$\Rightarrow P = b_O \sum K_{iZ} + b_1 \sum K_{iZ} x_i + b_2 \sum K_{iZ} y_i$

Balancing the applied Moment $M_X = -\sum K_{iZ}(b_O + b_1 x_i + b_2 y_i)y_i$

$\Rightarrow M_X = -b_O \sum K_{iZ} y_i - b_1 \sum K_{iZ} x_i y_i - b_2 \sum K_{iZ} y_i^{\,2}$

Balancing the applied Moment $M_Y = \sum K_{iZ}(b_O + b_1 x_i + b_2 y_i)x_i$

$\Rightarrow M_Y = b_O \sum K_{iZ} x_i + b_1 \sum K_{iZ} x_i^{\,2} + b_2 \sum K_{iZ} x_i y_i$

It is possible to choose the centre O such that

$\sum K_{iZ} x_i = \sum K_{iZ} y_i = \sum K_{iZ} x_i y_i = 0$.

So the three equations become

$$P = b_O \sum K_{iZ} \qquad \Rightarrow \qquad b_O = \frac{P}{\sum K_{iZ}}$$

$$M_X = -b_2 \sum K_{iZ} y_i^{\,2} \qquad \Rightarrow \qquad b_2 = \frac{-M_X}{\sum K_{iZ} y_i^{\,2}}$$

$$M_Y = b_1 \sum K_{iZ} x_i^{\,2} \qquad \Rightarrow \qquad b_1 = \frac{M_Y}{\sum K_{iZ} x_i^{\,2}}$$

The load on Pile i is then

$$P = \sum K_{iZ}(b_O + b_1 x_i + b_2 y_i)$$

$$= K_{iZ}\left(\frac{P}{\sum K_{iZ}} + \frac{M_Y}{\sum K_{iZ} x_i^{\,2}} x_i - \frac{M_X}{\sum K_{iZ} y_i^{\,2}} y_i\right)$$

$$= \frac{PK_{iZ}}{\sum K_{iZ}} + \frac{M_Y K_{iZ}}{\sum K_{iZ} x_i^{\,2}} x_i - \frac{M_X K_{iZ}}{\sum K_{iZ} y_i^{\,2}} y_i$$

To effect $\sum K_{iZ} x_i = \sum K_{iZ} y_i = \sum K_{iZ} x_i y_i = 0$, the location of O and the orientation of the axes X-X and Y-Y must then be the "principal axes" of the pile group.

Conventionally, designers may like to use moments along defined axes instead of moments about defined axes. If we rename the axes and U-U and V-V after translation and rotation of the axes X-X and Y-Y such that the condition $\sum K_{iZ} u_i = \sum K_{iZ} v_i = \sum K_{iZ} u_i v_i = 0$ can be satisfied, then the pile load become

$$P_{iZ} = \frac{PK_{iZ}}{\sum K_{iZ}} + \frac{M_U K_{iZ}}{\sum K_{iZ} u_i^{\,2}} u_i + \frac{M_Y K_{iZ}}{\sum K_{iZ} v_i^{\,2}} v_i$$

If all piles are identical, i.e. all K_{iZ} are equal, then the formula is reduced

$P_{iZ} = \frac{P}{N} + \frac{M_U}{\sum u_i^{\,2}} u_i + \frac{M_V}{\sum v_i^{\,2}} v_i$ where N is the number of piles.

Or if we do not wish to rotate the axes to U and V, then only

$\sum K_{iZ} x_i = \sum K_{iZ} y_i = 0$ and the moment balancing equations becomes

$$M_X = -b_O \sum K_{iZ} y_i - b_1 \sum K_{iZ} x_i y_i - b_2 \sum K_{iZ} {y_i}^2$$

$$\Rightarrow M_X = -b_1 \sum K_{iZ} x_i y_i - b_2 \sum K_{iZ} {y_i}^2$$

and $\quad M_Y = b_O \sum K_{iZ} x_i + b_1 \sum K_{iZ} {x_i}^2 + b_2 \sum K_{iZ} x_i y_i$

$$\Rightarrow M_Y = b_1 \sum K_{iZ} {x_i}^2 + b_2 \sum K_{iZ} x_i y_i$$

Solving

$$P = b_O \sum K_{iZ} \Rightarrow b_O = \frac{P}{\sum K_{iZ}}$$

$$b_1 = \frac{M_X \sum K_{iZ} x_i y_i + M_Y \sum K_{iZ} {y_i}^2}{-\left(\sum K_{iZ} x_i y_i\right)^2 + \left(\sum K_{iZ} {x_i}^2 \sum K_{iZ} {y_i}^2\right)}$$

$$b_2 = \frac{-M_Y \sum K_{iZ} x_i y_i - M_X \sum K_{iZ} {x_i}^2}{-\left(\sum K_{iZ} x_i y_i\right)^2 + \left(\sum K_{iZ} {x_i}^2 \sum K_{iZ} {y_i}^2\right)}$$

So the pile load becomes

$$P_{iZ} = \frac{PK_{iZ}}{\sum K_{iZ}} + \frac{M_X \sum K_{iZ} x_i y_i + M_Y \sum K_{iZ} {y_i}^2}{-\left(\sum K_{iZ} x_i y_i\right)^2 + \left(\sum K_{iZ} {x_i}^2 \sum K_{iZ} {y_i}^2\right)} K_{iZ} x_i$$

$$+ \frac{-M_Y \sum K_{iZ} x_i y_i - M_X \sum K_{iZ} {x_i}^2}{-\left(\sum K_{iZ} x_i y_i\right)^2 + \left(\sum K_{iZ} {x_i}^2 \sum K_{iZ} {y_i}^2\right)} K_{iZ} y_i \,.$$

If all piles are identical, i.e. all K_{iZ} are equal, then the formula is reduced

$$P_{iZ} = \frac{P}{N} + \frac{M_X \sum x_i y_i + M_Y \sum {y_i}^2}{-\left(\sum x_i y_i\right)^2 + \left(\sum {x_i}^2 \sum {y_i}^2\right)} x_i + \frac{-M_Y \sum x_i y_i - M_X \sum {x_i}^2}{-\left(\sum x_i y_i\right)^2 + \left(\sum {x_i}^2 \sum {y_i}^2\right)} y_i$$

For a symmetrical layout where $\sum x_i y_i = 0$, the equation is further reduced to

$$P_{iZ} = \frac{P}{N} + \frac{M_Y}{\sum {x_i}^2} x_i + \frac{-M_X}{\sum {y_i}^2} y_i$$

Appendix J

Mathematical Simulation of Curves related to Shrinkage and Creep Determination

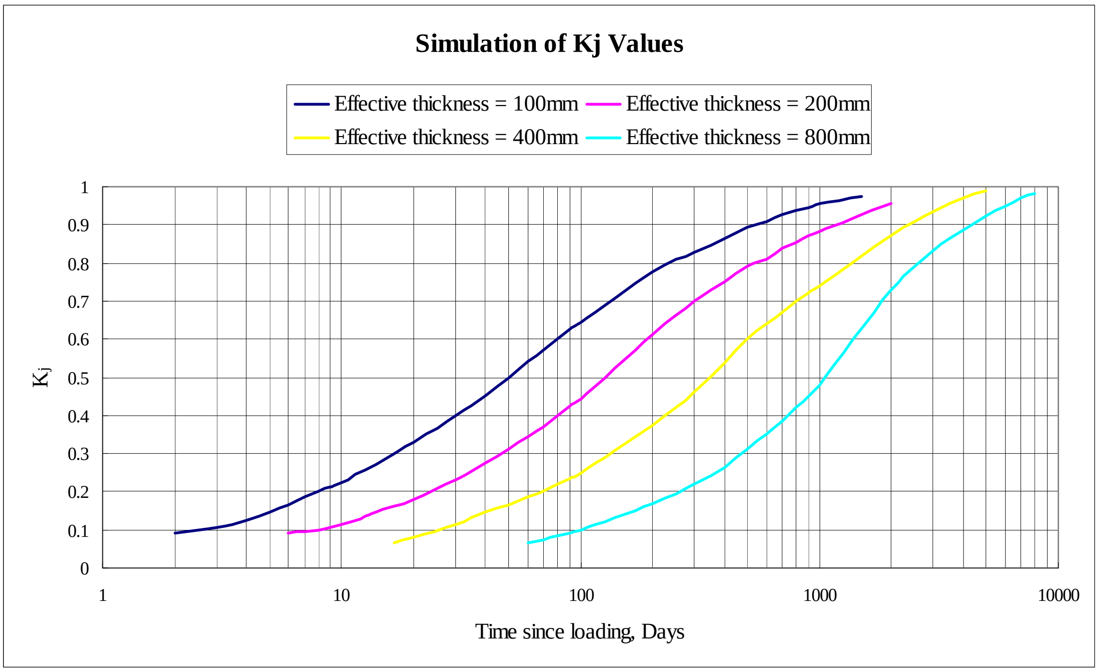
Simulation of Curves for Shrinkage and Creep Determination

Simulation of K_j values

Figure 3.5 of the Code is expanded and intermediate lines are added for reading more accurate values. The intermediate values are scaled off from the expanded figure and listed as follows (h_e = 50 mm which is seldom used is ignored) :

h_e = 100 mm		h_e = 200 mm		h_e = 400 mm		h_e = 800 mm	
Days	K_j	Days	K_j	Days	K_j	Days	K_j
2	0.09	6	0.09	16.6	0.065	60	0.065
3	0.108	7	0.095	20	0.08	70	0.075
4	0.125	8	0.1	30	0.115	80	0.084
5	0.145	9	0.105	40	0.145	90	0.092
6	0.165	10	0.112	50	0.165	100	0.099
7	0.185	11	0.12	60	0.185	200	0.17
8	0.2	12	0.13	70	0.2	300	0.22
9	0.213	13	0.138	80	0.22	400	0.265
10	0.225	14	0.145	90	0.235	500	0.31
20	0.33	20	0.18	100	0.25	600	0.35
30	0.4	30	0.23	200	0.375	700	0.386
40	0.45	40	0.275	300	0.46	800	0.42
50	0.5	50	0.31	400	0.54	900	0.45
60	0.543	60	0.345	500	0.6	1000	0.48
70	0.57	70	0.37	600	0.64	2000	0.73
80	0.6	80	0.4	700	0.67	3000	0.83
90	0.625	90	0.425	800	0.7	4000	0.888
100	0.645	100	0.445	900	0.72	5000	0.923
200	0.775	200	0.61	1000	0.74	6000	0.95
300	0.827	300	0.7	2000	0.87	7000	0.97
400	0.865	400	0.75	3000	0.935	8000	0.98
500	0.892	500	0.79	4000	0.97		
600	0.91	600	0.81	5000	0.99		
700	0.927	700	0.84				
800	0.937	800	0.855				
900	0.945	900	0.87				
1000	0.955	1000	0.883				
1500	0.975	2000	0.955				

Curves are plotted accordingly in Microsoft Excel as shown :



These curves are divided into parts and polynomial equations (x denote days) are simulated by regression done by the Excel as follows :

(i) Effectiveness thickness $h_e = 100\,\text{mm}$

for $2 \le x \le 10$

$K_j = -1.5740740764\times10^{-6}x^6 + 7.1089743699\times10^{-5}x^5 - 1.2348646738\times10^{-3}x^4 + 1.0396454943\times10^{-2}x^3 - 4.4218106746\times10^{-2}x^2 + 1.0785366750\times10^{-1}x - 1.4422222154\times10^{-2}$;

for $10 < x \le 100$

$K_j = -8.2638888726\times10^{-12}x^6 + 2.9424679436\times10^{-9}x^5 - 4.1646100361\times10^{-7}x^4 + 2.9995170408\times10^{-5}x^3 - 1.1964688098\times10^{-3}x^2 + 3.0905446162\times10^{-2}x + 9.3000049487\times10^{-3}$

for $100 < x \le 1000$

$K_j = -9.9999999553\times10^{-18}x^6 + 3.7871794729\times10^{-14}x^5 - 5.7487179303\times10^{-11}x^4 + 4.4829720169\times10^{-8}x^3 - 1.9268813492\times10^{-5}x^2 + 4.6787198128\times10^{-3}x + 3.3059999890\times10^{-1}$

(ii) Effectiveness thickness $h_e = 200\,\text{mm}$

for $1 \le x \le 10$

$K_j = -5.5555555584\times10^{-7}x^6 + 1.9230769236\times10^{-5}x^5 - 2.3632478631\times10^{-4}x^4 + 1.1888111887\times10^{-3}x^3 - 1.8372455154\times10^{-3}x^2 + 5.1966197721\times10^{-3}x + 5.0666667394\times10^{-2}$

for $10 < x \le 100$

$K_j = -6.0905886799\times10^{-12}x^6 + 2.0287559012\times10^{-9}x^5 - 2.6706836340\times10^{-7}x^4 +$

$1.7840233064E\times10^{-5}x^3 - 6.6454331705\times10^{-4}x^2 + 1.7736234727\times10^{-2}x - 1.3696178365\times10^{-2}$

for $100 < x \le 1000$

$K_j = -4.1666665317\times10^{-19}x^6 + 4.6185897038\times10^{-15}x^5 - 1.2899038408\times10^{-11}x^4 + 1.6179152071\times10^{-8}x^3 - 1.0631842073\times10^{-5}x^2 + 3.8848713316\times10^{-3}x + 1.4793333214\times10^{-1}$

(iii) Effectiveness thickness $h_e = 400\,\text{mm}$

for $1 \le x \le 16.6$

$K_j = 1.4187214466\times10^{-6}x^4 - 3.5464080361\times10^{-5}x^3 + 3.3384218737\times10^{-4}x^2 - 2.2688256448\times10^{-5}x + 2.7836053347\times10^{-2}$

for $16.6 < x \le 100$

$K_j = -1.5740740764\times10^{-6}x^6 + 7.1089743699\times10^{-5}x^5 - 1.2348646738\times10^{-3}x^4 + 1.0396454943\times10^{-6}x^3 - 4.4218106746\times10^{-2}x^2 + 1.0785366750\times10^{-1}x - 1.4422222154\times10^{-2}$

for $100 < x \le 1000$

$K_j = -9.3749999678\times10^{-18}x^6 + 3.1193910157\times10^{-4}x^5 - 4.0436698591\times10^{-11}x^4 + 2.6279902314\times10^{-8}x^3 - 9.8112164735\times10^{-6}x^2 + 2.8475810022\times10^{-3}x + 4.1166665811\times10^{-2}$

for $1000 < x \le 5000$

$K_j = -8.3333333334\times10^{-16}x^4 + 1.4166666667\times10^{-11}x^3 - 9.6666666667\times10^{-8}x^2 + 3.3333333333\times10^{-4}x + 4.9000000000\times10^{-1}$

(iv) Effectiveness thickness $h_e = 800\,\text{mm}$

for $3 \le x \le 60$

$K_j = 9.5889348301\times10^{-12}x^5 - 1.5604725262\times10^{-8}x^4 + 1.8715280898\times10^{-6}x^3 - 7.5635030550\times10^{-5}x^2 + 1.8805930655\times10^{-3}x + 1.4981311831\times10^{-2}$

for $60 < x \le 100$

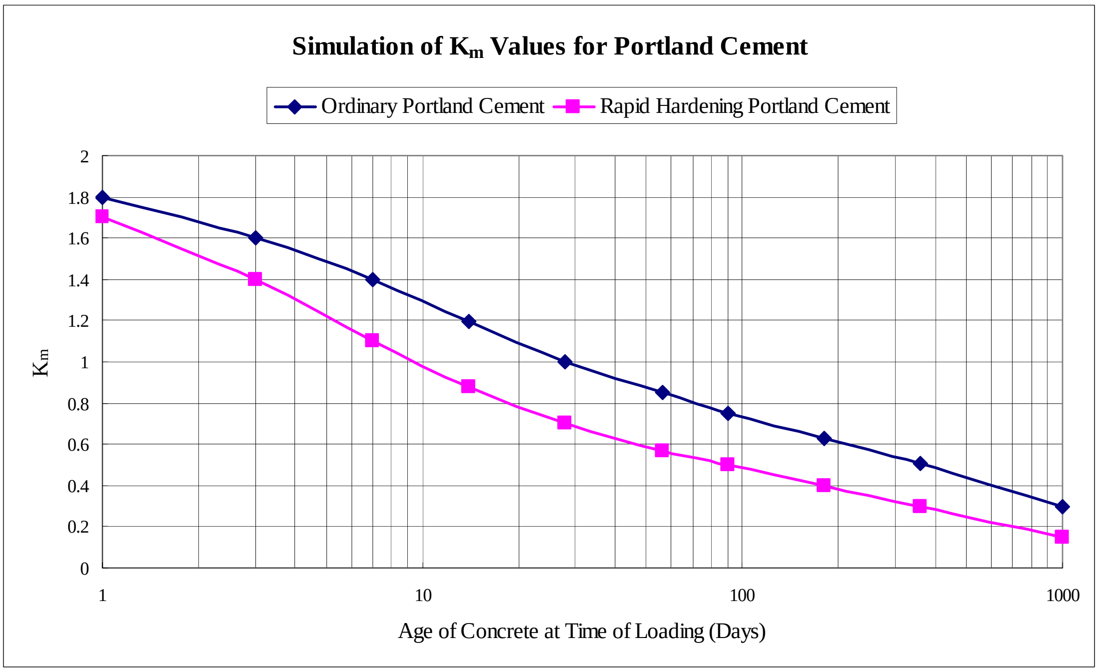
$K_j = -5.4210108624\times10^{-20}x^4 + 1.3010426070\times10^{-17}x^3 - 5.0000000012\times10^{-6}x^2 + 1.6500000000\times10^{-3}x - 1.6000000000\times10^{-2}$

for $100 < x \le 1000$

$K_j = -3.9583333158\times10^{-18}x^6 + 1.4818910202\times10^{-14}x^5 - 2.1967147366\times10^{-11}x^4 + 1.6383442558\times10^{-8}x^3 - 6.5899851301\times10^{-6}x^2 + 1.8249511657\times10^{-3}x - 3.1900000544\times10^{-2}$

Simulation of K_m values

Values of Figure 3.2 of the Code for Ordinary Portland Cement are read, Excel chart is plotted and polynomial equations are simulated as :



for $1 \le x \le 7$

$K_m = 8.3333333333 \times 10^{-3}x^2 - 1.3333333333 \times 10^{-1}x + 1.925$

for $7 < x \le 28$

$K_m = 7.3129251701 \times 10^{-4}x^2 - 4.4642857143 \times 10^{-2}x + 1.6766666667$

for $28 < x \le 90$

$K_m = 3.8967199783 \times 10^{-5}x^2 - 8.6303876389 \times 10^{-3}x + 1.2111005693$

for $90 < x \le 360$

$K_m = 2.3662551440 \times 10^{-6}x^2 - 1.9722222222 \times 10^{-3}x + 9.0833333333 \times 10^{-1}$